

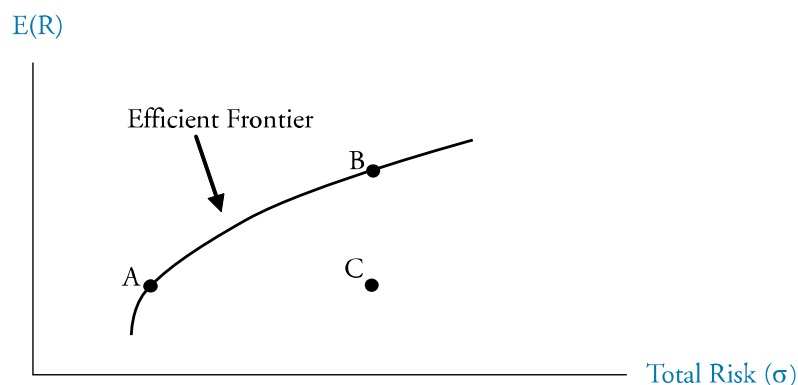
# Measures of Financial Risk

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## Mean-Variance Framework

- The traditional mean-variance model estimates the amount of financial risk for portfolios in terms of the portfolio's expected return (mean) and risk (standard deviation)
- The **efficient frontier** is the set of portfolios that dominates all other portfolios in the investment universe of risky assets with respect to risk and return

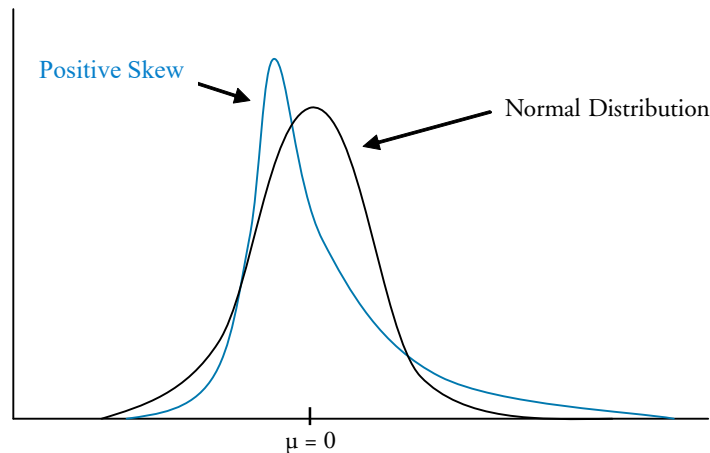
## Mean-Variance Framework (cont.)



## Mean-Variance Limitations

- A necessary assumption for this model is that return distributions for the portfolios are elliptical distributions (i.e., normal or near-normal)
- The mean-variance framework is unreliable when the underlying return distribution is not normal or elliptical
  - Standard deviation does not capture the probability of obtaining undesirable return outcomes when the underlying return density function is not symmetrical

## Mean-Variance Limitations (cont.)



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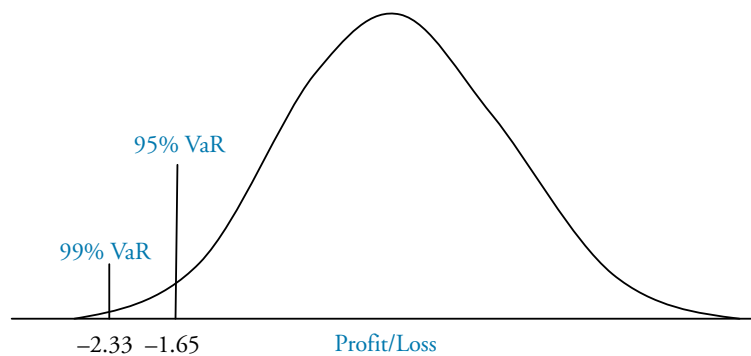
## Value at Risk

- VaR measurements work well with elliptical return distributions such as the normal distribution
- VaR is able to calculate the risk for non-normal distributions; however, VaR estimates may be unreliable in this case
- Limitations in implementing VaR for determining risk also result from selection of arbitrary confidence level and holding period, and the inability to calculate the magnitude of losses

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## Value at Risk (cont.)



## Coherent Risk Measures

- In order to properly measure risk, one must first clearly define what is meant by a measure of risk
- The properties of a **coherent risk measure** are [ $\rho(\cdot)$  = risk measure for random events]:
  - Monotonicity:  $Y \geq X \rightarrow \rho(Y) \leq \rho(X)$
  - Subadditivity:  $\rho(X + Y) \leq \rho(X) + \rho(Y)$
  - Positive homogeneity:  $\rho(hx) = h\rho(X)$  for  $h > 0$
  - Translation invariance:  $\rho(X + n) = \rho(X) - n$

## Coherent Risk Measures (cont.)

- The property of **subadditivity** states that a portfolio made up of sub-portfolios will have equal or less risk than the sum of the risks of each individual sub-portfolio
- This assumes that when individual risks are combined, there may be some diversification benefits
- VaR violates this coherent risk measure when the return distribution is not elliptical

## Expected Shortfall

- Value at risk can be defined as the minimum percent loss, equal to a pre-specified worst case quantile return
- **Expected shortfall** (ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return
- In other words, expected shortfall is the mean percent loss among the returns falling below the  $q$ -quantile
- Expected shortfall is also known as **conditional VaR** or **expected tail loss** (ETL)

## Expected Shortfall (cont.)

Expected shortfall (ES) is a more accurate risk measure than VaR for the following reasons:

- ES satisfies all the properties of coherent risk measurements
- ES is more appropriate for solving portfolio optimization problems than the VaR method
- ES gives an estimate of the magnitude of a loss for unfavorable events. VaR provides no estimate of how large a loss may be.
- ES has less restrictive assumptions regarding risk/return decision rules

## Sample Exam Question

With 180 observations, Bank X's VaR at the 95% confidence level is \$5 million. Loss observations beyond this point are (in millions): \$6, \$7, \$11, \$13, \$15, \$18, \$21, \$24, and \$35. What is the conditional VaR?

- a. \$9 million.
- b. \$32 million.
- c. \$14 million.
- d. \$17 million.

## Answer

- Answer: **d**
- Conditional VaR (or expected loss) is the average of the losses that are beyond the VaR level

## Example: Coherent Risk Measures

Which coherent risk measure property infers that if a random future value  $R_1$  is always greater than a random future value  $R_2$ , then the risk of the return distribution for  $R_1$  will be less than the risk of the return distribution for  $R_2$ ?

## Answer

- Monotonicity:  $Y \geq X \rightarrow \rho(Y) \leq \rho(X)$
- Monotonicity infers that if a random future value  $R_1$  is always greater than a random future value  $R_2$ , then the risk of the return distribution for  $R_1$  is less than the risk of the return distribution for  $R_2$