LM01 Rates and Returns

Interpretation of interest rates

Interest rates can be interpreted as:

- **Required rate of return:** If you invest $100 today on the condition that you get $110 after a year, then your required rate of return is 10%.
- **Discount rate**: For the same example, if you discount the future cash flow of $110 using a discount rate of 10%, you get a present value of $100.
- **Opportunity cost:** If you spend the $100 today instead of investing, then you lose the opportunity of earning 10% interest. In this sense interest rates can also be thought of as opportunity costs.

Components of interest rates

Interest rates have the following components:

- Real risk-free rate: Return on an investment with zero risk, assuming no inflation.
- Inflation premium: Extra return required to compensate for inflation.
- Default risk premium: Extra return required to compensate for the risk that the borrower will not make the promised payments.
- Liquidity premium: Extra return required to compensate for the risk of receiving less than the fair value of an investment if it must be sold quickly for cash.
- Maturity premium: This is the premium that investors demand on a security with a long maturity. The maturity premium compensates investors for the increased sensitivity of the market value of debt to a change in market interest rates as maturity is extended.

Interest rate = real risk-free rate + inflation premium + default risk premium + liquidity premium + maturity premium.
Nominal interest rate = real risk-free rate + inflation premium

In a country ABC, the real risk-free rate is 4% and the expected inflation is 3%. Company X domiciled in this country issues a 5-year bond with an estimated default risk premium of 2%, liquidity premium of 1%, and maturity premium of 1%. Calculate the interest rate of this bond.

Solution:

The interest rate for this bond will be 4 + 3 + 2 + 1 + 1 = 11%.

Major return measures

Holding period return (HPR) is the percentage return earned on an investment over a given period.

$$\text{HPR single period} = \text{capital gain} + \text{periodic income} = \frac{P_T - P_0 + D_T}{P_0}$$

where:
P_T = price at the end of the period
P_0 = price at the begining of the period
I_T = Income earned during period T.

A stock valued at $40 at the start of the period pays out $2 as dividend and has a value of $44 at the end of the period. Compute the HPR.
Solution:
HPR single period $= \frac{P_T - P_0 + D_T}{P_0} = \frac{44-40+2}{40} = 15\%$

Arithmetic mean return is a simple arithmetic average of a series of returns.

$$AM = \frac{(R_1 + R_2 + R_3 + \ldots + R_T)}{T}$$

Geometric mean return is the compounded annual rate of return for a series of returns.

$$GM = [(1 + R_1) * (1 + R_2) * \ldots * (1 + R_T)]^{\frac{1}{T}} - 1$$

An investor invested $100 in a mutual fund which had the following returns over a three-year period: -6%, 8%, 14%. Compute the HPR, arithmetic mean return, and geometric mean return.
Solution:
Ending value = (100)*(0.94)*(1.08)*(1.14) = $115.73
HPR = (0.94)*(1.08)*(1.14)-1 = 15.73%
Arithmetic mean return $= \frac{(-6\% + 8\% + 14\%)}{3} = 5.33\%$
Geometric mean return $= \sqrt[3]{(0.94) * (1.08) * (1.14)} - 1 = 4.99\%$

The **harmonic mean** is a special type of weighted mean in which an observation's weight is inversely proportional to its magnitude. The harmonic mean is used to find average purchase price for equal periodic investments.

$$X_H = n / \sum_{i=1}^{n} \left(\frac{1}{X_i}\right)$$

An investor purchased $1,000 worth of stock A each month for the past three months at prices of $5, $6, and $7. Calculate the average purchase price of the stock.
Solution:
Average purchase price = 3 / (1/5 + 1/6 + 1/7) = 5.88

Which mean to use?

- Arithmetic mean: Should be used with single period or cross-sectional data.
- Geometric mean: Should be used with time-series data.
- Harmonic mean: Should be used to find average purchase price for equal periodic

investments.

- Trimmed mean: Should be used when the data has extreme outliers.
- Winsorized mean: Should be used when the data has extreme outliers.

Comparison of money-weighted and time-weighted rates of return

Money-weighted return is the internal rate of return on money invested that considers all the cash inflows and cash outflows. It is similar to the internal rate of return (IRR).

An investor purchases XYZ stock at the start of the first year for $20 and ABC stock at the end of the first year for $30. Both the stocks pay year-end dividend of $1 per share. If the investor sold both the shares for $60 at the end of the second year, compute the money-weighted rate of return.

Solution:

Step1:

Determine the cash flows

Time period	Particulars	Cash flow
T = 0	Purchase of XYZ stock	-$20
T = 1	Purchase of ABC stock Dividend from XYZ stock	-$30 +$1
T = 2	Sale of both shares Dividend from both stocks	+$60 +$2

Step2:

$PV_{inflows} = PV_{outflows}$

$$\frac{62}{(1+r)^2} = \frac{-29}{(1+r)} - 20$$

Alternatively,

Input CF_0 = -20, CF_1 = -29, CF_2 = 62

The money-weighted rate of return is 17.91%.

Time-weighted rate of return is the compound growth rate at which $1 invested in a portfolio grows over a given measurement period.

An investor buys a stock for $10 at time t=0. At the end of Year 1, he receives a dividend of $1 and purchases another stock for $12. At the end of Year 2, he receives a dividend of $0.5 per share and sells both shares for $13. Calculate the time-weighted rate of return.

Solution:

1. Break the measurement period into two sub-periods based on the timing of the cash flows.

Holding period 1	Beginning value = $10 Dividends paid = $1 Ending value = $12

Holding period 2	Beginning value = \$24 (12 x 2) Dividends paid = \$1 (0.5 x 2) Ending value = \$26 (13 x 2)

2. Compute the HPY for each sub-period.
   $HPY_1 = (12 - 10 + 1)/10 = 30\%$
   $HPY_2 = (26 - 24 + 1)/24 = 12.5\%$
3. Calculate the compounded annual rate by taking the geometric mean of the two sub-periods.
   $(1 + TWRR)^2 = 1.30 \times 1.125$; TWRR = 20.93%

Money-weighted v/s time-weighted returns

- The money-weighted rate of return is impacted by the timing and amount of cash flows.
- The time-weighted rate of return is not impacted by the timing and amount of cash flows.
- The time-weighted return is an appropriate performance measure if the portfolio manager does not control the timing and amount of investment.
- On the other hand, money-weighted return is an appropriate measure if the portfolio manager has control over the timing and amount of investment.

Annualized return measures and continuously compounded returns

Annualized return converts the returns for periods that are shorter or longer than a year, to an annualized number for easy comparison.

$$\text{Annualized return} = \left(1 + r_{\text{period}}\right)^{t} - 1$$

where: t = number of periods in a year

The **continuously compounded return** associated with a holding period return can be calculated as:

- Natural logarithm of one plus that holding period return, or
- Natural logarithm of the ending price over the beginning price

If the holding period return of a stock was 10% for a period of one year, what is the equivalent continuously compounded rate of return for the year?

Solution:

r = ln (0.1 + 1) = 0.0953 = 9.53%

Key strokes for calculating ln(1.1) are

1.1 [ln]

Other major return measures and their applications

Gross return is the return earned by an asset manager prior to deducting management

fees and taxes. It measures investment skill.

Net return accounts for all managerial and administrative expenses is what the investor is concerned with.

Pre-tax nominal return is the return before accounting for inflation and taxes; this is the default, unless otherwise stated.

After-tax nominal return is the return after accounting for taxes.

Real return is the return after accounting for taxes and inflation.

Leveraged return is the return earned by the investor on his money after accounting for interest paid on borrowed money.