2027 CFA® EXAM PREP

SchweserNotes™

Quantitative Methods, Economics, and Corporate Finance

Level I Book 1

KAPLAN SCHWESER

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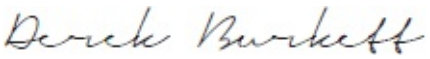

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Book 1: Quantitative Methods, Economics, and Corporate Finance

SchweserNotes™ 2027

Level I CFA®

SCHWESERNOTES™ 2027 LEVEL I CFA® BOOK 1: QUANTITATIVE METHODS, ECONOMICS, AND CORPORATE FINANCE

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Learning Outcome Statements (LOS)

1. **Returns of Financial Assets and Instruments**

The candidate should be able to:

a. describe, compare, and interpret returns.
b. describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation.

2. **Types of Financial Returns**

The candidate should be able to:

a. calculate, compare, and interpret different types of returns for financial assets, instruments, and indicators.

3. **Benchmarking Returns**

The candidate should be able to:

a. calculate and compare money-weighted and time-weighted rates of return.
b. describe the choices and the implications of the different weighting methods used in index construction and management, and calculate, interpret, and explain the value and the returns of an index.

4. **The Time Value of Money in Finance**

The candidate should be able to:

a. calculate and interpret the present value of fixed-income and equity instruments based on expected future cash flows.
b. calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given their present value and cash flows.
c. explain the cash flow additivity principle and its importance for the condition of no arbitrage, and explain its use in calculating implied forward interest rates, forward exchange rates, and option values.

5. **Statistical Characteristics of Asset Returns**

The candidate should be able to:

a. calculate, interpret, and evaluate various measures of (1) central tendency and location and (2) dispersion.
b. describe, interpret, and evaluate measures of skewness and kurtosis.
c. calculate, interpret, and evaluate covariance and correlation.
d. calculate, interpret, and evaluate semi-deviation and coefficient of variation.

6. **Statistical Distributions for Financial Asset Prices and Returns**

The candidate should be able to:

a. calculate, interpret, and evaluate unconditional expected values for mean, variance, and covariance.
b. calculate, interpret, and evaluate the principal moments of key statistical distributions used in finance.
c. c
d. formulate investment problems through Bayesian updating.

7. Estimation and Hypothesis Testing

The candidate should be able to:

a. explain the central limit theorem and the application of confidence intervals and sampling methodologies.
b. explain hypothesis testing and its components, including statistical significance, Type I and Type II errors, and the power of a test; construct appropriate hypothesis tests; and interpret the results.
c. compare and contrast parametric and non-parametric tests, describe situations in which each is the more appropriate type of test, construct appropriate hypothesis tests, and interpret the results.

8. The Return and Risk of a Financial Portfolio

The candidate should be able to:

a. calculate, interpret, and evaluate the expected return, variance, standard deviation, covariance, and correlation of portfolio returns.
b. describe, calculate, and interpret the minimum-variance portfolio and portfolios that lie on the efficient frontier.
c. explain the selection of an optimal portfolio, given an investor's risk aversion and the capital allocation line, and how this extends to the market portfolio and the capital market line.

9. Simulation of Financial Asset Prices and Returns

The candidate should be able to:

a. describe historical simulation and explain how it can be used in investment applications.
b. describe bootstrap resampling, and explain how it can be used in investment applications.
c. describe Monte Carlo simulation and explain how it can be used in investment applications.

10. Applications of Simple Linear Regression in Finance

The candidate should be able to:

a. describe, interpret, and explain simple linear regression, including coefficient estimation using the least squares criterion.

b. describe and compare the assumptions of simple linear regression, identify violations through analyzing residuals, evaluate the estimated model's goodness-of-fit and regression coefficients, and results of ANOVA estimates.
c. calculate and interpret predicted values, the standard error of the estimate, and prediction intervals for the dependent variable in a simple linear regression model and describe different functional forms.
d. calculate and interpret the variable estimates of the capital asset pricing model (CAPM).

11. Introduction to Financial Data Science

The candidate should be able to:

a. describe how big data, machine learning, and artificial intelligence are used in financial data science, fintech, and investment management.

12. The Firm and Market Structures

The candidate should be able to:

a. determine and interpret breakeven and shutdown points of production, as well as how economies and diseconomies of scale affect costs under perfect and imperfect competition.
b. describe characteristics of perfect competition, monopolistic competition, oligopoly, and pure monopoly.
c. explain supply and demand relationships under monopolistic competition, including the optimal price and output for firms as well as pricing strategy.
d. explain supply and demand relationships under oligopoly, including the optimal price and output for firms as well as pricing strategy.
e. identify the type of market structure within which a firm operates and describe the use and limitations of concentration measures.

13. Understanding Business Cycles

The candidate should be able to:

a. describe the business cycle and its phases.
b. describe credit cycles.
c. describe how resource use, consumer and business activity, housing sector activity, and external trade sector activity vary over the business cycle and describe their measurement using economic indicators.

14. Fiscal Policy

The candidate should be able to:

a. compare monetary and fiscal policy.
b. describe roles and objectives of fiscal policy as well as arguments as to whether the size of a national debt relative to GDP matters.

c. describe tools of fiscal policy, including their advantages and disadvantages.
d. explain the implementation of fiscal policy and difficulties of implementation as well as whether a fiscal policy is expansionary or contractionary.

15. Monetary Policy

The candidate should be able to:

a. describe the roles and objectives of central banks.
b. describe tools used to implement monetary policy tools and the monetary transmission mechanism, and explain the relationships between monetary policy and economic growth, inflation, interest, and exchange rates.
c. describe qualities of effective central banks; contrast their use of inflation, interest rate, and exchange rate targeting in expansionary or contractionary monetary policy; and describe the limitations of monetary policy.
d. explain the interaction of monetary and fiscal policy.

16. Introduction to Geopolitics

The candidate should be able to:

a. describe geopolitics from a cooperation versus competition perspective.
b. describe geopolitics and its relationship with globalization.
c. describe functions and objectives of the international organizations that facilitate trade, including the World Bank, the International Monetary Fund, and the World Trade Organization.
d. describe geopolitical risk.
e. describe tools of geopolitics and their impact on regions and economies.
f. describe the impact of geopolitical risk on investments.

17. International Trade

The candidate should be able to:

a. describe the benefits and costs of international trade.
b. compare types of trade restrictions, such as tariffs, quotas, and export subsidies, and their economic implications.
c. explain motivations for and advantages of trading blocs, common markets, and economic unions.

18. Capital Flows and the FX Market

The candidate should be able to:

a. describe the foreign exchange market, including its functions and participants, distinguish between nominal and real exchange rates, and calculate and interpret the percentage change in a currency relative to another currency.

b. describe exchange rate regimes and explain the effects of exchange rates on countries' international trade and capital flows.
c. describe common objectives of capital restrictions imposed by governments.

19. Exchange Rate Calculations

The candidate should be able to:

a. calculate and interpret currency cross-rates.
b. explain the arbitrage relationship between spot and forward exchange rates and interest rates, calculate a forward rate using points or in percentage terms, and interpret a forward discount or premium.

20. Organizational Forms, Corporate Issuer Features, and Ownership

The candidate should be able to:

a. compare the organizational forms of businesses.
b. describe key features of corporate issuers.
c. compare publicly and privately owned corporate issuers.

21. Investors and Other Stakeholders

The candidate should be able to:

a. compare the financial claims and motivations of lenders and shareholders.
b. describe a company's stakeholder groups and compare their interests.
c. describe environmental, social, and governance factors of corporate issuers considered by investors.

22. Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits

The candidate should be able to:

a. describe the principal-agent relationship and conflicts that may arise between stakeholder groups.
b. describe corporate governance and mechanisms to manage stakeholder relationships and mitigate associated risks.
c. describe potential risks of poor corporate governance and stakeholder management and benefits of effective corporate governance and stakeholder management.

23. Working Capital and Liquidity

The candidate should be able to:

a. explain the cash conversion cycle and compare issuers' cash conversion cycles.
b. explain liquidity and compare issuers' liquidity levels.
c. describe issuers' objectives and compare methods for managing working capital and liquidity.

24. Capital Investments and Capital Allocation

The candidate should be able to:

a. describe types of capital investments.
b. describe the capital allocation process, calculate net present value (NPV), internal rate of return (IRR), and return on invested capital (ROIC), and contrast their use in capital allocation.
c. describe principles of capital allocation and common capital allocation pitfalls.
d. describe types of real options relevant to capital investments.

25. Capital Structure

The candidate should be able to:

a. calculate and interpret the weighted-average cost of capital for a company.
b. explain factors affecting capital structure and the weighted-average cost of capital.
c. explain the Modigliani–Miller propositions regarding capital structure.
d. describe optimal and target capital structures.

26. Business Models

The candidate should be able to:

a. describe key features of business models.
b. describe various types of business models.

READING 1

RETURNS OF FINANCIAL ASSETS AND INSTRUMENTS

MODULE 1.1: RATES OF RETURN

LOS 1.a: Describe, compare, and interpret returns.

We can evaluate returns on a financial asset, a financial instrument, or a financial indicator. These categories overlap somewhat, but we can define them as follows:

Financial assets are items investors can own that represent value, and primarily include:

- cash
- equity ownership
- debt, which is an asset of the party to whom a debt is owed
- securities that are hybrids of debt and equity

Financial instruments are standardized means of trading financial assets in markets, and primarily include:

- equity and debt securities
- tradable standardized contracts, such as some derivatives

Financial indicators are observable measures of value that do not generate cash flows, and include:

- interest rates
- currency exchange rates
- market indexes

Returns in a Single Period

The two sources of asset returns are price changes and cash income. We speak of the **price return** (or **capital appreciation return**) as the percentage change in an asset's price between two points in time:

$$r_{price} = \frac{P_1 - P_0}{P_0}, \text{ where } P_0 \text{ is the beginning price and } P_1 \text{ is the ending price.}$$

The **capital distribution return** is the cash income an asset generates during a period, as a percentage of the asset's beginning price. Examples of capital distribution return we will see later in the Level I curriculum include the **dividend yield** of an equity security and the **current yield** of a fixed income security:

$$r_{distribution} = \frac{income}{P_0}$$

Total return on an asset during a period is the sum of its price return and capital distribution return for that period:

$$r_{total} = r_{price} + r_{distribution}$$

$$= \frac{P_1 - P_0 + income}{P_0}$$

PROFESSOR'S NOTE

We often refer to this as an asset's **holding period return**. As we will see later in this reading, returns are typically stated on an annualized basis rather than on a holding period basis.

EXAMPLE: Price, capital distribution, and total return

A stock is valued at €20 at the beginning of the period, pays €1 in dividends over the period, and at the end of the period is valued at €22. Calculate its price return, capital distribution return, and total return for the period.

Answer:

$$\text{Price return} = \frac{22 - 20}{20} = 10\%$$

$$\text{Capital distribution return} = \frac{1}{20} = 5\%$$

$$\text{Total return} = 10\% + 5\% = 15\%, \text{ or } \frac{22 - 20 + 1}{20} = 15\%$$

A return calculated from known past prices and income is an **actual return**, or **ex post return**. We can also estimate an **expected return**, or **ex ante return**, for a future period. Investors make decisions based on expected returns, but actual returns most often differ from what was expected. The difference between expected returns and actual returns represents the concept of *risk*, which we will address in the next LOS.

Returns based on cash that an investor has received are **realized returns**. These include income received (capital distributions) and the proceeds from selling an investment. We may also speak of the **unrealized return** on an asset that an investor still holds, based on its current market price. The distinction between realized and unrealized returns is important for tax reasons. With some exceptions, tax authorities generally assess taxes on income in the period an investor receives it, and on price gains in the period when an investor realizes them by selling an asset.

Returns Across Multiple Periods

The **arithmetic mean return** is the simple average of a series of periodic returns:

$$\text{arithmetic mean return} = \frac{(R_1 + R_2 + R_3 + \ldots + R_n)}{n}$$

The **geometric mean return**, or **compound average return**, is a compound rate:

$$\text{geometric mean return} = \sqrt[n]{(1 + R_1) \times (1 + R_2) \times (1 + R_3) \times \ldots \times (1 + R_n)} - 1$$

EXAMPLE: Arithmetic and geometric mean return

For the last three years, the returns for Acme Corporation common stock have been −9.34%, 23.45%, and 8.92%. Calculate the arithmetic average and compound average rate of return over the three-year period.

Answer:

$$\text{arithmetic mean return} = \frac{-9.34 + 23.45 + 8.92}{3} = 7.68\%$$

geometric mean return

$$= \sqrt[3]{(1 - 0.0934) \times (1 + 0.2345) \times (1 + 0.0892)} - 1$$
$$= \sqrt[3]{0.9066 \times 1.2345 \times 1.0892} - 1$$
$$= \sqrt[3]{1.21903} - 1$$
$$= 1.06825 - 1 = 6.825\%$$

PROFESSOR'S NOTE

You can get a third root (or other higher-order root) with your calculator as follows:

- On the TI, enter 1.21903 [y^x] 3 [1/x] − 1 [=]
- On the HP, enter 1.21903 [ENTER] 3 [1/x] [y^x] 1 −

Notice in the example that the geometric mean return is less than the arithmetic mean return. The **arithmetic-geometric mean inequality principle** states that the geometric mean is always less than or equal to the arithmetic mean, and the difference increases as the variability of the observations increases. The only time the arithmetic and geometric means are equal is when there is no variability in the observations (i.e., all observations are equal).

An arithmetic mean return assumes the amount invested at the start does not change. However, that amount does change if an investment experiences any price appreciation or depreciation, or has any cash flows. For this reason, an arithmetic mean return can be misleading, as the following example shows.

EXAMPLE: Geometric and arithmetic mean returns

An investment of $1,000 loses 25% in Period 1 and gains 25% in Period 2. Is the return over both periods more accurately represented by the arithmetic mean return or the geometric mean return?

Answer:

In Period 1, the investment's value decreases to $1,000(1 − 0.25) = $750. In Period 2, its value increases to $750(1 + 0.25) = $937.50.

The arithmetic mean return is $\frac{-25\% + 25\%}{2} = 0\%$. This is misleading because it suggests the investment's value did not change over the two periods.

The geometric mean return is $\sqrt{(1-0.25)(1+0.25)}-1=-3.175\%$. This accurately reflects the actual change in value over the two periods: \$1,000(1 − 0.03175)(1 − 0.03175) = \$937.51 (difference due to rounding).

Compounding Periods and Annualized Returns

Interest rates and market returns are typically stated as **annualized returns**, regardless of the actual length of the period over which they occur. To annualize a return that is realized over a specific number of days, use the following formula:

$$\text{annualized return} = (1 + \text{return during the period})^{365/\text{days}} - 1$$

EXAMPLE: Annualized return, shorter than one year

A saver deposits \$100 into a bank account. After 90 days, the account balance is \$100.75. What is the saver's annualized rate of return?

Answer:

$$\text{return over 90 days} = \frac{100.75}{100} - 1 = 0.0075 = 0.75\%$$

$$\text{annualized return} = (1 + 0.0075)^{365/90} - 1 = 0.0308 = 3.08\%$$

EXAMPLE: Annualized return, longer than one year

An investor buys a 500-day government bill for \$970 and redeems it at maturity for \$1,000. What is the investor's annualized return?

Answer:

$$\text{return over 500 days} = \frac{1000}{970} - 1 = 0.0309 = 3.09\%$$

$$\text{annualized return} = (1 + 0.0309)^{365/500} - 1 = 0.0225 = 2.25\%$$

In our earlier example of arithmetic and geometric means, the geometric mean resulted in an annual rate of return because the holding periods were years. If the holding periods are other than years, the geometric mean is *not* the same as the annual return. The root for the geometric mean is the number of *periods*, while the root for the annual return is the number of *years* from the start of the first holding period to the end of the last holding period.

EXAMPLE: Geometric mean vs. annual return

For the last four semiannual periods, the 6-month holding period returns on an investment were 2.0%, 0.5%, −1.0%, and 1.5%. Calculate the geometric mean and the annual rate of return.

Answer:

$$\text{geometric mean} = \sqrt[4]{(1+0.02)(1+0.005)(1-0.01)(1+0.015)} - 1$$

$$= 0.007435 = 0.7435\%, \text{ which is the geometric mean of the six-month holding period returns.}$$

$$\text{annual return} = \sqrt[2]{(1+0.02)(1+0.005)(1-0.01)(1+0.015)} - 1$$

$$= 0.0149 = 1.49\%$$

The four semiannual periods equal two years, so to get an annual return, we use *two* as the root.

When we know the number of periods in a year (as we do with daily, weekly, monthly, or quarterly periods), we can easily annualize the returns for a period using the following formula:

$$\text{annualized return} = (1 + \text{return during the period})^{c} - 1$$

where:
c is the number of periods that make up a year.

EXAMPLE: Comparing returns on an annualized basis

An investor observes the following three securities:

- Security A produced a monthly return of 0.5%.
- Security B produced a quarterly (three-month) return of 1.5%.
- Security C produced a semiannual (six-month) return of 3.0%.

Which security produced the highest annualized return?

Answer:

- Security A annualized return = $(1 + 0.005)^{12} - 1 = 0.0617 = 6.17\%$
- Security B annualized return = $(1 + 0.015)^{4} - 1 = 0.0614 = 6.14\%$
- Security C annualized return = $(1 + 0.03)^{2} - 1 = 0.0609 = 6.09\%$

On an annualized basis, Security A produced the highest return.

While annualizing returns provides us with a consistent standard for comparing the performance of different investments, we must recognize that it assumes returns are repeatable. In the previous example, the annualized return on Security A is based on one month's return of 0.5%. To realize a return of 6.17% over a full year, the security would have to produce the same 0.5% return in each of another 11 months. Its actual performance is likely to differ—and even if its monthly returns average 0.5%, they are likely to exhibit some variability.

Continuous Compounding

In time value of money calculations (which we will address in more detail in our reading on The Time Value of Money in Finance), more frequent compounding has an impact on future value and present value computations. For example, we can calculate the present value of $1,000 to be received one year from now using a stated annual interest rate of 6% with a range of compounding periods.

With semiannual compounding:

$$\text{PV} = 1{,}000\left(1 + \frac{0.06}{2}\right)^{-2} = 942.60$$

With quarterly compounding:

$$\text{PV} = 1{,}000\left(1 + \frac{0.06}{4}\right)^{-4} = 942.18$$

With monthly compounding:

$$\text{PV} = 1{,}000\left(1 + \frac{0.06}{12}\right)^{-12} = 941.91$$

With daily compounding:

$$\text{PV} = 1{,}000\left(1 + \frac{0.06}{365}\right)^{-365} = 941.77$$

The mathematical limit of shortening the compounding period is known as continuous compounding. Given the return for a period, we can use the natural logarithm (ln, or LN on your financial calculator) to state its associated **continuously compounded return** or **logarithmic return**:

$$R_{cc} = \ln(1 + \text{return during the period}) = \ln\left(\frac{\text{ending value}}{\text{beginning value}}\right)$$

EXAMPLE: Calculating continuously compounded returns

A stock was purchased for $100 and sold one year later for $120. Calculate the investor's annual rate of return on a continuously compounded basis.

Answer:

$$\ln\left(\frac{120}{100}\right) = 18.232\%$$

If we were given the return (20%) instead, the calculation is this:

ln(1 + 0.20) = 18.232%

A useful property of continuously compounded rates of return is that they are additive for multiple periods. That is, a continuously compounded return from $t = 0$ to $t = 2$ is the sum of the continuously compounded return from $t = 0$ to $t = 1$ and the continuously compounded return from $t = 1$ to $t = 2$.

EXAMPLE: Continuously compounded return over multiple periods

The value of an investment is 1,000 at $t = 0$, 1,150 at $t = 1$, and 1,250 at $t = 2$. Calculate its continuously compounded rates of return for each period and for the time from $t = 0$ to $t = 2$.

Answer:

$$\text{From } t = 0 \text{ to } t = 1\text{: } \ln\left(\frac{1{,}150}{1{,}000}\right) = 0.13976 = 13.976\%$$

$$\text{From } t = 1 \text{ to } t = 2\text{: } \ln\left(\frac{1{,}250}{1{,}150}\right) = 0.08338 = 8.338\%$$

$$\text{From } t = 0 \text{ to } t = 2\text{: } \ln\left(\frac{1{,}250}{1{,}000}\right) = 0.22314 = 22.314\%$$

Or, 13.976% + 8.338% = 22.314%.

MODULE QUIZ 1.1

1. If an investment loses 3% of its value over 120 days, its annualized return is *closest* to:
   A. −8.0%.
   B. −8.5%.
   C. −9.0%.

2. If a stock's initial price is $20 and its price increases to $23, its continuously compounded rate of return is *closest* to:
   A. 13.64%.
   B. 13.98%.
   C. 15.00%.

3. An ex ante return is described *most accurately* as a return:

A. expected over a future period.
B. realized in a specified past period.
C. that would be realized in a year if past performance continues.

MODULE 1.2: COMPONENTS AND MEASURES OF RETURN

LOS 1.b: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation.

Interest rates measure the time value of money, although risk differences among financial securities lead to differences in their equilibrium interest rates. Equilibrium interest rates are the **required rate of return** for a particular investment, in the sense that the market rate of return is the return that investors and savers require to get them to willingly lend their funds. Interest rates are also referred to as **discount rates** and, in fact, the terms are often used interchangeably. If an individual can borrow funds at an interest rate of 10%, then that individual should discount payments to be made in the future at that rate to get their equivalent value in current dollars or other currencies.

We can also view interest rates as the **opportunity cost** of current consumption. If the market rate of interest on 1-year securities is 5%, earning an additional 5% is the opportunity forgone when current consumption is chosen rather than saving (postponing consumption).

For any asset that has **risk**, or uncertainty about the amount or timing of its future cash flows, the interest rate (investors' required rate of return) will reflect the degree of risk. We can think of an interest rate as consisting of a **risk-free rate** and a **risk premium**.

Risk-Free Rates of Return

The risk-free rate (which we sometimes refer to as the **nominal risk-free rate**) has two components: a real risk-free rate and an expected inflation rate. When we speak of a "real" rate of return, we are referring to an investor's increase in purchasing power (i.e., after adjusting for inflation). The **real risk-free rate** is an interest rate that contains no expectation of inflation and no uncertainty about future cash flows. What the real risk-free rate represents in economic terms is

time preference, the degree to which current consumption is preferred to equal future consumption.

Because expected inflation in future periods is not zero, the rates we observe on instruments that are considered free of default risk (e.g., US Treasury bills) are essentially risk-free rates, but not real risk-free rates. Treasury bill rates are nominal risk-free rates because they contain an **inflation premium**.

The relation between nominal and real risk-free rates is as follows:

$$(1 + \text{nominal risk-free rate}) = (1 + \text{real risk-free rate})(1 + \text{expected inflation rate})$$

or,

$$\text{real risk-free rate} = \frac{1 + \text{nominal risk-free rate}}{1 + \text{inflation rate}} - 1$$

Often, including in many parts of the CFA curriculum, this relation is approximated as follows:

$$\text{real risk-free rate} \approx \text{nominal risk-free rate} - \text{expected inflation rate}$$

Risk Premia

Securities may have one or more types of risk. The risk premium includes each type of risk, so each added risk increases the required rate of return. A risk premium in the interest rate on a bond may include (but is not limited to) the following:

- **Default risk premium.** This reflects the risk that a borrower will not make the promised payments in a timely manner.
- **Liquidity risk premium.** This reflects the risk of receiving less than fair value for an investment if it must be sold quickly for cash.
- **Maturity risk premium.** The prices of longer-term bonds are more sensitive to interest rate changes than those of shorter-term bonds. Longer-maturity bonds have more maturity risk than shorter-term bonds and require a maturity risk premium.

Examples of risk premia in the required rate of return on a stock include (but are not limited to) the following:

- **Size risk premium.** This reflects the difference between the returns on large-capitalization stocks and small-capitalization stocks.

- **Value risk premium.** This reflects the difference between the returns on value stocks (those with high ratios of book value to market value) and stocks with lower book-to-market value ratios.

Later in the Level I curriculum, we will discuss and use the concept of a **market risk premium**, which we define as the difference between the return on the "market portfolio" of all risky assets and the return on a risk-free asset.

Common Measures of Return

As we saw with risk-free returns, a **real return** is a nominal return adjusted for inflation. Consider an investor who earns a nominal return of 7% over a year when inflation is 2%. The investor's approximate real return is simply 7 − 2 = 5%. The investor's exact real return is slightly lower: 1.07 / 1.02 − 1 = 0.049 = 4.9%.

A real return measures the increase in an investor's purchasing power—how much more goods she can purchase at the end of one year due to the increase in the value of her investments. If she invests $1,000 and earns a nominal return of 7%, she will have $1,070 at the end of the year. If the price of the goods she consumes has increased 2%, from $1.00 to $1.02 per unit, she will be able to consume 1,070 / 1.02 = 1,049 units. She has given up consuming 1,000 units today, but instead, she is able to purchase 1,049 units at the end of one year. Her purchasing power has gone up 4.9%; this is her real return.

We often speak of an investment's **excess return**, which is how much more (or less) it earns compared to a benchmark or a reference rate. For example, if the chosen benchmark (such as the nominal risk-free rate) is 3%, and an investment earns 8%, we can approximate its excess return as 8% − 3% = 5%. More exactly, its excess return is 1.08 / 1.03 − 1 = 4.85%.

Gross return refers to the total return on a security portfolio before deducting fees for the management and administration of the investment account. **Net return** refers to the return after these fees have been deducted. Both gross and net returns deduct commissions on trades and other costs that are necessary to generate the investment returns.

gross return = total return − transaction costs

net return = gross return − management fees and administrative costs

Pretax nominal return refers to the return before paying taxes. **After-tax nominal return** refers to the return after the tax liability is deducted. Tax rates on investments differ and may be affected by tax laws where the investment is made as well as those in the investor's home jurisdiction. Dividend income, interest income, short-term capital gains, and long-term capital gains may all be taxed at different rates. Many countries tax long-term capital gains at lower rates than short-term capital gains. Investors typically can use capital losses on some investments to offset capital gains on other investments for tax purposes. Capital gains are usually not taxed until the period when the investor realizes the gains.

Assuming a jurisdiction has one tax rate for capital gains and another single tax rate for capital distributions like interest and dividends, we can express the after-tax nominal rate of return as follows:

$$\text{after-tax return} = (\text{price return})(1 - \text{capital gains tax rate}) + (\text{capital distribution return})(1 - \text{capital distribution tax rate})$$

EXAMPLE: Pretax and after-tax nominal returns

An investor buys a stock for $160 at the beginning of the year and sells it for $200 at the end of the year. The stock paid $8 in dividends during the year. The investor is subject to tax rates of 25% on capital gains and 40% on capital distributions. Calculate the investor's pretax and after-tax nominal rates of return.

Answer:

Price return = ($200 − $160) / $160 = 25%

Capital distribution return = $8 / $160 = 5%

The pretax return is 25% + 5% = 30%.

The after-tax return is 25%(1 − 0.25) + 5%(1 − 0.40) = 21.75%.

In finance, *leverage* refers to using borrowed funds or derivatives to increase the amount of an asset an investor can control. A **leveraged return** refers to a return to an investor that is a multiple of the return on the underlying asset. The leveraged return is calculated as the gain or loss on the investment as a percentage of an investor's cash investment.

To illustrate the effect of leverage on returns, consider a fund that can invest the amount V_E in a portfolio without leverage, and earn the rate of return r_P on that

portfolio. The fund's unleveraged return (as a money amount) is simply $r_P \times V_E$. Now, let's say this fund can borrow the amount V_D at an interest rate of r_D, and can still earn r_P by investing the proceeds. The fund's leveraged return (again as a money amount), after subtracting the interest cost, then becomes $r_P \times (V_E + V_D) - (r_D \times V_D)$.

Thus, stated as a rate of return on the initial value of V_E, the leveraged rate of return is as follows:

$$\text{leveraged return} = \frac{r_P(V_D + V_E) - r_D V_D}{V_E}$$

EXAMPLE: Effect of leverage on a positive return

Mammoth Fund can earn an 8% return on an unleveraged portfolio of $800 million. The fund can borrow up to $200 million at a 5% interest rate. Calculate the fund's leveraged return if it borrows and invests this amount.

Answer:

$$\text{leveraged return} = \frac{r_P(V_D + V_E) - r_D V_D}{V_E}$$

$$= \frac{0.08(\$200 \text{ million} + \$800 \text{ million}) - 0.05(\$200 \text{ million})}{\$800 \text{ million}} = 8.75\%$$

Using leverage increased the fund's rate of return from 8% to 8.75%.

EXAMPLE: Effect of leverage on a negative return

Continuing the previous example, calculate Mammoth Fund's leveraged return if it experienced a loss of 4% on its investment portfolio.

Answer:

$$\text{leveraged return} = \frac{-0.04(\$200 \text{ million} + \$800 \text{ million}) - 0.05(\$200 \text{ million})}{\$800 \text{ million}}$$

$$= -6.25\%$$

Using leverage decreased the fund's rate of return from −4% to −6.25%.

These examples highlight the key point investment managers must understand about leverage: *It magnifies gains, but it also magnifies losses.*