Portfolio Construction

Historical Risk and Return

Exam Focus

- Describe characteristics of the major asset classes that investors consider in forming portfolios.

US Asset Class Risk and Return (%)

Asset Class		1930s	1940s	1950s	1960s	1970s	1980s	1990s	2000s	2010–2017	1926–2017
Large stocks	Return	–0.1	9.2	19.4	7.8	5.9	17.6	18.2	–1.0	13.9	**10.2**
	Risk	41.6	17.5	14.1	13.1	17.2	19.4	15.9	16.3	13.6	**19.8**
Small stocks	Return	1.4	20.7	16.9	15.5	11.5	15.8	15.1	6.3	14.8	**12.1**
	Risk	78.6	34.5	14.4	21.5	30.8	22.5	20.2	26.1	19.4	**31.7**
LT corp. bonds	Return	6.9	2.7	1.0	1.7	6.2	13.0	6.4	7.7	8.3	**6.1**
	Risk	5.3	1.8	4.4	4.9	8.7	14.1	6.9	11.7	8.8	**8.3**
LT govt. bonds	Return	4.9	3.2	–0.1	1.4	5.5	12.6	8.8	7.7	6.8	**5.5**
	Risk	5.3	2.8	4.6	6.0	8.7	16	8.9	12.4	10.9	**9.9**
Treasury bills	Return	0.6	0.5	1.9	3.9	6.3	8.9	4.9	2.8	0.2	**3.4**
	Risk	0.2	0.1	0.2	0.4	0.6	0.9	0.4	0.6	0.1	**3.1**
Inflation	Return	–2.0	5.4	2.2	2.5	7.4	5.1	2.9	2.5	1.7	**2.9**
	Risk	2.5	3.1	1.2	0.7	1.2	1.3	0.7	1.6	1.1	**4.0**

Source: 2018 SBBI Yearbook

Historical Risk and Return

United States: Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	6.5	8.4	20.0
	Bonds	2.0	2.5	10.4
Premiums	Equities vs. bonds	4.4	6.5	20.7

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Historical Risk and Return

World: Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	5.2	6.6	17.4
	Bonds	2.0	2.5	11.0
Premiums	Equities vs. bonds	3.2	4.4	15.3

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Historical Risk and Return

World (excl. US): Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	4.5	6.2	18.9
	Bonds	1.7	2.7	14.4
Premiums	Equities vs. bonds	2.8	3.8	14.4

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Other Investment Characteristics

- Negative skew
- Positive excess kurtosis
- Liquidity
    - Bid-ask spread
    - Price impact of trades

Risk Aversion

Exam Focus

- Explain risk aversion and its implications for portfolio selection.
- Explain the selection of an optimal portfolio, given an investor's utility (or risk aversion) and the capital allocation line.

Risk Aversion

- **Consider two alternatives:**
  - Receive $2 if heads and $0 if tails; expected payoff = $1
  - **Or**, receive $1 (with certainty)
    - **Risk averse** prefers $1 payment
    - **Risk neutral** is indifferent between the two
    - **Risk seeking** would prefer the gamble

Risk Aversion: Example

Abner is risk averse. Which of the following statements is *most likely* correct?

A. He will choose relatively safe investments.

B. He may hold some very risky investments.

C. His risk tolerance is relatively low.

-1

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Investor Utility

- Utility function: $U = E(r) - \frac{1}{2}A\sigma^2$
  - Utility increases with higher expected return
  - A is coefficient of risk aversion:
    A > 0 for risk averse, = 0 for risk neutral, < 0 for risk seeking
- Example: US large stocks utility = $0.102 - (0.5 \times A \times 0.198^2)$
  - Shows the tradeoff between risk and expected return
  - Greater risk aversion (A); more expected return to compensate for risk

Risk Aversion and Utility: Example

Which investment will a risk-averse investor with a risk aversion coefficient of 2 choose?

U =

Investment	Expected Return E(r) %	Standard Deviation σ %	Utility A = 2
1	12	30	
2	15	35	0.0275
3	21	40	0.0500
4	24	45	0.0375

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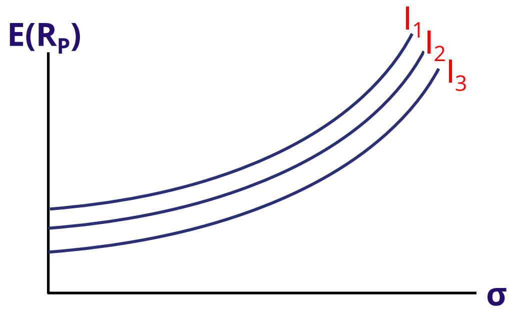
Indifference Curves



- An investor is indifferent about any point on I_1
- Any point on   is preferable to I_2, and I_2 is preferable to
- Risk-averse investors have   -sloping indifference curves
- Higher risk aversion =   curves   risk aversion coefficient)

-5

Combining Risky and Risk-Free Assets

$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

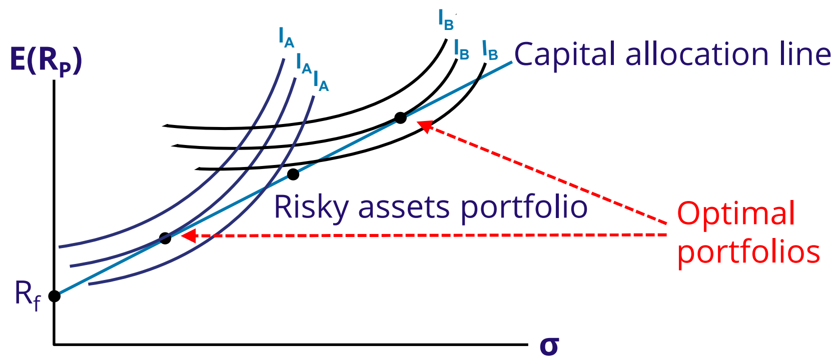
$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

If B's variance is 0:

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2} = \sigma_A w_A$$

Investor's Optimal Portfolio

Investor A is more risk averse than Investor B (steeper indifference curves).



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Solutions

Risk Aversion: Example

Abner is risk averse. Which of the following statements is *most likely* correct?

A. He will choose relatively safe investments.

B. He may hold some very risky investments.

C. His risk tolerance is relatively low.

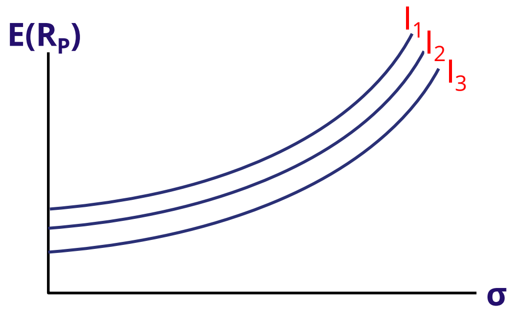
Risk Aversion and Utility: Example

Which investment will a risk-averse investor with a risk aversion coefficient of 2 choose?

$$U = 0.12 - \left(0.5 \times 2 \times 0.3^2\right)$$

Investment	Expected Return E(r) %	Standard Deviation σ %	Utility A = 2
1	12	30	0.0300
2	15	35	0.0275
3	21	40	0.0500
4	24	45	0.0375

Indifference Curves



- An investor is indifferent about any point on I_1
- Any point on I_1 is preferable to I_2, and I_2 is preferable to I_3
- Risk-averse investors have upward-sloping indifference curves
- Higher risk aversion = steeper curves (higher risk aversion coefficient)

Portfolio Standard Deviation

Exam Focus

- Calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data.
- Calculate and interpret portfolio standard deviation.

Arithmetic Mean Return: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

Mean Return A:

Mean Return B:

-2

Variance

Population variance:

$$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{T}$$

Typically, we are analyzing a sample, not a population, so we should calculate sample variance instead:

$$s^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T - 1}$$

Variance: Example

Sample Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

Var_A = = s.d. =

Var_B = = s.d. =

-6

Covariance

- Extent to which two variables move together over time
- Positive covariance = variables tend to move together
- Negative covariance = variables tend to move in opposite directions
- Zero covariance = no linear relationship
- Size of covariance not reflective of strength of relationship

$$\text{Cov}_{1,2} = \frac{\sum_{t=1}^{n}\left\{\left[R_{t,1} - \bar{R}_1\right]\left[R_{t,2} - \bar{R}_2\right]\right\}}{n-1}$$

Covariance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%
Mean	4%	6%

Covariance of returns for Assets A and B:

=

-2

Correlation

- Covariance can be standardized by converting it to correlation
- Size of correlation **is** reflective of strength of relationship
- Correlation is bounded by –1 and +1:

+1 = perfect positive correlation

–1 = perfect negative correlation

0 = uncorrelated

$$\rho_{1,2} = \frac{Cov_{1,2}}{\sigma_1 \sigma_2}$$

Correlation: Example

$Cov_{AB} = -0.0028$; $\sigma_A = 0.07$; $\sigma_B = 0.08$

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$$

$\rho_{A,B} =$

-2

Portfolio Returns Standard Deviation

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2w_A w_B \text{Cov}_{A,B}}$$

Note: $\text{Cov}_{A,B} = \rho_{A,B}\sigma_A\sigma_B$

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2w_A w_B \rho_{A,B}\sigma_A\sigma_B}$$

Correlation = 1: $\sigma_{Rp} = w_A\sigma_A + w_B\sigma_B$

Correlation < 1: $\sigma_{Rp} < w_A\sigma_A + w_B\sigma_B$

Return and Risk of a Two-Asset Portfolio: Example 1

Asset	Weight%	E(r)%	σ%	Covariance (Decimal)
S&P 500 Index	80	9.93	16.21	0.005
MSCI Index	20	18.20	33.11	

Compute the two-asset portfolio's expected return and risk:

R_P = + =

σ_P^2 = + + =

σ_P = =

-3

Return and Risk of a Two-Asset Portfolio: Example 2

Asset	Weight %	E(r) %	σ%	Correlation
FTSE 100 Index	60	5.5	13.2	–0.01
Gilts	40	0.7	4.2	

Compute the two-asset portfolio's expected return and risk:

R_P = + =

σ_P^2 = + +

=

σ_P = =

-3

Solutions

Arithmetic Mean Return: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

Mean Return A: (11 + 4 – 3) / 3 = 4%

Mean Return B: (6 – 2 + 14) / 3 = 6%

Variance: Example

Sample Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

$$Var_A = \frac{(11-4)^2 + (4-4)^2 + (-3-4)^2}{3-1} = 49 \text{ (or } 0.0049) \qquad \text{s.d.} = 7\%$$

$$Var_B = \frac{(6-6)^2 + (-2-6)^2 + (14-6)^2}{3-1} = 64 \text{ (or } 0.0064) \qquad \text{s.d.} = 8\%$$

Covariance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%
Mean	4%	6%

Covariance of returns for Assets A and B:

$$\frac{(11-4)(6-6)+(4-4)(-2-6)+(-3-4)(14-6)}{3-1} = -28 \text{ or } -0.0028$$

Correlation: Example

$Cov_{AB} = -0.0028$; $\sigma_A = 0.07$; $\sigma_B = 0.08$

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$$

$$\rho_{A,B} = \frac{-0.0028}{(0.07)(0.08)} = -0.5$$

Return and Risk of a Two-Asset Portfolio: Example 1

Asset	Weight%	E(r)%	σ%	Covariance (Decimal)
S&P 500 Index	80	9.93	16.21	0.005
MSCI Index	20	18.20	33.11	

Compute the two-asset portfolio's expected return and risk:

$$R_P = (0.8 \times 0.0993) + (0.2 \times 0.1820) = 11.58\%$$

$$\sigma_P^2 = (0.8^2 \times 0.1621^2) + (0.2^2 \times 0.3311^2) + (2 \times 0.8 \times 0.2 \times 0.005) = 0.022802$$

$$\sigma_P = \sqrt{0.022802} = 0.151 \text{ or } 15.1\%$$

Return and Risk of a Two-Asset Portfolio: Example 2

Asset	Weight %	E(r) %	σ%	Correlation
FTSE 100 Index	60	5.5	13.2	−0.01
Gilts	40	0.7	4.2	

Compute the two-asset portfolio's expected return and risk:

$$R_P = (0.6 \times 0.055) + (0.4 \times 0.007) = 3.58\%$$

$$\sigma_P^2 = (0.6^2 \times 0.132^2) + (0.4^2 \times 0.042^2) + (2 \times 0.6 \times 0.4 \times -0.01 \times 0.132 \times 0.042)$$

$$= 0.006528$$

$$\sigma_P = \sqrt{0.006528} = 0.081 \text{ or } 8.1\%$$

The Efficient Frontier

Exam Focus

- Describe the effect on a portfolio's risk of investing in assets that are less than perfectly correlated.
- Describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum-variance portfolio.

Portfolio Returns Standard Deviation

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \text{Cov}_{A,B}}$$

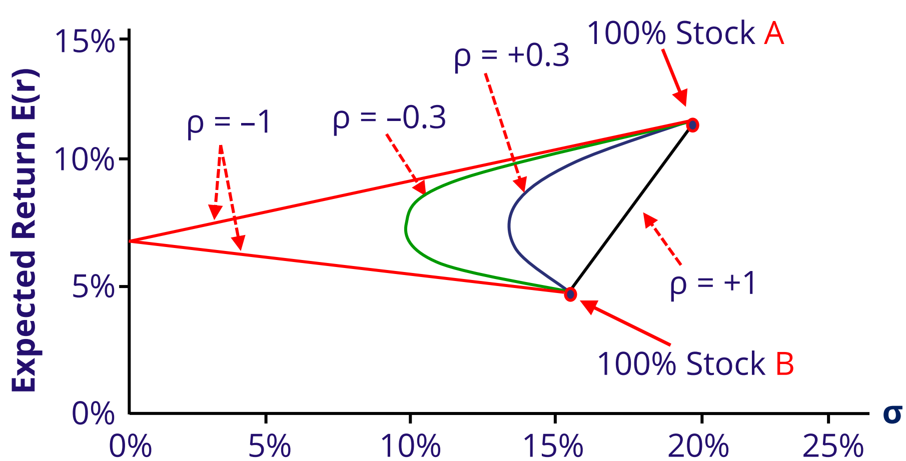
Note: $\text{Cov}_{A,B} = \rho_{A,B}\sigma_A\sigma_B$

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B}\sigma_A\sigma_B}$$

Correlation = 1: $\sigma_{Rp} = w_A\sigma_A + w_B\sigma_B$

Correlation < 1: $\sigma_{Rp} < w_A\sigma_A + w_B\sigma_B$

Correlation and Risk Reduction



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Portfolio Diversification: Example

A portfolio manager reallocates a portion of the portfolio to a new stock that has the same standard deviation of returns as the existing portfolio but has a correlation coefficient with the existing portfolio that is less than +1. Adding this stock will *most likely* have what effect on the standard deviation of the revised portfolio's returns? The standard deviation will:

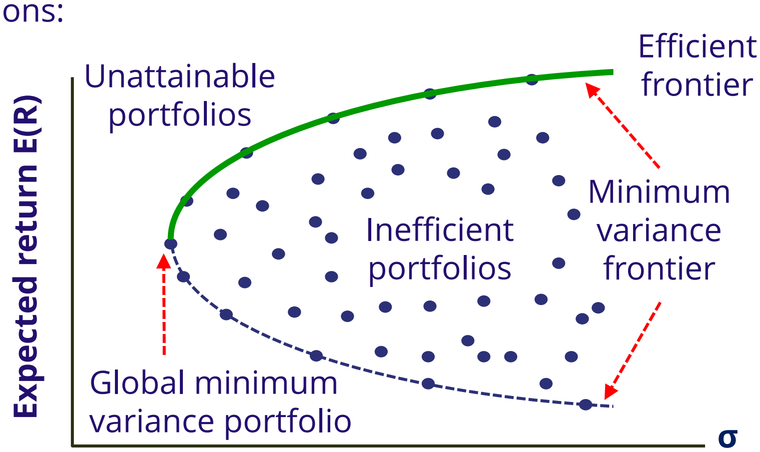
A. increase.

B. decrease.

C. remain the same.

Efficient Frontier

This is computed from expectations of assets' (or asset classes') risk, returns, and correlations:



Efficient Frontier: Example

Which of the following portfolios *most likely* falls below the Markowitz efficient frontier?

Portfolio	Expected Return	Standard Deviation
A	7%	14%
B	9%	26%
C	12%	22%

A. Portfolio A.

B. Portfolio B.

C. Portfolio C.

-1

Solutions

Portfolio Diversification: Example

A portfolio manager reallocates a portion of the portfolio to a new stock that has the same standard deviation of returns as the existing portfolio but has a correlation coefficient with the existing portfolio that is less than +1. Adding this stock will *most likely* have what effect on the standard deviation of the revised portfolio's returns? The standard deviation will:

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Efficient Frontier: Example

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C	12%	22%

A. Portfolio A.

B. Portfolio B.

C. Portfolio C.

Systematic Risk and Beta

Exam Focus

- Describe the implications of combining a risk-free asset with a portfolio of risky assets.
- Explain the capital allocation line (CAL) and the capital market line (CML).
- Explain systematic and nonsystematic risk, including why an investor should not expect to receive additional return for bearing nonsystematic risk.

Exam Focus

- Explain return generating models (including the market model) and their uses.
- Calculate and interpret beta.

Combining Risky and Risk-Free Assets

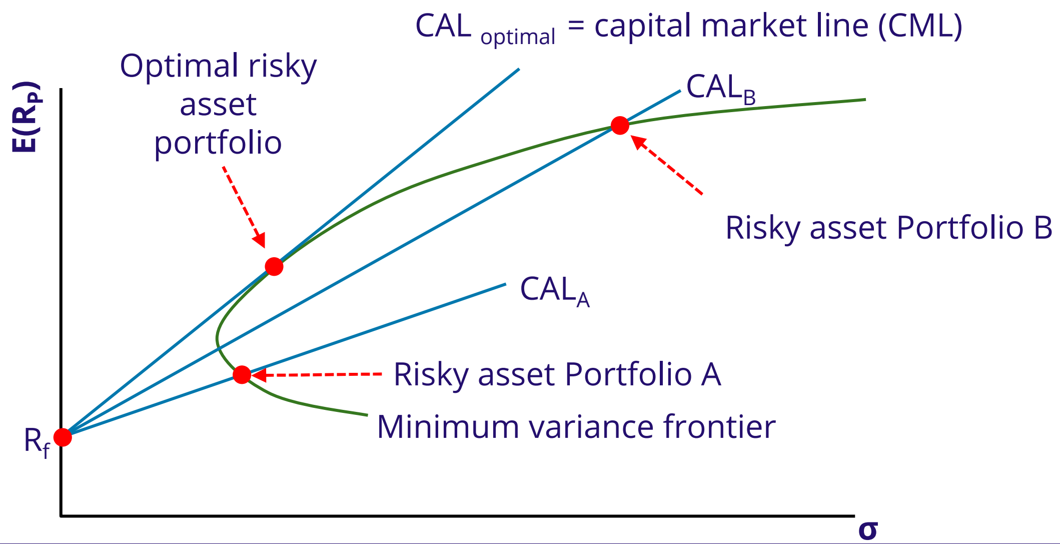
$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

If B's variance is 0:

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2} = \sigma_A w_A$$

Adding a Risk-Free Security to the Model



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Equation of CML

Linear equation for CML: $$E(R_P) = R_f + \underbrace{\left(\frac{E(R_M) - R_f}{\sigma_M}\right)}_{\text{Gradient}} \sigma_P$$

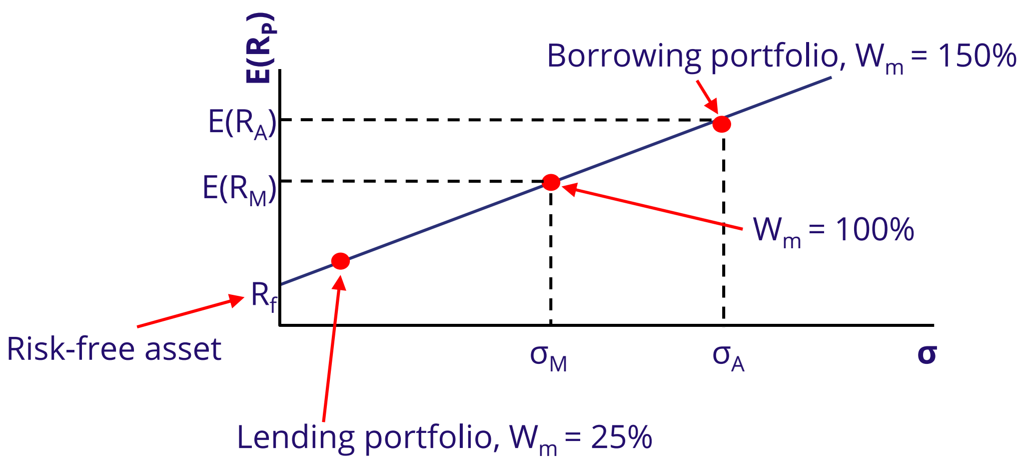
Intercept

Gradient = Sharpe ratio

Equation rearranged: $$E(R_P) = R_f + \left(E(R_M) - R_f\right)\left(\frac{\sigma_P}{\sigma_M}\right)$$

Portfolio risk relative to market risk

Borrowing and Lending Portfolios



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Systematic and Unsystematic Risk

Systematic risk + unsystematic risk = total risk

Systematic risk (market risk)

- Caused by **macro factors:** interest rates, GDP growth, supply shocks
- Measured by covariance of returns with returns on the market portfolio

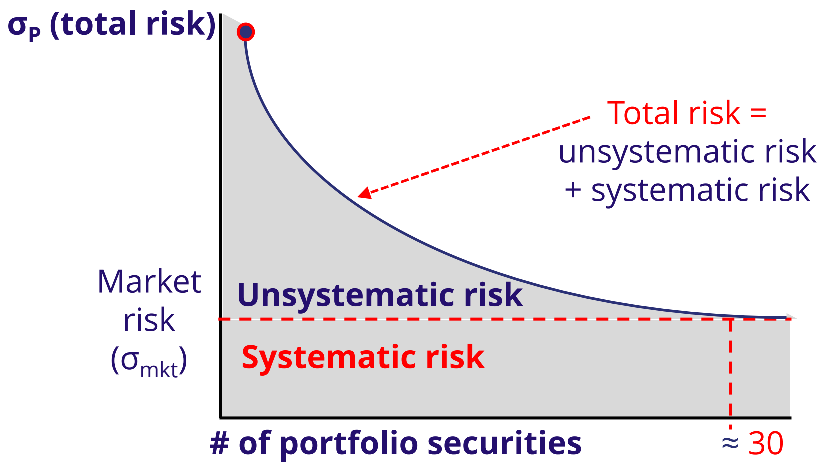
Unsystematic risk (firm-specific risk, diversifiable risk)

- Can be reduced/eliminated by holding a **well-diversified portfolio**

Capital asset model result

- Only **systematic** risk must be rewarded with higher expected returns

A Well-Diversified Portfolio



Types of Risk: Example

The issuer-specific component of the variability in a stock's total return that is unrelated to overall market variability is known as:

A. systematic risk.

B. non-diversifiable risk.

C. unsystematic risk.