Measuring Returns, Volatility, and Correlation

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Volatility, Variance, and Implied Volatility

- The **volatility** of a variable is the standard deviation of that variable's continuously compounded return
- The **variance rate** of a variable is the square of its standard deviation (variance and standard deviation are computed using historical data).
- Risk managers may also compute **implied volatility**, which is the volatility that forces a model price (i.e., option pricing model) to equal the market price.

Nonnormal Distributions

- The third moment is the skewness and the fourth moment is the kurtosis.
- For a normal distribution, which has thin tails and is symmetric, there is no skewness or excess kurtosis.
- Financial returns often follow a nonnormal distribution, and as such, there is skewness and excess kurtosis.

Jarque-Bera Test

- The Jarque-Bera (JB) test statistic can be used to test whether a distribution is normal, meaning that there is zero skewness and no excess kurtosis (K – 3 = 0).
- If the result falls below the critical value, the null will not be rejected and the distribution will be deemed normal.
- If the result is above the critical value, the null will be rejected. Financial returns are likely to follow a more normal distribution over longer time periods.

Jarque-Bera Test (cont.)

- The test statistic, where *T* is the sample size, is:

$$JB = (T-1)\left(\frac{\hat{S}^2}{6} + \frac{\left(\hat{K}-3\right)^2}{24}\right)$$

The Power Law

- The power law is an alternative approach to using probabilities from a normal distribution.
- When X is **large**, probability of variable V exceeding X is:

$$P(V > X) = K \times X^{-\alpha}$$

- K and α are constants obtained from regression.
- Applies to variables such as incomes, visits to a website, etc.

The Power Law: Example

- Suppose that α = 4 and K = 40 for a particular financial variable, V. What is the probability of the variable exceeding 10?

$$P(V > 10) = 40 \times 10^{-4} = 0.4\%$$

Spearman's Rank Correlation

- Spearman's
  - Nonparametric approach
  - Based on rankings of values
  - Four-step process
    1. Order the set pairs of variables *X* and *Y* with respect to the set *X*
    2. Determine the ranks of X_i and Y_i for each time period *i*
    3. Calculate the difference of the variable rankings and square the difference
    4. Sum squares and plug in formula

Spearman's Rank Correlation (cont.)

Year	*X*	*Y*	*X Rank*	*Y Rank*	d_i	d_i^2
2012	–20.0%	10.0%	1	2	–1	1
2014	–10.0%	30.0%	2	4	–2	4
2010	25.0%	–20.0%	3	1	2	4
2013	40.0%	20.0%	4	3	1	1
2011	60.0%	40.0%	5	5	0	0
					Sum =	10

Kendall's Tau

- Kendall's Tau
  - Ordinal like Spearman's Rank
  - Requires classification of concordant and discordant pairs
    - A concordant pair of observations is when the rankings of two pairs are in agreement: Xt < Xt* and Yt < Yt* or Xt > Xt* and Yt > Yt* when t ≠ t*
    - A discordant pair of observations is when the rankings of two pairs are not in agreement: Xt < Xt* and Yt > Yt* or Xt > Xt* and Yt < Yt* when t ≠ t*
    - Plug into formula, but a little "brain intensive" for exam