## 2. INTEREST RATES AND TIME VALUE OF MONEY

The time value of money equates cash flows from different dates. Since cash now is preferred over future cash, we use an interest rate, r, to compare financial instruments with varied payment timings.

If USD 9,524 today equals USD 10,000 in a year, the difference, USD 476, is the compensation for the delay. This results in an interest rate of 5%.

Interest rates can be interpreted in **three** ways.

1) **Required rates of return:** It refers to the minimum rate of return that an investor must earn on his/her investment.

2) **Discount rates:** Interest rate can be interpreted as the rate at which the future value is discounted to estimate its value today.

3) **Opportunity cost:** Interest rate can be interpreted as the opportunity cost which represents the return forgone by an investor by spending money today rather than saving it. For example, an investor can earn 5% by investing $1000 today. If he/she decides to spend it today instead of investing it, he/she will forgo earning 5%.

**Interest rate = r =** Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

### Determinants of Interest Rates

- **Real risk-free interest rate:** It reflects the single-period interest rate for a completely risk-free security when no inflation is expected.
- **Inflation premium:** It reflects the compensation for expected inflation.
  ***Nominal risk-free rate*** *= Real risk-free interest rate + Inflation premium*
  - E.g., interest rate on a 90-day U.S. Treasury bill (T-bill) refers to the nominal interest rate.
- **Default risk premium:** It reflects the compensation for default risk of the issuer.
- **Liquidity premium**: It reflects the compensation for the risk of loss associated with selling a security at a value less than its fair value due to high transaction costs.
- **Maturity premium**: It reflects the compensation for the high interest rate risk associated with long-term maturity.

**Practice: Example 1 from the CFA Institute's Curriculum.**

## 3. RATES OF RETURN

Risk and return are the two most important characteristics of an investment.

### Return

Financial assets generate two types of return for investors.

1) Periodic income i.e. cash dividends or interest payments.
2) Capital gain or loss i.e. changes in the price of a financial asset.

**Note:**
Some assets provide return only through price movement or through periodic income (e.g., non-dividend paying stocks, retirement annuities etc.)

## 1) Holding Period Return (HPR)

A holding period return refers to the return earned by an investor from holding an asset for a single specified period of time e.g. 1 day, 1 week, 1 month, 5 years etc.

$$Total\ return = Capital\ gain\ (or\ loss) yield + Dividend\ yield$$

$$R = \frac{(P_1 - P_0) + I_1}{P_0} - 1$$

*where,*

$P_0$ = *price at the beginning of period*

$P_1$ = *price at the end of period*

$I$ = *income*

**NOTE:**

HPR for period longer than one year. For example, the holding period return for 3-year can be computed as follows:

Three year holding period return = $[(1 + R_1) \times (1 + R_2) \times (1 + R_3)] - 1$

where, $R_i$ = annual return

## 2) Arithmetic or Mean Return

Arithmetic mean (AM) is the sum of all returns divided by the total number of observations.

$$\bar{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i.T-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

- The AM is the ***average*** of the returns earned on a ***unit*** of investment at the ***beginning*** of each holding period.
- AM reflects a ***constant*** dollar investment at the beginning of each time period.

**Advantages:**

- The arithmetic mean return is easy to compute.
- It is the most commonly used value in statistics.
- It has known statistical properties i.e. standard deviation (S.D.).

**S.D.** reflects the dispersion of the observations around the mean. It can be used to determine whether the mean return is statistically different from zero.

## 3) Geometric Mean Return

$$\overline{R}_{Gi} = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times (1 + R_{iT-1}) \times (1 + R_{iT})} - 1$$

*where,*

$R_{it}$ = *return in period t*

$T$ = *total number of periods*

- Geometric mean (GM) measures the average or compound growth rate over ***multiple*** periods.
- The GM return reflects compounding of returns and it is assumed that the investment amount is not reset at the beginning of each year. Thus, GM reflects a "buy-and-hold" strategy.

***Advantage:***

- GM return provides a more accurate measure of return that an investor will earn than an AM return.

**Important to Note:**

- If all returns (values) are identical ➔ the geometric mean = arithmetic average.
- If the return values are volatile ➔ the geometric mean < arithmetic average.
- When the actual holding period returns are not equal, ➔ AM is biased upwards.
- The greater the volatility of returns, the greater the difference between geometric mean and arithmetic average.

**Geometric mean versus Arithmetic mean:**

- The geometric mean return represents the growth rate or compound rate of return on an investment.
- The arithmetic mean return represents an average single-period return on an investment.
- The geometric mean is *always* ≤ arithmetic mean.

- When there is no variability in the observations (i.e. when all the observations in the series are the same), geometric mean = arithmetic mean
- The greater the variability of returns over time, the more the geometric mean will be lower than the arithmetic mean.
- The geometric mean return decreases with an increase in standard deviation (holding the arithmetic mean return constant).

In addition, the geometric mean ranks the two funds differently from that of an arithmetic mean.

**Practice: Example 2,3,4,and 5 from the CFA Institute's Curriculum.**

## 4) The Harmonic Mean

$$Harmonic\ Mean\ \bar{X}_H = n / \sum_{i=1}^{n} \left(\frac{1}{X_i}\right)$$

with $X_i > 0$ for i = 1,2, …, n.

- It is a special case of the weighted mean in which each observation's weight is inversely proportional to its magnitude.

**Cost Averaging** is an investment strategy involving periodic investments of fixed amount of money. Harmonic mean is appropriate when averaging the ratios, and the ratios are repeatedly applied to a fixed quantity to yield a variable number of units.

In cost averaging, the ratios to be averaged are prices per share at the date of the purchase, and then apply those prices to a constant amount of money to yield a variable number of shares.

**Important to note:**

- Harmonic mean formula cannot be used to compute average price paid when different amounts of money are invested at each date.
- When all the observations in the data set are the same, geometric mean = arithmetic mean = harmonic mean.
- When there is variability in the observations, harmonic mean < geometric mean < arithmetic mean.

**Practice: Example 6 and 7 from the CFA Institute's Curriculum.**

Besides arithmetic, geometric, and harmonic means, **trimmed** and **winsorized means** are used to reduce the influence of outliers in a dataset.

**Outliers** are extreme values (outliers) in a dataset may reflect a rare value in the population or an error.

## 5) Trimmed Mean

Trimmed Mean is the arithmetic mean of the distribution computed after excluding a stated small % of the lowest and highest values.

## 6) Winsorized Mean

In a winsorized mean, a stated % of the lowest values is assigned a specified low value and a stated % of the highest values is assigned a specified high value and then a mean is computed from the restated data.

E.g., in a 95% winsorized mean,

- The bottom 2.5 % of values are set = $2.5^{th}$ percentile value.
- The upper 2.5% of values are set = $97.5^{th}$ percentile value.

**Practice: Exhibit 8 and Question-Set from the CFA Institute's Curriculum.**

# 4. MONEY-WEIGHTED AND TIME-WEIGHTED RETURN

The money-weighted rate of return (MWR) measures the compound growth rate in the value of all funds invested in the account over the entire evaluation period. It represents an internal rate of return (IRR) of an investment. Like IRR,

- Amounts invested are cash *outflows* for the investor.
- Amounts returned or withdrawn by the investor are a cash *inflow* for the investor.
- The money that remains at the end of an investment cycle is a cash *inflow* for the investor.

The IRR is the discount rate at which the sum of present values of these cash flows = 0. IRR is computed as follows:

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

*where,*
*T = number of periods*
*$CF_t$ = cash flow at time t*

***Example:*** Assume,

- Amount invested in a mutual fund at the beginning of 1st year = $100
- Amount invested in a mutual fund at the beginning of 2nd year = $950
- Amount withdrawn at the end of 2nd year = $350
- Balance at the end of year 3 = +1270

$CF_0$ = –100
$CF_1$ = –950
$CF_2$ = +350
$CF_3$ = +1,270

$$\frac{CF_0}{(1+IRR)^0} + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} = \frac{-100}{1} + \frac{-950}{(1+IRR)^1} + \frac{+350}{(1+IRR)^2} + \frac{+1270}{(1+IRR)^3} = 0$$

IRR = 26.11%

**Important to note:**

- When funds are contributed to an account prior to a period of strong or positive (negative) performance, MWR> (<) AM and GM.
- When funds are withdrawn from an account prior to a period of strong or positive (negative) performance, MWR< (>) AM and GM.

**Limitation of money weighted return:** It cannot be used to compare return between different individuals or different investment opportunities.

**Refer to:** Example below Exhibit 11, CFA Institute's Curriculum.

**Practice:** Example 8 from the CFA Institute's Curriculum.

## Time-Weighted Returns

The time-weighted rate of return (TWR) measures the compound rate of growth over a stated evaluation period of one unit of money initially invested in the account.

- In TWR, the account needs to be valued whenever an external cash flow occurs.
- TWR measures the actual rate of return earned by the portfolio manager.
- TWR is preferred to use to evaluate the performance of the portfolio manager *when the manager has no control over the deposits and withdrawals made by clients.*

**Three steps to compute an exact TWR of return on a portfolio are as follows:**

i) Compute portfolio value prior to any significant cash inflows/outflows.
ii) Compute holding period return for each sub-period.
iii) Link together sub-period returns to compute to compute the TWR for the entire evaluation period.

**Note:** For investment period >1year, geometric mean of annual return is used to calculate the TWR.

TWR requires determining a value for the account each time any cash flow occurs which can be costly.

One way to handle this issue is to calculate a reasonable approximation by valuing the portfolio at frequent interval. Higher the frequency, more accurate the approximation. However, marking to market an account on a daily basis is administratively more cumbersome

Time weighted return = $r_{twr} = [(1 + rt,1) \times (1 + rt,2) \times \ldots \times (1 + rt,n)]^{1/N} - 1$

**Practice: Example 9 and 10 CFA Institute's Curriculum.**

## 5. ANNUALIZED RETURN

A return for a period less than one year can be annualized by compounding it by the number of periods in a year.

$$r_{annual} = (1 + r_{period})^{c} - 1$$

*where,*
*c = number of periods in a year*
*e.g. c = 4 for a quarter, c= 12 for a month.*

**Note**

- A monthly return is compounded 12 times
- A weekly return is compounded 52 times
- A quarterly return is compounded 4 times
- A daily returns is compounded 365 times

$$r_{weekly} = (1 + r_{daily})^{5} - 1;$$
$$r_{weekly} = (1 + r_{annualy})^{1/52} - 1$$

**Example:**
Suppose, weekly return = 0.2%.Then the compound annual return is computed as follows:

$$r_{annual} = (1 + r_{weekly})^{52} - 1 = (1 + 0.2\%)^{52} - 1 = (1.002)^{52} - 1 = 0.1095 = 10.95\%$$

Suppose, return for 15 days = 0.4%, then the compound annual return is:

$$r_{annual} = (1 + r_{15})^{365/15} - 1 = (1 + 0.4\%)^{365/15} - 1 = (1.004)^{365/15} - 1 = 0.1020 = 10.20\%$$

Suppose, return for 18-month = 20%, then the compound annual return is:

$$r_{annual} = (1 + r_{18_{month}})^{2/3} - 1 = (1 + 0.20)^{2/3} - 1 = (0.1292) = 12.920\%$$

***Benefit:*** By annualizing returns, we can compare different assets and over different time periods.

***Limitation:*** When returns are annualized, it is assumed that money can be reinvested repeatedly and earn a similar return.

### Continuously Compounded Rates of Return

***It is important to note*** that when a stock's continuously compounded return is normally distributed, then future stock price is necessarily lognormally distributed.

$$S_T = S_0 \exp(r_{0,T})$$

Where,
exp = e
$r_{0,t}$ = Continuously compounded return from 0 to T

- Since $S_T$ is proportional to the log of a normal random variable → $S_T$ is lognormal.

**Price relative** = Ending price / Beginning price = $S_{t+1}/ S_t = 1 + R_{t, t+1}$

*where,*
$R_{t,\,t+1}$ *= holding period return on the stock from t to t + 1.*

**Continuously compounded return associated with a holding period from t to t + 1:**

$r_{t,\,t+1}$= ln(1 + holding period return)

*Or*

$r_{t,\,t+1}$ = ln(price relative) = ln $(S_{t+1} / S_t)$ = ln $(1 + R_{t,t+1})$

**NOTE:** *The continuously compounded return < associated holding period return.*

**Continuously compounded return associated with a holding period from 0 to T:**

$R_{0,T}$= ln $(S_T / S_0)$

*Or*

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

Where,
$r_{T\text{-}l,\,T}$ = One-period continuously compounded returns

**Example:** Suppose, one-week holding period return = 0.04.

Equivalent continuously compounded return =
One-week continuously compounded return = ln (1.04)
= 0.039221

**Practice:** Example 11, 12 and 13 from the CFA Institute's Curriculum.

## 6. OTHER MAJOR RETURN MEASURES AND THEIR APPLICATIONS

### Gross and Net Return

**Gross return** = Return – trading expenses (e.g. commissions) – any other expense directly related to the generation of returns

- Gross return is an appropriate measure to evaluate and compare the investment skill of asset managers because it excludes any fees related to the management and administration of an investment.
- Expenses that are not directly related to the generation of returns include management expenses, custodial fees, taxes etc.

**Net Return** = Gross Return – all managerial and administrative expenses

- Net return reflects the return that is actually earned by investors.

### Pre-Tax and After-Tax Nominal Return

Capital gains and income return are usually taxed differently. Typically,

- Long-term capital gains receive preferential tax treatment.
- Interest income is taxed as ordinary income.
- Dividend income can be taxed as ordinary income, can have a lower tax rate, or can be exempt from taxes.

**After-tax nominal return** = Total return –any allowance for taxes on dividends, interest & realized gains

Tax liability can be minimized by:

- Selecting securities that receive favorable tax treatment.
- Reducing trading turnover.

### Real Returns

$$(1 + r) = (1 + r_{rF}) \times (1 + \pi) \times (1 + RP)$$
$$(1 + r_{real}) = (1 + r_{rF}) \times (1 + RP) \; or$$
$$(1 + r_{real}) = (1 + r) \div (1 + \pi)$$

*where,*

$r$ = *Nominal return*

$r_{rF}$ = *Real risk-free return* ➔ *It reflects a compensation for postponing current consumption.*

$\pi$ = *Inflation* ➔ *It reflects a compensation for loss of purchasing power.*

*RP* = *Risk premium* ➔ *It is a compensation for assuming risk.*

**After-Tax Real Return:** The after-tax real return represents the return that is received by an investor in return for postponing consumption and assuming risk after paying taxes on investment returns.

**Benefits of using Real Returns:**

- Real returns can be used to compare returns across time periods.
- Real returns can be used to compare returns among countries when returns are expressed in local currencies.
- After-tax real return can be used as a benchmark for making investment decisions. However, it is not commonly estimated because of the difficulty to estimate a general tax component applicable to all investors. Tax component depends on various factors i.e.
  - Investor's marginal tax rate.
  - Investment time horizon i.e. long-term v/s short-term
  - Type of account the asset is held in e.g. tax-exempt, tax-deferred, or normal.

## Leveraged Return

There are **two** ways to leverage a return.

1) **Using futures contracts**. For example, if 5% of the notional value of the asset is invested, then the leveraged return i.e. the return on investor's own money (whether positive or negative) is 20 times the actual return of the underlying security.
2) **Borrowing money to purchase the asset**. For example, if 50% of the amount invested is borrowed, then the asset return (both gains and losses) to the investor is doubled.

**Practice:** Example 14 and 15 from the CFA Institute's Curriculum.

**Practice:** End of Chapter Practice Problems from the CFA Institute's Curriculum and Questions from FinQuiz Question-bank.