FRM Part I Exam

By AnalystPrep

Questions with Answers - Valuation and Risk Models

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Table of Contents

Reading 47: Measures of Financial Risk

Q.939 Billy Marquette has recently joined a small company that provides private commercial jets to royal families, government officials, and directors of big firms. Marquette is a retired commercial pilot with a very basic understanding of finance. On his first day, he is handed a report on risk management measures. The excerpt from the report says "due to volatility in oil prices, the company has a weekly 90% VaR of €20,000". Which of the following is the most appropriate explanation of the excerpt?

A. There is a 90% probability that the company will experience a loss of €2,000 on a weekly basis.

B. There is a 10% probability that the company will experience a loss of €20,000 in any given week.

C. There is a 90% probability, in any given week, that the company will experience a loss of more than €20,000.

D. There is a 10% probability, in any given week, that the company will experience a loss in excess of €20,000.

The correct answer is **D.**

Value at Risk (VaR) is a statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific time frame. This metric is most commonly used by investment and commercial banks to determine the extent and occurrence ratio of potential losses in their institutional portfolios. Risk managers use VaR to measure and control the level of risk exposure. One can look at VaR as providing an estimate of the potential loss that could be made on an investment portfolio over a certain period of time, given normal market conditions and a certain level of confidence. In this case, a 90% VaR of €20,000 means that there is a 10% chance that the company will experience a loss exceeding €20,000 in any given week. This is because the 90% confidence level implies that losses will not exceed the VaR estimate 90% of the time. Therefore, the remaining 10% represents the probability that losses could exceed this amount, which is exactly what choice D states.

Choice A is incorrect. The statement does not imply a 90% probability of a weekly loss of €2,000. The VaR figure given is €20,000, not €2,000.

Choice B is incorrect. While it correctly identifies the 10% probability associated with VaR, it incorrectly states that this represents the likelihood of a loss exactly equal to €20,000 in any given week. In reality, VaR represents the minimum loss that could occur with a certain probability.

Choice C is incorrect. This choice incorrectly interprets the 90% confidence level as implying a 90% chance of losses exceeding €20,000 in any given week. In fact, there's only a 10% chance (100%-90%) that losses will exceed this amount.

Things to Remember

- Value at Risk (VaR) is a measure of the potential loss that could occur on an investment portfolio over a specific time frame, given normal market conditions and a certain level of confidence.
- VaR is commonly used by banks and financial institutions to assess and manage risk exposure.
- A 90% VaR of €20,000 means that there is a 10% chance that losses will exceed €20,000 in any given week.
- VaR provides an estimate of the minimum loss that could occur with a certain probability, not an exact loss amount.
- Risk managers use VaR to control and monitor the level of risk within a firm or investment portfolio.

Q.3328 An investment company has a portfolio which has the following ordered performance by historical data. Calculate the expected shortfall $ES_{0.95}$.

Probabilty	1%	5%	10%	12%	15%
Profit/Loss	−500	−300	−100	−90	−50

A. 300

B. 340

C. 400

D. 425

The correct answer is **B.**

Given a discrete distribution, the ES is the equivalent of:

$$ES_{\alpha} = \frac{1}{1-\alpha}\sum_{p=0}^{1-\alpha}[p^{th}\text{ highest loss}] \times [\text{probability of } p^{th}\text{ highest loss}]$$

At $\alpha = 0.95$,

$$ES_{0.95} = \frac{[(0.01 \times 500) + (0.04 \times 300)]}{0.05} = 340$$

Note: The sum of probabilities in the numerator must sum to $(1-\alpha)$

Alternative Approach

To calculate the expected shortfall, we must ask ourselves, "If we are in the worst 5% of the loss distribution, what is the expected loss?" The first column of the given table makes it clear that the 5% tail of the distribution is composed of a 1% probability that the loss is 500 and a 4% probability that the loss is 300. Conditional on being in the tail of the distribution, there is, therefore, a 1/5 chance that the loss is 500 million and a 4/5 chance that it is 300. The expected shortfall (in millions of dollars) is, therefore:

$$\left(\frac{1}{5}\right) \times 500 + \left(\frac{4}{5}\right) \times 300 = 340$$

Q.3329 An investment company has a portfolio which has the following ordered performance by historical data. Calculate the expected shortfall, $ES_{0.99}$, i.e., at 99% level of confidence

Probabilty	1%	5%	10%	12%	15%
Profit/Loss	−500	−300	−100	−90	−50

A. 168

B. 400

C. 460

D. 500

The correct answer is **D.**

There is only one number 500 beyond 1%, therefore, the average is 500.

Further Explanation

The expected shortfall (also called conditional VaR) is the expected tail loss. It is the average of the worst 100*(1-a)% of losses. For a discrete distribution, ES is derived as:

$$ES_{\alpha} = \frac{1}{1-\alpha}\sum_{p=0}^{\alpha}(p^{th} \text{ loss} \times \text{probability of pth loss})$$

In words, to determine the expected shortfall at a level of confidence a, we must find the average of all the outcomes whose probability is less than or equal to 1 − a.

At a 99% confidence level, the significance level is 1%. To establish the expected shortfall at 1%, we must find the average of all the outcomes whose probability is less than or equal to 1%. in this case,

$$\text{Expected shortfall} = \frac{(0.01 * 500)}{0.01} = 500$$

Q.3397 A hypothetical portfolio of securities exhibits the following expected losses shown:

Name	Loss (million dollar)	Probability (%)
1	10	40%
2	20	35%
3	50	15%
4	100	5%
5	200	2.5%
6	225	2%
7	250	0.5%

Calculate the expected shortfall at the 95% and 99% confidence level?

A. ES (95%) = $225 million; ES (99%) = $237.5 million

B. ES (95%) = $215 million; ES (99%) = $237.5 million

C. ES (95%) = $217.5 million; ES (99%) = $250 million

D. ES (95%) = $225 million; ES (99%) = $250 million

The correct answer is **B.**

Note that the given data are in expected losses. The expected shortfall (also called conditional VaR) is the expected tail loss. It is the average of the worst 100*(1-α)% of losses. For discrete distribution, ES is derived as:

$$ES_{\alpha} = \frac{1}{1-\alpha}\sum_{p=0}^{\alpha}(\text{pth loss} \times \text{probability of pth loss})$$

In other words, to determine the expected shortfall at a level of confidence α, we must find the average of all the outcomes whose probability is less than or equal to 1 - α.

At a 95% confidence level, the significance level is 5%. To establish the expected shortfall at 5%, we must find the average of all the outcomes whose probability is less than or equal to 5%

$$ES_{0.95} = \frac{200 \times 2.5\% + 225 \times 2\% + 250 \times 0.5\%}{5\%} = 215 \text{ million dollars}$$

At a 99% confidence level, the significance level is 1%. To establish the expected shortfall at 1%, we must find the average of all the outcomes whose probability is less than or equal to 1%

$$ES_{0.99} = \frac{225 \times 0.5\% + 250 \times 0.5\%}{1\%} = 237.5 \text{ million dollars}$$

Q.3398 The VaR of a loan portfolio is computed at various confidence levels:

Confidence Level	VaR
95.0%	2%
95.5%	5%
96.0%	6%
96.5%	7%
97.0%	9%
97.5%	10%
98.0%	13%
98.5%	15%
99.0%	20%
99.5%	30%

What is the expected shortfall at the 97.5% confidence level?

A. 0.1

B. 0.15

C. 0.195

D. 0.2

The correct answer is **C.**

The expected shortfall at the 97.5% confidence level is computed by averaging all value of risk greater than the 97.5% confidence level.
Expected shortfall at the 97.5% confidence level is therefore,

$$\frac{13\% + 15\% + 20\% + 30\%}{4} = 19.5\%$$

Q.4639 A hypothetical portfolio has an annual 1% VaR of $45,000. Which of the following statements is the **most likely** correct about the portfolio?

A. The loss over the next year is expected to be at most $45,000 in 1% of the cases.

B. There is only a 1% chance that we will gain more than $45,000 over the next year.

C. The likelihood of losing more than $45,000 over the next year is 1%.

D. The likelihood of losing no more than $45,000 over the next year is 1%.

The correct answer is **C.**

The Value at Risk (VaR) metric is a commonly used risk management tool that quantifies the potential loss that could happen in an investment portfolio over a specified period of time at a given confidence level. In this case, an annual 1% VaR of $45,000 implies that there is a 1% chance that the portfolio could experience a loss exceeding $45,000 over the next year. This is based on the statistical analysis of historical market trends and volatilities. The VaR metric is often used by financial institutions and investment firms to measure the extent of potential losses that could be incurred in extreme market conditions. Therefore, the statement 'The likelihood of losing more than $45,000 over the next year is 1%' accurately reflects the interpretation of the given VaR value.

Choice A is incorrect. The statement is misleading as it suggests that the maximum loss in 1% of the cases will be $45,000. However, VaR does not provide a maximum loss figure but rather a minimum loss figure at a certain confidence level.

Choice B is incorrect. This choice incorrectly interprets VaR as a measure of potential gains rather than potential losses. VaR measures the risk of potential losses, not gains.

Choice D is incorrect. This statement misinterprets VaR by suggesting that there's only 1% chance of losing no more than $45,000 over the next year which contradicts with the concept of VaR which states that there's 1% chance we could lose more than $45,000 over next year.

Things to Remember

- Value at Risk (VaR) is a statistical measure used to quantify the level of financial risk within a portfolio.
- VaR provides an estimate of the maximum potential loss that a portfolio may experience over a specified time horizon at a given confidence level.
- The confidence level in VaR represents the probability that the actual loss will not exceed the VaR estimate.
- VaR is widely used in risk management to assess and manage the potential downside risk of investment portfolios.
- Historical simulation, Monte Carlo simulation, and parametric methods are common approaches to calculate VaR.

Q.4640 The investment returns and the corresponding probabilities are given in the following table:

Returns	Probability
20%	0.1
30%	0.3
−10%	0.2
15%	0.3
7%	0.1

What is the standard deviation of the investment returns?

A. 0.142

B. 0.154

C. 0.132

D. 0.138

The correct answer is **A.**

The variance of the return R is given by:

$$Var(R) = E(R^2) - [E(R)]^2$$

Now,

$$E(R^2) = 0.1 \times (20\%)^2 + 0.3 \times (30\%)^2 + 0.2 \times (-10\%)^2 + 0.3 \times (15\%)^2 + 0.1 \times (7\%)^2 = 0.04024$$
$$E(R) = 0.1 \times 20\% + 0.3 \times 30\% + 0.2 \times -10\% + 0.3 \times 15\% + 0.1 \times 7\% = 0.142$$
$$\therefore Var(R) = 0.04024 - (0.142)^2 = 0.020076$$

Thus the standard deviation is given by:

$$\sqrt{0.020076} = 0.1417 = 14.17\%$$

Q.4641 An investor invests his funds in two correlated assets, A and B. The standard deviation of asset A is 20%, and that of B is 15%. The portfolio variance is 2.84%. Given that the investor has three times as much money in asset A than he has in asset B, what is the correlation coefficient between assets A and B?

A. 0.0962

B. 0.2133

C. 0.3994

D. 0.8078

The correct answer is **C.**

The variance of a portfolio is given by:

$$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2\rho w_A w_B \sigma_A \sigma_B$$

Where

W_A: the weight of asset A

W_B: the weight of asset B

σ_A: standard deviation of asset A

σ_B: standard deviation of asset B

ρ: correlation coefficient between asset A and B

Now let the amount invested in asset be B be w_B and thus:

$$w_A = 3w_B$$
$$w_B + 3w_B = 1$$
$$\therefore w_B = \frac{1}{4} \Rightarrow w_A = \frac{3}{4}$$

Now,

$$w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2\rho w_A w_B \sigma_A \sigma_B = 0.0284$$
$$\left(\frac{3}{4}\right)^2 \times 0.2^2 + \left(\frac{1}{4}\right)^2 \times 0.15^2 + 2 \times \frac{3}{4} \times \frac{1}{4} \times 0.2 \times 0.15 \times \rho = 0.0284$$
$$\Rightarrow \rho = \frac{0.0284 - \left(\frac{3}{4}\right)^2 \times 0.2^2 - \left(\frac{1}{4}\right)^2 \times 0.15^2}{2 \times 0.15 \times 0.2 \times \frac{1}{4} \times \frac{3}{4}} = 0.39944$$

Q.4643 The losses from a portfolio for one year are normally distributed with mean -10 and standard deviation 20. What is the value of the 99% expected shortfall?

A. 52.85

B. 37.4

C. 42.85

D. 26.43

The correct answer is **C.**

We start with the formula for the expected shortfall (ES) at a certain confidence level:

$$ES = \mu + \sigma \left(\frac{e^{-\frac{U^2}{2}}}{(1 - X)\sqrt{2\pi}} \right)$$

Here, μ is the mean of the distribution, σ is the standard deviation, U is the z-score that corresponds to our desired confidence level X.

For a confidence level of 99%, we find the z-score by inverting the cumulative distribution function (CDF), Φ^{-1}, of the standard normal distribution:

$$U = \Phi^{-1}(0.99) = 2.33$$

This z-score represents the point at which 99% of the distribution's values fall below it.

With our z-score U and the known values for the mean $\mu = -10$ and standard deviation $\sigma = 20$, we can now plug these into the ES formula:

$$ES = -10 + 20 \left(\frac{e^{-\frac{2.33^2}{2}}}{(1 - 0.99)\sqrt{2\pi}} \right) = 42.85$$

This means that, at the 99% confidence level, the expected shortfall of the portfolio—the average loss in the worst 1% of cases—is approximately $42.85 million.

Q.6218 An investor is evaluating two portfolios to determine which lies on the efficient frontier. For each point on the efficient frontier, at least one portfolio can be constructed from all available investments with the expected risk and return corresponding to that point. How does the inclusion of a risk-free asset impact the efficient frontier?

A. The efficient frontier becomes a straight line, representing a linear relationship between expected return and standard deviation.

B. The efficient frontier bends upwards, with higher risk for each level of expected return.

C. The efficient frontier remains a curve but shifts upwards, providing higher returns for each level of risk.

D. The efficient frontier shifts downwards, providing lower returns for each level of risk.

The correct answer is **A.**

By including a risk-free asset, the efficient frontier becomes a straight line because it creates a linear relationship between the expected return and standard deviation. This is due to the ability to mix the risk-free asset with a risky market portfolio, offering the highest Sharpe ratio.

B is incorrect because the efficient frontier does not bend upwards; instead, it becomes a straight line when a risk-free asset is available. This change allows for leverage, which increases the slope without changing the curvature.

C is incorrect because the efficient frontier does not remain a curve. The addition of risk-free assets transforms the efficient frontier into a straight line.

D is incorrect because the efficient frontier does not shift downwards. A downward shift would imply lower returns for each level of risk, which is contrary to the inclusion of a risk-free asset that enhances investment opportunities.

Things to Remember

- The introduction of a risk-free asset allows for the construction of a Capital Market Line (CML), which is a straight line on the risk-return graph, extending from the risk-free rate to the tangent point on the efficient frontier.

- The CML represents portfolios that combine the risk-free asset with the market portfolio, resulting in a linear risk-return trade-off.

- The slope of the CML is the Sharpe ratio, which measures the excess return per unit of risk.

- The inclusion of a risk-free asset enables investors to achieve any desired combination of risk and return along the CML by adjusting their proportion of investment in the risk-free asset and the market portfolio.

Q.6219 Financial analysts often assume that returns of risky financial assets such as equities follow a normal distribution. However, empirical evidence shows that actual returns exhibit characteristics different from the normal distribution. Which of the following is a significant difference between the normal distribution and the actual return distribution of equities?

A. The returns of equities usually exhibit lighter tails compared to a normal distribution.

B. The returns of equities often exhibit lower kurtosis compared to a normal distribution.

C. The returns of equities generally display skewness and heavier tails than those of a normal distribution.

D. The returns of equities usually show fewer extreme values compared to what is predicted by a normal distribution.

The correct answer is **C.**

Empirical evidence shows that the returns of equities do not follow a normal distribution because they generally display skewness and heavier tails, which means a higher probability of extreme values compared to a normal distribution. This phenomenon is often referred to as "fat tails."

A is incorrect because equities exhibit heavier, not lighter, tails compared to a normal distribution. This implies a higher likelihood of extreme losses or gains.

B is incorrect because equity returns typically exhibit higher kurtosis, meaning they have more pronounced peaks and heavier tails than the normal distribution. Lower kurtosis would suggest less extreme values and a more flat-topped distribution.

D is incorrect because equities show more, not fewer, extreme values than predicted by a normal distribution, which is indicative of the heavy-tailed nature of actual return distributions.

Things to Remember

- Skewness refers to the asymmetry of the return distribution. Positive skewness indicates a distribution with a longer right tail, while negative skewness indicates a longer left tail.

- Kurtosis measures the "tailedness" of the distribution. Higher kurtosis means more frequent extreme returns (fat tails) compared to the normal distribution.

- Equity returns often exhibit **leptokurtosis**, characterized by a high peak and fat tails, leading to more frequent extreme outcomes than predicted by the normal distribution.

- Assuming that financial returns are normally distributed can lead to underestimating the probability of distributive tail events, which can result in inadequate risk management strategies.

Q.6220 A financial risk manager is using the VaR measure to evaluate the risk of a portfolio over a specific time period and confidence level. VaR is commonly used due to its simplicity and the ease of understanding the associated risk under normal market conditions. However, what are the major limitations of VaR despite its widespread use?

A. VaR can be subadditive, which violates one of the desirable properties of a coherent risk measure.

B. VaR does not capture the potential losses in the tail beyond the confidence level.

C. VaR assumes normal distribution of returns, which may not accurately reflect the actual distribution of asset returns.

D. VaR does not consider the correlations between different assets during periods of market stress.

The correct answer is **B.**

VaR does not account for or capture the potential losses beyond the specified confidence level. This can be crucial as significant losses beyond the VaR threshold can still occur, which VaR doesn't predict, thus leaving a part of the risk unaddressed.

A is incorrect because VaR can sometimes be non-subadditive, meaning the VaR of a combined portfolio can be greater than the sum of the VaRs of its individual components. This violates the subadditivity property of coherent risk measures. Therefore, the claim that VaR can be subadditive is incorrect; rather, the correct limitation is that it can be non-subadditive.

C is incorrect because VaR can be calculated using various methods, including historical simulation, which does not assume normal distribution. The assumption of normally distributed returns is a limitation specific to the parametric (delta-normal) approach, not VaR as a whole.

D is incorrect because VaR does account for correlations between different assets when calculated using the historical simulation or Monte Carlo simulation methods. These methods use actual or simulated return distributions that inherently include asset correlations.

Things to Remember

- VaR is a measure that estimates the maximum potential loss of a portfolio at a given confidence level over a specific time period.
- VaR is affected by arbitrary parameters, like the confidence level and holding period, which can both influence its accuracy.
- VaR can give a false sense of security as it assumes normal market conditions and may fail to predict extreme market events and crises, where correlations between assets can change drastically.
- Alternative risk measures, such as Conditional VaR (CVaR), address some of VaR's limitations by estimating the expected loss beyond the VaR threshold, providing a fuller picture of tail risk.

Q.6221 Which of the following statements correctly describes the property of subadditivity in the context of a coherent risk measure?

A. A risk measure is subadditive if adding a risk-free asset to a portfolio does not change its risk measure.

B. A risk measure is subadditive if the risk of a combined portfolio does not exceed the sum of the risks of the individual portfolios.

C. A risk measure is subadditive if increasing the size of a portfolio proportionally increases its risk measure.

D. A risk measure is subadditive if the risk of a portfolio decreases linearly with additional diversification.

The correct answer is **B.**

A coherent risk measure must exhibit subadditivity, meaning that the risk measure of a combined portfolio should not exceed the sum of the risk measures of its individual components. This property encourages diversification because merging two portfolios should not increase the overall risk more than the sum of the individual risks.

A is incorrect because subadditivity does not relate to adding a risk-free asset; that property is more aligned with translation invariance.

C is incorrect because subadditivity is not about proportional increases in risk measure; this describes homogeneity.

D is incorrect because the property of subadditivity does not specify that risk decreases linearly with diversification; it simply states that the combined risk should not exceed the sum of individual risks.

Things to Remember

- The subadditivity property captures the benefits of diversification in risk management. Combined portfolios should have a risk measure that reflects these benefits.
- Subadditivity is crucial for risk managers when evaluating the cumulative risk of different asset classes to benefit from diversification effects.

Q.6222 Value at Risk (VaR) is a widely used risk management tool that estimates the potential loss in value of a portfolio over a defined period for a given confidence interval. Despite its popularity, there are significant limitations to VaR that question its effectiveness as a comprehensive risk measure. Why is VaR not considered a coherent risk measure?

A. VaR fails to account for the current size of the portfolio.

B. VaR fails the monotonicity property of coherent risk measures.

C. VaR does not account for the skewness and kurtosis of return distributions.

D. VaR fails the subadditivity test required for coherent risk measures.

The correct answer is **D.**

VaR is not considered a coherent risk measure because it fails the subadditivity test. Subadditivity would require that the risk measure of a combined portfolio should not exceed the sum of the risk measures of the individual portfolios, which VaR does not always satisfy. This failure undermines the principle of diversification in risk assessment.

A is incorrect because VaR does account for the size of the portfolio, as it measures potential losses in monetary terms based on given parameters.

B is incorrect because VaR generally satisfies monotonicity; if one portfolio is riskier than another, its VaR will generally be higher.

C is incorrect because VaR can be calculated using methods that take into account the skewness and kurtosis of return distributions, such as historical simulation or Monte Carlo simulation. These methods use actual or simulated return distributions, which include skewness and kurtosis.

Things to Remember

- A coherent risk measure satisfies four properties: subadditivity, translation invariance, positive homogeneity, and monotonicity.
- Risk managers often complement VaR with measures such as ES, which is a coherent risk measure, to capture the tail risk better.

Q.6685 A portfolio manager operates within the mean-variance framework, having constructed an efficient frontier using only risky assets. Upon introducing a risk-free asset available for both borrowing and lending at a single rate, the investment opportunity set is fundamentally reshaped. The manager must now advise a diverse client base, ranging from extremely risk-averse to highly risk-seeking investors. How does the introduction of this risk-free asset alter the composition of the optimal portfolio of risky assets for all investors, regardless of their individual risk tolerance?

A. All investors hold a unique portfolio of risky assets tangent to the new frontier.

B. All investors hold the same portfolio of risky assets tangent to the new frontier.

C. Risk-averse investors combine the risk-free asset with a unique risky portfolio.

D. Risk-seeking investors leverage a unique risky portfolio using the risk-free rate.

The correct answer is **B.**

When we introduce a risk-free asset that allows for both borrowing and lending, we create a new, linear efficient frontier. This new line is tangent to the original curved frontier of risky assets, and that single point of tangency represents what we call the 'market portfolio'—the one optimal portfolio of risky assets. The critical insight here is that every investor, no matter their tolerance for risk, should hold this same market portfolio. They then tailor their overall risk exposure by allocating funds between this market portfolio and the risk-free asset. A conservative investor will lend at the risk-free rate (buy the risk-free asset), while an aggressive investor will borrow at that rate to leverage their position in the market portfolio. The composition of the risky part of their holdings remains identical.

A is incorrect because the theory posits that all investors will hold the same portfolio of risky assets (the market portfolio), not a unique one. The uniqueness comes from how they combine this single risky portfolio with the risk-free asset to match their risk tolerance.

C is incorrect. While risk-averse investors will indeed combine the risk-free asset with the risky portfolio, the risky portfolio they use is not unique to them. It is the same market portfolio that all other investors hold.

D is incorrect. Similarly, while risk-seeking investors may leverage the risky portfolio, the underlying risky portfolio itself is not unique. It is the same market portfolio that is optimal for all investors in this framework.

Things to Remember

- This concept is a cornerstone of the **Capital Asset Pricing Model (CAPM)**, which builds directly on the mean-variance framework and the efficient frontier. The line extending from the risk-free rate through the market portfolio is known as the Capital Market Line (CML).

- This conclusion that all investors hold the same market portfolio relies on several strong assumptions, including that all investors have the same expectations about returns, standard deviations, and correlations, and can borrow and lend at the same risk-free rate.

- In practice, investors have different views and constraints, leading them to hold different risky portfolios. However, the model provides a powerful theoretical benchmark for portfolio construction.
- The "market portfolio" theoretically includes all available risky assets in the market, weighted by their market capitalization.

Q.6686 A risk analyst at a large investment bank is reviewing the daily returns of a major equity index over the last 20 years. When comparing the empirical distribution of these returns to a normal distribution with the same mean and standard deviation, she observes that the actual distribution is more peaked around the mean and has a significantly higher frequency of extreme movements (both positive and negative) than the normal distribution would predict. Which statement correctly characterizes the relationship between the observed distribution of the index returns and the theoretical normal distribution, based on these common findings?

A. Both small changes and large changes occur more frequently than the normal distribution predicts.

B. Both small changes and large changes occur less frequently than the normal distribution predicts.

C. Small changes occur more frequently, while large changes occur less frequently than predicted.

D. Small changes occur less frequently, while large changes occur more frequently than predicted.

The correct answer is **A.**

The typical distribution of financial asset returns is not normal; it is leptokurtic, meaning it is more peaked around the mean and has "fatter tails". The peakedness means that small changes (returns close to the mean) happen more often than a normal distribution would suggest. The fatter tails mean that large changes or extreme events (both gains and losses) also happen much more often than predicted by the normal distribution. Consequently, intermediate-sized moves happen less often.

B is incorrect. This is the opposite of what is observed. Both small and large changes are more frequent in reality.

C is incorrect. While it correctly identifies that small changes are more frequent, it incorrectly states that large changes are less frequent. The "fat tails" phenomenon means large changes are significantly more common than predicted by a normal distribution.

D is incorrect. While it correctly identifies that large changes are more frequent, it incorrectly states that small changes are less frequent. The higher peak of the actual distribution indicates a higher probability of returns near the mean.

Things to Remember

- This characteristic of financial returns is crucial for risk management because models assuming a normal distribution will systematically underestimate the probability of extreme, high-impact events. This was a contributing factor in the 2008 financial crisis, where many VaR models based on normal assumptions failed to anticipate the severity of losses.
- The term for this shape is **leptokurtosis**. A distribution with positive excess kurtosis (kurtosis greater than 3) is leptokurtic.
- Because of fatter tails, risk measures that specifically focus on the tail of the distribution, like Value-at-Risk (VaR) and Expected Shortfall (ES), were developed to provide a better assessment of potential losses than standard deviation alone.

Q.6687 A firm's risk manager is analyzing two portfolios, A and B, composed of different assets. Both portfolios are found to have a one-day 97.5% VaR of $5 million. A stress test reveals that for Portfolio A, losses in the worst 2.5% of scenarios are uniformly distributed between $5 million and $7 million. For Portfolio B, losses in the worst 2.5% of scenarios are uniformly distributed between $5 million and $15 million. The manager decides to calculate Expected Shortfall (ES) to better distinguish the risk profiles. Given this information, how would the one-day 97.5% Expected Shortfall (ES) of the two portfolios compare?

A. The ES for Portfolio A would be significantly lower than the ES for Portfolio B.

B. The ES for Portfolio A would be significantly higher than the ES for Portfolio B.

C. The ES for both portfolios would be equal to their VaR of $5 million.

D. The ES for both portfolios would be equal and greater than their VaR.

The correct answer is **A.**

Expected Shortfall (ES), also known as Conditional VaR, is defined as the expected loss, conditional on the loss being greater than the VaR level. It specifically addresses VaR's limitation by quantifying the average magnitude of tail losses. For Portfolio A, the losses beyond VaR are between $5M and $7M, so the average (expected) loss in that tail will be $6M. For Portfolio B, the losses beyond VaR are between $5M and $15M, so the average loss in that tail will be $10M.

Since ES measures this expected tail loss, the ES for Portfolio A ($6M) would be significantly lower than the ES for Portfolio B ($10M), correctly identifying Portfolio B as the riskier one despite having the same VaR.

B is incorrect. Portfolio B has a wider and more severe range of losses in its tail, which will result in a higher, not lower, Expected Shortfall.

C is incorrect. By definition, Expected Shortfall must be greater than VaR, as it is the average of all losses in the tail which begins at the VaR level. It could only be equal in a theoretical case where all tail losses are exactly equal to the VaR amount.

D is incorrect. While the ES for both portfolios will be greater than their VaR, they will not be equal. The difference in their tail loss distributions means their expected shortfalls will be different, reflecting their different levels of tail risk.

Things to Remember

- ES provides a more complete picture of tail risk than VaR. Because of its superior theoretical properties (coherence), regulators are increasingly favoring it. The Fundamental Review of the Trading Book (FRTB) rules, for instance, replace VaR with ES for internal models-based approaches.
- While VaR corresponds to a single point (a percentile) on the loss distribution, ES is an average over a range (the entire tail beyond the VaR point).
- This makes ES sensitive to the severity of extreme losses, a desirable property for a risk measure. A single, extremely large potential loss in the tail will significantly increase ES but might not affect VaR at all if it's beyond the specified percentile.

Q.6688 A team of quantitative analysts is developing a new internal risk measure and must ensure it is coherent. They are testing the property of homogeneity. They begin with a base portfolio and calculate its risk measure. They then create a second portfolio by exactly doubling the position size of every asset in the base portfolio, with no change in market conditions or liquidity. According to the homogeneity property of a coherent risk measure, what should be the relationship between the risk measure of the new, larger portfolio and the original portfolio?

A. The new risk measure should be exactly double the original risk measure.

B. The new risk measure should be less than double the original risk measure.

C. The new risk measure should be equal to the original risk measure plus a constant.

D. The new risk measure should be equal to the original risk measure squared.

The correct answer is **A.**

The homogeneity property of a coherent risk measure states that changing the size of a portfolio by multiplying the amounts of all its components by a factor λ should result in the risk measure being multiplied by the same factor λ. In this scenario, the portfolio size is doubled (λ=2). Therefore, a coherent risk measure must also double. This property reflects the intuitive idea that if you double your bets, you double your risk, assuming no other changes like market liquidity impact.

B is incorrect. This would imply diversification benefits from simply scaling up a portfolio, which contradicts the homogeneity axiom. This might happen in reality due to non-linearities, but it violates the theoretical property.

C is incorrect. This describes the property of translation invariance, which deals with adding a cash amount to a portfolio, not scaling the entire portfolio.

D is incorrect. This suggests a non-linear, exponential relationship between portfolio size and risk, which is inconsistent with the linear scaling required by the homogeneity axiom.

Things to Remember

- The four properties of a coherent risk measure are: **monotonicity, translation invariance, homogeneity**, and **subadditivity**.
- **Homogeneity** can be formally expressed as: $\text{Risk}(\lambda P) = \lambda \times \text{Risk}(P)$ for any $\lambda > 0$, where P is a portfolio.
- There's an important practical caveat: this property may not hold for very large scaling factors (λ). A portfolio that is 100 times larger may be more than 100 times as risky because of liquidity constraints; it becomes much harder to unwind large positions without adversely affecting market prices.

Q.6689 A bank's risk department analyzes two distinct, independent portfolios, A and B. For a 99% confidence level, the VaR of Portfolio A is $2 million and the VaR of Portfolio B is $3 million. The sum of the individual VaRs is $5 million. After merging the two portfolios, the department calculates the 99% VaR of the combined portfolio $(A + B)$ and finds it to be $6 million. Which property of a coherent risk measure has been violated by Value-at-Risk (VaR) in this scenario?

A. The subadditivity property.

B. The monotonicity property.

C. The translation invariance property.

D. The homogeneity property.

The correct answer is **A.**

The subadditivity property stipulates that for any two portfolios, A and B, the risk measure for the merged portfolio should be no greater than the sum of the risk measures for the individual portfolios. This can be written as $\text{Risk}(A + B) \leq \text{Risk}(A) + \text{Risk}(B)$. This property mathematically reflects the benefits of diversification. In this scenario, the VaR of the combined portfolio ($6 million) is greater than the sum of the individual VaRs ($2 million + $3 million = $5 million). This is a direct violation of the subadditivity axiom, which is a primary reason VaR is not considered a coherent risk measure.

B is incorrect. The monotonicity property states that if a portfolio consistently produces worse outcomes than another, it should have a higher risk measure. This scenario does not provide information to assess monotonicity.

C is incorrect. Translation invariance relates to the effect of adding cash to a portfolio, which is not what occurs in this scenario.

D is incorrect. The homogeneity property relates to scaling the size of a single portfolio by a constant factor, not merging two different portfolios.

Things to Remember

- VaR's failure to be subadditive is a significant flaw. It implies that, under certain circumstances, breaking a firm into separate units could appear to reduce total risk (as measured by the sum of VaRs), discouraging sensible risk aggregation and diversification.
- This violation typically occurs with portfolios that have skewed, "fat-tailed" distributions, such as those containing short option positions or high-yield bonds, where extreme losses are possible but infrequent.
- In contrast, **Expected Shortfall (ES)** is a coherent risk measure because it always satisfies the subadditivity property, ensuring that the measured risk of a merged portfolio never exceeds the sum of the individual risks.

Q.6690 An asset manager is building an efficient frontier based on the mean-variance framework. The initial analysis includes a universe of 50 large-cap domestic stocks. The manager then decides to expand the investment universe to include 50 international stocks and 20 real estate investment trusts (REITs). The new assets exhibit different expected returns and volatilities, and crucially, their returns are imperfectly correlated with the initial set of domestic stocks. What is the most likely effect on the efficient frontier of risky assets when these additional, imperfectly correlated investments are included in the optimization?

A. The new efficient frontier will offer superior risk-return combinations.

B. The new efficient frontier will offer inferior risk-return combinations.

C. The new efficient frontier will offer identical risk-return combinations.

D. The new efficient frontier will offer higher returns only for the same level of risk.

The correct answer is **A.**

The efficient frontier represents the set of portfolios with the highest possible expected return for a given level of risk. When you introduce additional assets into the investment universe, especially those that are not perfectly correlated with the existing assets, you increase the number of possible portfolio combinations. This expansion of the opportunity set allows for greater diversification benefits. As a result, it becomes possible to construct new portfolios that either have a higher expected return for the same level of risk, or a lower level of risk for the same expected return. This pushes the efficient frontier "up and to the left" on a risk-return graph, representing a set of superior risk-return combinations.

B is incorrect. Adding more assets can never make the efficient frontier worse. In the worst-case scenario (if the new assets offer poor risk-return profiles), they would simply not be included in the optimal portfolios, and the frontier would remain unchanged.

C is incorrect. The frontier will only remain identical if the new assets offer no diversification benefits or are dominated by existing portfolios, which is highly unlikely when adding diverse asset classes.

D is incorrect. This is only partially true. The new frontier will offer higher returns for a given level of risk, but it will also offer lower risk for a given level of return. The overall trade-off becomes superior, not just one dimension of it.

Things to Remember

- The key driver of improvement in the efficient frontier is **correlation**. The lower the correlation between the new assets and the existing ones, the greater the diversification benefit and the more significant the improvement in the available risk-return trade-offs.

- This principle is the fundamental argument for global diversification. By investing across different countries and asset classes, investors can build portfolios that lie on a more favorable efficient frontier than if they restricted themselves to a single market or asset type.

- The shape of the efficient frontier is a curve because the benefits of diversification are subject to diminishing returns.

Q.6691 A risk measure's coherence can be assessed by examining how it assigns weights to the different percentiles of a loss distribution. A measure is deemed coherent only if these weights satisfy a specific condition. Consider a proposed risk measure where the analysis of its weighting scheme reveals that the weight assigned to the 98th percentile of the loss distribution is 1.0, while the weight assigned to the 99th percentile is 0.0. Based on this weighting scheme, which statement correctly assesses the coherence of this risk measure?

A. The measure is not coherent because the weights are not a non-decreasing function of the percentile.

B. The measure is coherent because the weights are a non-decreasing function of the percentile.

C. The measure is not coherent because only Expected Shortfall is a coherent spectral risk measure.

D. The measure is coherent because it correctly identifies a specific percentile as the risk threshold.

The correct answer is **A.**

A fundamental theorem states that a risk measure defined by weights assigned to percentiles is coherent if and only if the weights are a non-decreasing function of the percentile. This means that for any two percentiles p_1 and p_2, if $p_1 > p_2$, then the weight $w(p_1)$ must be greater than or equal to the weight $w(p_2)$. In this scenario, we have $p_1 = 99\%$ and $p_2 = 98\%$. The corresponding weights are $w99\% = 0.0$ and $w98\% = 1.0$. Since $99\% > 98\%$ but $w99\% < w98\%$, the non-decreasing condition is violated. This specific weighting pattern, where all weight is concentrated on a single percentile, is characteristic of Value-at-Risk (VaR), which is known to be non-coherent for this very reason.

B is incorrect. The weights are demonstrably a decreasing function at this point (dropping from 1.0 to 0.0 as the percentile increases), which is a direct violation of the coherence condition.

C is incorrect. While Expected Shortfall is a coherent risk measure, it is not the only one. Spectral risk measures are a broader class of coherent measures where weights can increase in various ways to reflect risk aversion. The reason for non-coherence here is the specific violation of the weighting rule, not the measure's identity.

D is incorrect. Identifying a specific percentile is the mechanism of VaR, which is precisely what makes it non-coherent. Coherence requires consideration of the entire tail beyond a certain point, not just a single point.

Things to Remember

- This weighting condition provides a powerful and general way to understand coherence. **Expected Shortfall (ES)** is coherent because its weights are zero up to the VaR percentile and then a constant positive value for all higher percentiles, which satisfies the non-decreasing condition.

- **Spectral risk measures** are a family of coherent risk measures defined by different non-decreasing weighting functions. This allows users to create custom coherent risk measures that reflect their specific degree of risk aversion by, for example, giving progressively higher weights to more extreme tail losses.

Q.6692 A regulator is drafting new capital adequacy rules and is focused on ensuring the mandated risk measure promotes sound risk management. A key principle is that diversification should be encouraged, not penalized. An analyst's report demonstrates a scenario where merging two separate portfolios leads to a required capital charge (based on the risk measure) that is greater than the sum of the capital charges for the individual portfolios. The regulator must choose between Value-at-Risk (VaR) and Expected Shortfall (ES). How do VaR and Expected Shortfall (ES) respectively relate to the subadditivity axiom, which is central to the regulator's diversification principle?

A. Expected Shortfall always satisfies subadditivity, whereas Value-at-Risk can violate it.

B. Value-at-Risk always satisfies subadditivity, whereas Expected Shortfall can violate it.

C. Both Value-at-Risk and Expected Shortfall always satisfy the subadditivity axiom.

D. Both Value-at-Risk and Expected Shortfall can violate the subadditivity axiom.

The correct answer is **A.**

The subadditivity axiom is the formal property that reflects the benefits of diversification: $\text{Risk}(A + B) \leq \text{Risk}(A) + \text{Risk}(B)$. A risk measure that satisfies this property will never penalize the merging of portfolios. Expected Shortfall (ES) has all the properties of a coherent risk measure, including subadditivity. In contrast, Value-at-Risk (VaR) fails to be a coherent risk measure precisely because there are circumstances under which it violates the subadditivity property. The scenario described by the analyst is a perfect example of such a violation. Therefore, ES consistently encourages diversification, while VaR can fail to do so.

B is incorrect. This statement reverses the properties of the two measures. It is VaR that can violate subadditivity, not ES.

C is incorrect. While ES always satisfies subadditivity, VaR does not, so this statement is false.

D is incorrect. Expected Shortfall is proven to be subadditive and therefore a coherent risk measure; it does not violate this axiom.

Things to Remember

- The choice between VaR and ES has significant regulatory implications. Because ES is coherent and VaR is not, major regulatory reforms like the **Fundamental Review of the Trading Book (FRTB)** have moved to replace VaR with ES for calculating market risk capital requirements for banks using internal models.
- This shift is intended to create a more stable financial system where risk measures do not create perverse incentives to break up diversified portfolios simply to lower a regulatory capital figure.
- The subadditivity of ES ensures that from a risk measurement perspective, the whole is never riskier than the sum of its parts.

Q.6693 A quantitative hedge fund designs its own internal capital allocation model using a spectral risk measure. This approach allows the fund to reflect its unique risk aversion profile. The fund's management is particularly concerned with avoiding catastrophic "black swan" events and wants the risk measure to be disproportionately influenced by the most extreme potential losses in the tail of the distribution. To implement this high degree of tail-risk aversion within a coherent spectral risk measure, how should the weights assigned to the percentiles of the loss distribution be structured?

A. Weights must be non-decreasing and increase more sharply at higher percentiles.

B. Weights must be non-decreasing but increase less sharply at higher percentiles.

C. Weights must be zero below a threshold and constant for all higher percentiles.

D. Weights must be zero for all percentiles except for one specific high percentile.

The correct answer is **A.**

Spectral risk measures are a class of coherent risk measures where the risk is calculated as a weighted average of the quantiles (percentiles) of the loss distribution. For the measure to be coherent, the weights must be a non-decreasing function of the percentile. The way these weights increase reflects the user's risk aversion. A high degree of risk aversion, particularly towards extreme tail losses, means that greater importance (weight) should be assigned to larger losses. Therefore, the weights should not only be non-decreasing but should also increase more steeply for the highest percentiles, placing a disproportionately large weight on the most extreme outcomes.

B is incorrect. An increase that becomes less sharp at higher percentiles would signify decreasing marginal concern for more extreme losses, which is contrary to high risk aversion.

C is incorrect. This describes the weighting scheme for Expected Shortfall (ES). While ES is a coherent spectral measure, it assigns equal weight to all losses in the tail. It does not reflect a *disproportionately* higher aversion to the most extreme losses within that tail compared to losses just beyond the VaR level.

D is incorrect. This describes the weighting scheme for Value-at-Risk (VaR), which assigns all weight to a single percentile. This measure is not coherent because it violates the non-decreasing weight condition and ignores the magnitude of tail losses entirely.

Things to Remember

- Spectral risk measures provide a flexible framework for creating coherent risk measures tailored to specific risk preferences. Both VaR and ES can be viewed within this framework, with VaR having an incoherent weighting scheme and ES having the simplest coherent one.
- The risk aversion profile is embedded in the "spectrum," which is the function that defines the weights. An investor who is risk-neutral about tail losses would use ES (equal weighting). An investor who is extremely risk-averse to tail losses would use a function that assigns rapidly increasing weights to the tail.
- For example, a pension fund with long-term liabilities might choose a spectral measure with high weights on far-tail events to ensure solvency under extreme stress.

Q.6694 An analyst is examining the diversification benefits of combining two risky assets, Asset X and Asset Y. She plots the set of all possible risk-return combinations for portfolios of X and Y under different assumptions for the correlation coefficient () between their returns. How does the curvature of the efficient frontier for this two-asset portfolio change as the correlation coefficient () decreases from +1.0 toward -1.0?

A. The frontier becomes more curved, shifting further to the left.

B. The frontier becomes less curved, shifting further to the right.

C. The frontier's curvature decreases, but its position is unchanged.

D. The frontier's curvature increases, but its position is unchanged.

The correct answer is **A.**

The degree of curvature in a two-asset efficient frontier is a direct visual representation of the diversification benefit. When two assets are perfectly correlated ($\rho = 1.0$), there is no diversification benefit; combining them simply results in a portfolio with a risk-return profile that is a linear combination of the two, forming a straight line. As the correlation decreases, the returns of the assets move less in tandem, allowing for a reduction in portfolio risk that is greater than the linear combination. This benefit appears as a curve, or bow, to the left in risk-return space. The lower the correlation, the more pronounced the curve, with the maximum curvature (and thus maximum diversification benefit) occurring at perfect negative correlation ($\rho = -1.0$).

B is incorrect. A frontier that is less curved and shifting to the right would represent a loss of diversification benefits, which occurs as correlation increases, not decreases.

C is incorrect. A decrease in curvature corresponds to an increase in correlation. Furthermore, a change in curvature is inseparable from a change in the frontier's position; the curve appears precisely because new, lower-risk portfolios become available to the left.

D is incorrect. While the curvature does increase as correlation falls, its position must also shift to the left to reflect the creation of portfolios with lower risk for a given level of return.

Things to Remember

- The portfolio variance formula for two assets, $\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2$, shows that the **correlation coefficient () is the key driver of diversification**. A lower reduces the overall portfolio variance.
- This principle is why asset allocation is often called the only "free lunch" in finance. By combining imperfectly correlated assets, an investor can potentially reduce portfolio risk without sacrificing expected return.
- In a multi-asset portfolio, the benefits depend on the entire correlation matrix. The goal is to find assets with low, or ideally negative, correlations to the existing portfolio components.

Q.6695 Consider two investment portfolios, Portfolio P and Portfolio Q. Due to their structuring with complex derivatives, it has been mathematically proven that in every possible future market state, the loss from Portfolio P is guaranteed to be greater than or equal to the loss from Portfolio Q. No scenario exists where Q loses more than P. According to the properties of a coherent risk measure, what must be the relationship between the risk measures of Portfolio P and Portfolio Q?