LM01 Returns of Financial Assets and Instruments

Version 1.0

Introductory Note

Financial Calculator: CFA Institute allows only two calculator models during the exam:
- Texas Instruments BA II Plus (including BA II Plus Professional) and
- Hewlett Packard 12C (including the HP 12C Platinum, 12C Platinum 25th Anniversary Edition, 12C 30th Anniversary Edition, and HP 12C Prestige)

Unless you are already comfortable with the HP financial calculator, we recommend using the Texas Instruments financial calculator. Explanations and keystrokes in our study materials are based on the Texas Instruments BA II Plus calculator. Before you start using the calculator to solve problems, we recommend that you set the number of decimal places to 'floating decimal.'

1. Introduction: Returns of Financial Assets and Instruments

The Quantitative Methods learning modules cover concepts that form the fundamental building blocks of investment decision-making and investment strategies. The concepts you learn here will reappear throughout the three levels of the curriculum.

This learning module covers:
- Types of returns provided by financial assets.
- Common return measures and how to calculate them.

2. Financial Returns

We begin our discussion by understanding the difference between 'financial assets,' 'financial instruments,' and 'financial indicators.'

- **Financial assets**: Financial assets represent various forms of value held by individuals or entities. For example, cash, equity, debt, etc.
- **Financial instruments**: Financial instruments are the means through which financial assets are standardized, packaged, and traded in financial markets. For example, stocks, bonds, derivatives, etc. The standardization facilitates the efficient exchange of these instruments between investors.
- **Financial indicators**: Financial indicators are observable but not directly tradable indicators of value. Unlike financial assets and instruments, financial indicators do not generate cash flows. For example, exchange rates, interest rates, market indexes, etc.

The S&P 500 index is a financial indicator, whereas an ETF based on the S&P 500 index is a financial instrument. Both represent a financial asset: the equity of the 500 largest publicly listed corporations in the United States.

Instructor's Note: The difference between financial assets and financial instruments is subtle, and both terms are often used interchangeably.

2.1 Total Return

Financial assets normally generate returns in two ways:

- **Capital appreciation return**: Also called 'price return,' it is the change in price between two different points of time. It can be expressed as:

  $$r_{price} = \frac{P_1 - P_0}{P_0}$$

  where:
  P_0 is the price when the initial investment is made.
  P_1 is the price at a later valuation date.

- **Capital distribution return**: It is the income earned through dividends, coupon payments, etc. It can be expressed as:

  $$r_{distribution} = \frac{Inc}{P_0}$$

  A metric used to represent the capital distribution of equities is 'dividend yield,' calculated as the annual dividends paid by a stock divided by its market price.

  A metric used to represent the capital distribution of bonds is 'current yield,' calculated as the annual coupon payments divided by the market price of the bond.

The **total return** takes into account both sources of return. It is expressed as:

$$r = r_{price} + r_{distribution}$$

$$r = \frac{P_1 - P_0}{P_0} + \frac{Inc}{P_0} = \frac{P_1 - P_0 + Inc}{P_0}$$

Some financial assets may provide return through only one of these sources. For example, non-dividend-paying stocks provide return solely through price movements, whereas a life annuity (an insurance product that provides regular income for the remainder of an individual's life in exchange for a lump sum premium) provides return only through the income component.

Expected returns vs. actual returns

Expected returns (or ex ante returns) are the anticipated returns on an investment over a specified period. Say you shortlist a stock for investment; based on historical data, current market conditions, and analysis of the stock, you forecast a return of 15% over the next one year. The 15% is the expected return on the stock. This return is not guaranteed.

Actual returns (or ex post returns) are the returns on an investment after the investment period has ended. Assume that after one year, the stock returns were 5%. The 5% is the actual return on the stock. Actual returns may differ from expected returns due to changes in economic conditions, market volatility, or unforeseen events affecting the asset.

Investors demand a risk premium to compensate for the difference between expected returns and actual returns. The greater the likelihood that actual returns will fall short of expected returns, the more the compensation investors will seek.

Unrealized returns vs. realized returns

Unrealized returns ("paper" gains/losses) are potential profits or losses on investments that you currently hold but have not yet sold. These values fluctuate based on market prices, representing the difference between the current market value and the original purchase price.

Realized returns are actual, locked-in profits or losses from selling an asset. In most cases, taxes are applied only on realized returns.

Example:
(*This is based on the case study from the curriculum.*)

The following information is provided for a stock.

Date	**Price per share**	**Dividend per share**
31 December 2016	267.60	-
31 December 2017	338.70	6.00
31 December 2018	435.70	9.00
31 December 2019	689.00	9.75
31 December 2020	1,243.50	10.50
31 December 2021	835.20	11.50
31 December 2022	631.30	12.50
31 December 2023	374.30	13.50

Calculate the price return, capital distribution return, and total return for this stock in 2020.

Solution:

Price return:

The price return is the change in price between two different points of time. The price at the beginning of 2020 is the same as the price at the end of 2019; hence, P_0 is 689.00 and P_1 is 1,243.50.

$$r_{price} = \frac{P_1 - P_0}{P_0} = \frac{1{,}243.50 - 689.00}{689.00} = 80.48\%$$

Capital distribution return:

The dividend for 2020 was 10.50. Therefore, the capital distribution return for 2020 can be calculated as:

$$r_{distribution} = \frac{Inc}{P_0} = \frac{10.50}{689.00} = 1.52\%$$

Total return:

The total return includes both sources of return.

$r = r_{price} + r_{distribution} = 80.48\% + 1.52\% = 82.00\%$

Holding period return is the return that an investor earns over a specified holding period. The holding period can range from days to years. In the above example, the holding period was one year, 2020. Using the same approach, we can calculate the holding period returns for 2021, 2022, and so on.

2.2 Arithmetic and Geometric Holding Period Return

Often investors need to calculate the aggregate returns across multiple holding periods. There are different methods to do this.

An **arithmetic mean return** is the simple average of all holding period returns. It is calculated as:

$$\bar{r}_i = \frac{r_{i1} + r_{i2} + \cdots + r_{i,T}}{T}$$

where:

T is the total number of periods

r_{it} is the return during a particular period t

A **geometric mean return** represents the average rate at which an asset's value grows over time, accounting for the compounding of returns. It is calculated as:

$$\bar{r}_{Gi} = [(1 + r_{i1}) \times (1 + r_{i2}) \times \ldots \times (1 + r_{iT})]^{\frac{1}{T}} - 1$$

where:

T is the total number of periods

r_{it} is the return during a particular period t

The geometric mean is a more accurate measure of investment performance, whereas, the arithmetic mean can be misleading. This is demonstrated in the following example. Suppose an investment yielded a 100% return in Year 1 and a -50% return in Year 2; i.e., if we started with an investment of 100, its value grew to 200 at the end of Year 1 and then dropped back to 100 at the end of Year 2. Let us calculate the arithmetic and geometric means for this scenario.

$$\text{Arithmetic mean } \bar{r}_i = \frac{100\% - 50\%}{2} = 25\%$$

$$\text{Geometric mean } \bar{r}_{Gi} = [(1 + 100\%) \times (1 - 50\%)]^{\frac{1}{2}} - 1 = 0\%$$

The geometric mean correctly indicates that the net return over two years is 0%, whereas the arithmetic mean incorrectly shows that the average return is 25%.

Example:
(*This is based on the case study from the curriculum.*)

The following information is provided for a stock.

Date	Price per share	Dividend per share	Total return (%)
31 December 2016	267.60	-	-
31 December 2017	338.70	6.00	28.81%
31 December 2018	435.70	9.00	31.30%
31 December 2019	689.00	9.75	60.37%
31 December 2020	1,243.50	10.50	82.00%
31 December 2021	835.20	11.50	−31.91%
31 December 2022	631.30	12.50	−22.92%
31 December 2023	374.30	13.50	−38.57%

Calculate the arithmetic and geometric average annual total return for this stock for 2017-2023.

Solution:

The arithmetic mean return can be calculated as:

$$\bar{r}_i = \frac{r_{i1} + r_{i2} + \cdots + r_{i,T}}{T}$$

$$\bar{r}_i = \frac{28.81\% + 31.30\% + 60.37\% + 82.00\% - 31.91\% - 22.92\% - 38.57\%}{7} = 15.58\%$$

The geometric mean return can be calculated as:

$$\bar{r}_{Gi} = [(1 + r_{i1}) \times (1 + r_{i2}) \times \ldots \times (1 + r_{iT})]^{\frac{1}{T}} - 1$$

$$\bar{r}_{Gi} = [(1 + 0.2881) \times (1 + 0.3130) \times (1 + 0.6037) \times (1 + 0.8200) \times (1 - 0.3191) \times (1 - 0.2292) \times (1 - 0.3857)]^{\frac{1}{7}} - 1 = 6.86\%$$

As discussed previously, the geometric mean is a more accurate measure of investment performance. Given the sequence of returns in the above example, $1 invested in this stock at the start of 2017 would have grown to:

$\$1 \times 1.2881 \times 1.3130 \times 1.6037 \times 1.8200 \times 0.6809 \times 0.7708 \times 0.6143 = 1.5915$

If the stock had grown at a constant 6.86% every year, $1 invested at the start of 2017 would have become:

$\$1 \times 1.0686^7 = 1.5911$

As expected, both scenarios give the same answer (the slight difference in the 4th decimal place is due to rounding.) The 6.86% represents the average growth in value or compound growth rate of the investment.

Arithmetic-geometric mean inequality principle: If the annual returns on an investment are constant, the geometric mean will be equal to the arithmetic mean. If the annual returns on an investment are volatile, the geometric mean will always be lower than the arithmetic mean (6.86% vs. 15.58% in our example.) The greater the variability of returns, the greater the difference between the arithmetic and geometric mean return.

2.3 Compounding: Daily, Monthly, Quarterly, and Annual Returns

To compare the performance of investments with different holding periods, we can annualize all available returns. The general equation to annualize returns is:

$$r_{\text{annualized}} = (1 + r_{\text{period}})^{c} - 1$$

The following example demonstrates this process.

Example:
(*This is based on the Knowledge Check example from the curriculum.*)

An investor wants to evaluate the returns of three recently formed ETFs. He gathers the following information.

ETF	**Time since Inception**	**Return since Inception (%)**
1	146 days	4.61
2	5 weeks	1.10
3	15 months	14.35

Assuming that a year has 365 days, or 52 weeks, or 12 months, which ETF has the highest annualized rate of return?

Solution:

The annualized return for ETF 1 is:

$1.0461^{365/146} - 1 = 11.93\%$

The annualized return for ETF 2 is:

$1.0110^{52/5} - 1 = 12.05\%$

The annualized return for ETF 3 is:

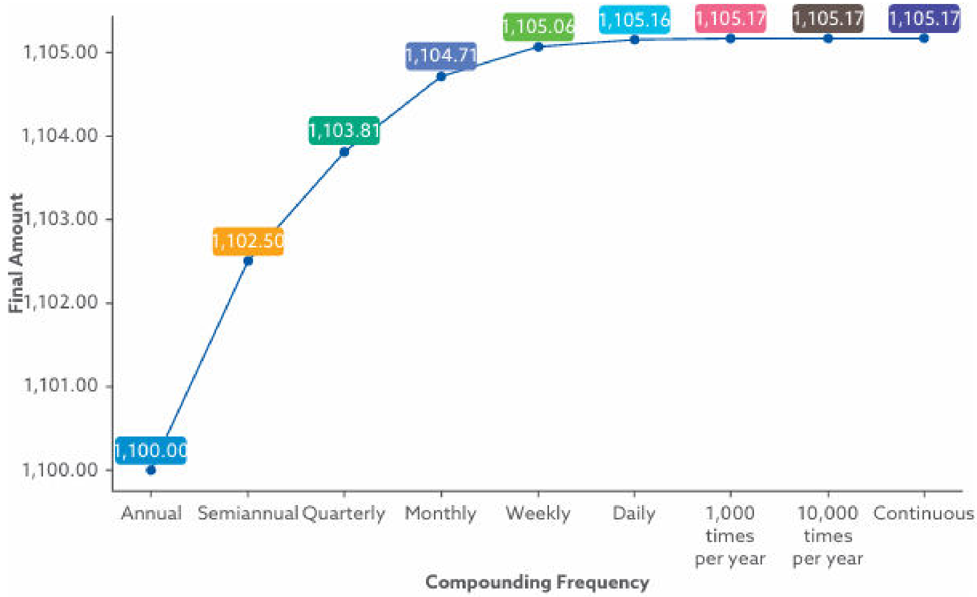
$1.1435^{12/15} - 1 = 11.32\%$

Therefore, ETF 2 has the highest annualized rate of return.

A major limitation of annualizing returns is the implicit assumption that money can be reinvested repeatedly while earning the same return. This process extrapolates short-term performance to the full twelve-month period and potentially overestimates annual performance. In our example, despite having the lowest value for the periodic rate, ETF 2 has the highest annualized rate of return because of the reinvestment rate assumption and the compounding of the periodic rate.

2.4 Continuous Compounding

Compound interest is usually calculated at set intervals, such as annually, quarterly, monthly, etc. The more frequently we compound returns, the higher the final amount will be, but there is a limit. Exhibit 10 from the curriculum demonstrates the impact of compounding frequency. We start with an initial amount of 1,000 and invest it at 10% at different frequencies of compounding.



As compounding becomes more frequent, the incremental gains from increasing frequency become infinitesimally small.

Continuous compounding is a theoretical financial concept where interest is calculated and reinvested into an account's principal balance an infinite number of times, rather than at specific intervals. It represents the maximum potential growth of an investment.

The continuously compounded return associated with a holding period return can be calculated as:

- Natural logarithm of one plus that holding period return, or
- Natural logarithm of the ending price over the beginning price

  i.e., $R_c = \ln(1 + R_t) = \ln(P_t/P_0)$

Example:

1. If the one-week holding period return is 4%, then the equivalent continuously compounded return is ln (1 + 0.04) = 3.922%.

2. A stock was purchased at t = 0 for $30. One period later at t = 1, the stock is valued at $34.50. The continuously compounded return over the period can be calculated as ln (34.50/30) = 13.976%.

A property of the continuously compounded rates of return is that they are additive. The continuously compounded return from period 0 to period T is the sum of the incremental one-period returns between 0 and T.

$$r_{0,T} = r_{0,1} + r_{2,3} + \cdots + r_{T-1,T}$$

Example:
(*This is based on the case study from the curriculum.*)

Exhibit 12 shows the month-end prices of a stock.

Date	Price per share
31 December 2019	689.0
31 January 2020	736.0
28 February 2020	695.4
31 March 2020	666.4
30 April 2020	688.4
29 May 2020	786.8
30 June 2020	765.4
31 July 2020	901.0
31 August 2020	882.6
30 September 2020	875.4
30 October 2020	1,014.5
30 November 2020	1,124.0
31 December 2020	1,243.5

1. What are the continuously compounded price returns for this stock for the semi-annual period 1 January 2020–30 June 2020?

Solution:

The continuously compounded price return for the period 1 January 2020 (using the price on 31 December, the previous year) through 30 June 2020 is

$$r_C = \ln\frac{P_1}{P_0} = \ln\frac{765.40}{689.00} = 10.52\%$$

This is the period return and not the annualized return.

2. What are the continuously compounded price returns for this stock for the semi-annual period 1 July 2020–31 December 2020?

Solution:

The continuously compounded price return for the period 1 July 2020–31 December 2020 is:

$$r_C = \ln\frac{P_1}{P_0} = \ln\frac{1{,}243.50}{765.40} = 48.53\%$$

3. What is the annual continuously compounded price return for this stock from 1 January 2020–31 December 2020?

Solution:

The continuously compounded price return for the period 1 January 2020–31 December 2020 is:

$$r_C = \ln\frac{P_1}{P_0} = \ln\frac{1{,}243.50}{689.00} = 59.04\%$$

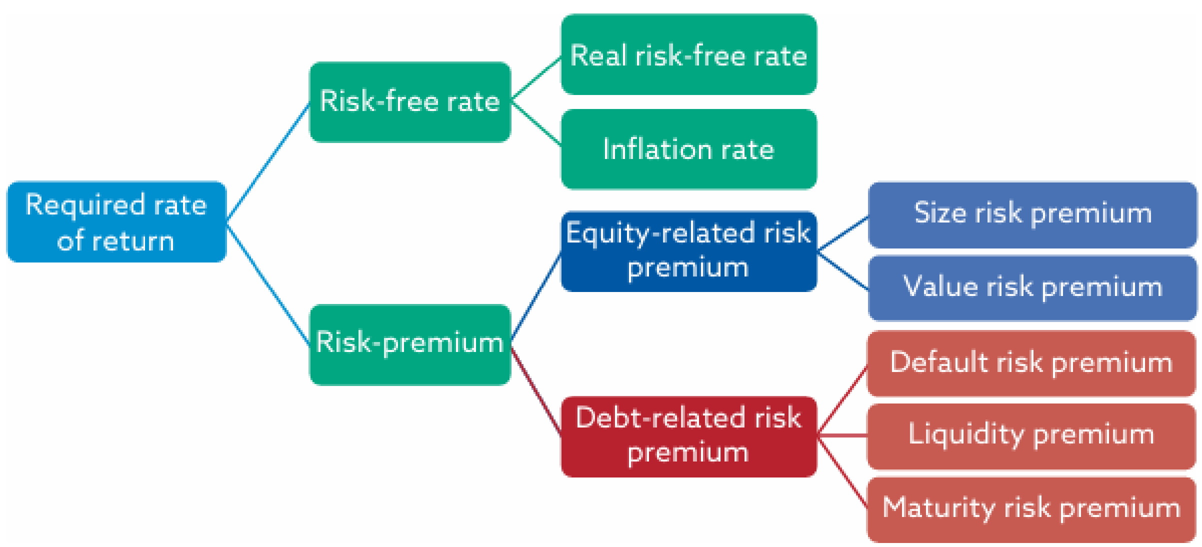
Note that adding the continuously compounded price return for the period 1 January 2020–30 June 2020 of 10.51% to the continuously compounded price return for the period 30 June 2020–31 December 2020 of 48.53% yields 59.04%, which is equivalent to the direction calculation. This finding points to a very desirable quality of log returns: They are additive.

3. Common Return Measures

3.1 The Required Rate of Return

The required rate of return is the minimum rate of return an investor must receive to accept an investment. This return depends on the risk characteristics of the investment. It is lower for safe investments and higher for risky investments.

We can break the required rate of return into different components as shown in Exhibit 13 of the curriculum.



The two main components are:

- Risk-free rate: This is the return on an investment without any default or reinvestment risk. E.g., short-term government bonds.

- Risk premium: It represents additional returns expected for bearing extra risks and varies according to the financial asset type, investment time horizon, and other specific risk factors. E.g., stocks, and corporate bonds. Stock investments have equity-related risk premiums; corporate bond investments have debt-related risk premiums.

Risk-free rate

Typically, the returns on short-term government bonds are considered as the risk-free rate. This is a nominal rate and represents the total return, combining the real risk-free rate (actual purchasing power growth) and the inflation premium.

The relationship between the nominal risk-free rate, r_f, the real risk-free rate, r_{rf}, and inflation, infl is:

$$(1 + r_f) = (1 + r_{rf})(1 + \text{infl})$$

We can rearrange this equation and express it in terms of the real risk-free rate as:

$$r_{rf} = \frac{1 + r_f}{1 + \text{infl}} - 1$$

The above formula is used for precise calculations. We can also use a quick approximation to estimate the real risk-free rate:

$$r_{rf} = r_f - \text{infl}$$

However, we should note that the approximation method does not give accurate results, especially over longer time periods.

Risk premiums

Risk premiums are additional returns that investors demand for investing in risky assets. The total return of any asset can be expressed as:

$$r_i = r_{rf} + \text{infl} + \Sigma\text{risk premia}$$

The set of risk premia depends on the cash flow characteristics of the investment. Examples of risk premia for stocks include:

- Size risk premium: Extra return for investing in small companies vs. large companies.
- Value risk premium: Extra return for investing in value stocks vs. growth stocks.

Examples of risk premia for bonds include:

- Default risk premium: Extra return for investing in a riskier corporate bond vs. a less risky government bond.
- Liquidity premium: Extra return for holding a bond that may be harder to sell quickly without impacting the price.
- Maturity risk premium: Extra return for holding longer maturity bonds, which are more sensitive to interest rates as compared to shorter maturity bonds.

Example:

(*This is based on the knowledge check question from the curriculum.)*

An analyst observes the historic geometric average returns shown below:

Asset Class	Geometric Average Return (%)
Equities	8.0
Corporate bonds	6.5
Treasury bills	2.5

The average inflation rate was 2.1% during that time.

1. Calculate the real rate of return for equities.

Solution:

The precise calculation of the real rate of return for equities is:

$$r_{rf} = \frac{1 + r_f}{1 + \text{infl}} - 1$$

$$r_{rf} = \frac{1 + 0.080}{1 + 0.0210} - 1 = 5.78\%$$

The approximate real rate of return for equities is:

$$r_{rf} = r_f - \text{infl}$$

$$r_{rf} = 8.0\% - 2.1\% = 5.9\%$$

Note that the difference between the precise real return and the approximate real return, 5.78% – 5.9% = −0.12%, compounds over time. Over 10 years, the compounded difference between the approximate and the precise return overestimates the compound growth effect by $1.0578^{10} - 1.0590^{10}$ = 1.7540 – 1.7740 = −2.0%.

Instructor's Note: Use the precise calculation on the exam if the answer choices are close to each other. If the answer choices are spread apart, the approximation may help you identify the correct choice.

2. Calculate the real rate of return for corporate bonds.

Solution:

The precise calculation of the real rate of return for corporate bonds is:

$$r_{rf} = \frac{1 + 0.065}{1 + 0.0210} - 1 = 4.31\%$$

The approximate real rate of return for corporate bonds is:

$$r_{rf} = 6.5\% - 2.1\% = 4.4\%$$

3. Calculate the risk premium for equities.

Solution:

The total return for equities can be expressed as:

$r_i = r_{rf} + \text{infl} + \Sigma\text{risk premia}$

The approximate sum of the real risk-free rate and the inflation rate is the nominal risk-free rate, which is the return on treasury bills. i.e.

$r_{rf} + \text{infl} = 2.5\%$

Therefore:

$8.0\% = 2.5\% + \Sigma\text{risk premia}$

$\Sigma\text{risk premia} = 8.0\% - 2.5\% = 5.5\%$

3.2 Excess Returns

Excess return is the difference between an asset's return and the return on some reference rate or benchmark.

Suppose we want to calculate the excess returns using the risk-free rate as the benchmark. The precise calculation is:

$$\dot{r}_{r,t} = \frac{1 + r_{i,t}}{1 + rf_t} - 1$$

where:
$\dot{r}_{r,t}$ = asset's excess return
$r_{i,t}$ = asset return
rf_t = risk-free rate

The approximate calculation is:

$$\dot{r}_{r,t} = r_{i,t} - rf_t$$

Using the data from our previous example, the equity excess return when the risk-free rate is used as the benchmark is:

$$\dot{r}_{r,t} = \frac{1 + r_{i,t}}{1 + rf_t} - 1 = \frac{1.08}{1.025} - 1 = 5.37\%$$

The equity excess return can be approximated as:

$$\dot{r}_{r,t} = r_{i,t} - rf_t = 8.0\% - 2.5\% = 5.5\%$$

Instructor's Note: On the exam, if you get the term 'risk-free rate' with no mention of whether the rate is real or nominal, then assume that we are talking about the nominal risk-free rate.

When the risk-free rate is used as the benchmark, the excess return is equal to the risk premium (a special case). However, excess return is a broader term, and we can also calculate excess returns using inflation or a market index such as the S&P500 as benchmarks.

3.3 Real Returns

Inflation rates can vary over time. Therefore, inflation-adjusted, real returns are useful when comparing returns across time periods. The real return of an asset can be calculated as:

$$r_{real} = \frac{1 + r_i}{1 + infl} - 1$$

Example:
(*This is based on the case study from the curriculum.*)

An investor bought 1,000 shares of a company at the start of 2022 for 835.20 per share and sold them at the end of the year for 631.30 per share. The investor also received a dividend of 12.50 per share during the year. The inflation rate for 2022 was 7.70%.

Calculate the investor's real total return before taxes.

Solution:

We first calculate the investor's nominal total return.

$$r = \frac{P_1 - P_0 + Inc}{P_0} = \frac{631.30 - 835.20 + 12.50}{835.20} = -22.92\%$$

(Here we assume that the dividend income occurs at the end of the period and is not reinvested into another investment.)

We then calculate the investor's real total return by adjusting for inflation.

$$r_{real} = \frac{1 + r_i}{1 + infl} - 1 = \frac{1 - 0.2292}{1 + 0.0770} - 1 = -28.43\%$$

In 2022, the investor suffered both a loss on the investment, −22.92%, and then the 7.70% annual inflation further reduced the investor's purchasing power to −28.43%. This is how much the investor purchasing power has been reduced through this investment.

3.4 Gross and Net Return

Investors often use asset managers to manage and invest their capital. These managers charge management fees and administrative expenses, usually as a percentage of the assets they manage. As a result, the returns earned by investors are reduced by these fees and expenses.

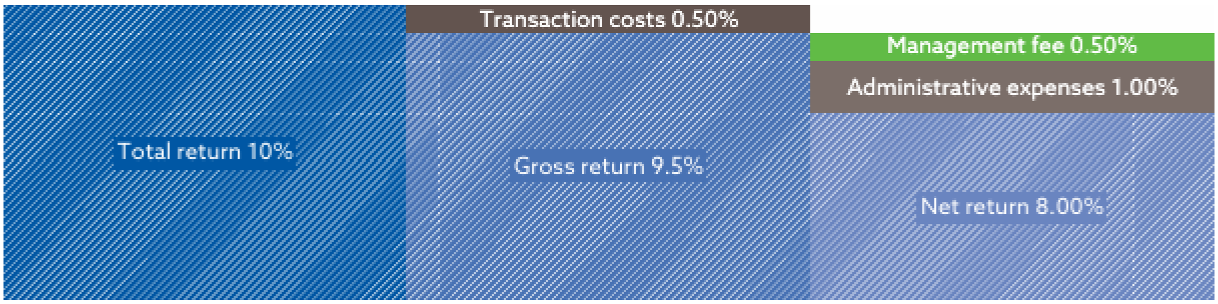
Gross Return

Gross return is the return earned by an asset manager prior to deducting management fees and taxes. It measures the investment skill of a manager.

Net Return

Net return is the return earned by the investor on an investment after all managerial and administrative expenses have been accounted for. This is the measure of return that should matter to an investor.

Exhibit 16 from the curriculum shows the relationship between total return, gross return, and net return.



Mathematically, the relationship can be expressed as:

Gross return = Total return – Transaction costs

Net return = Gross return – Management fees – Administrative expenses

Example:
(*This is based on the knowledge check example from the curriculum.*)

An investment fund, specialized in environmentally sustainable and socially responsible investments, reported a total annual return of −5% before costs. The fund's total assets were valued at CHF500 million. The fund incurs transaction costs of 0.05%, a management fee of 1.00%, and administrative expenses amounting to 0.50%—all as a percentage of its total asset value. The management fee is assessed on the assets at the end of the year.

1. Calculate the fund’s gross return.

Solution:

Gross return = Total return – Transaction costs

Gross return = -5% - 0.05% = -5.05%

2. Calculate the fund’s net return.

Solution:

Net return = Gross return – Management fees – Administrative expenses

Net return = -5.05% - 1.00% - 0.50% = -6.55%

3.5 Pre-Tax and After-Tax Nominal Return

The return measures we discussed so far were pre-tax nominal returns, i.e., before deducting any taxes. This is the default, unless otherwise stated.

However, most jurisdictions impose taxes on realized capital gains and interest/dividend income. Typically, the tax rates on capital gains are different than the tax rates on capital distributions.

The after-tax nominal return can be calculated as:

$$r_{net-tax} = r_{price} \times \left(1 - tax_{capital\ gains}\right) + r_{capital} \times (1 - tax_{capital\ distributions})$$

Example:
(*This is based on the case study from the curriculum.*)

In 2020, an investor purchased 1,000 shares of a stock at a price of 689.00 per share. By the end of the year, the price had risen to 1,243.50 per share. Additionally, the investor received a dividend of 10.50 per share during the year. The investor sold all their shares at the end of the year. The capital gains tax rate is 42%. The capital distributions, such as dividends, are taxed at the income tax rate of 27%.

1. Calculate the investor's realized capital gain for tax purposes.

Solution:

The realized capital gain per share is: 1,243.50 – 689.00 = 544.50.

Total capital gain on the 1,000 shares is: 544.50 × 1,000 = 554,500.

2. Calculate the capital gains tax that the investor must pay.

Solution:

Capital gains tax = Total capital gain × Capital gains tax rate

Capital gains tax = 554,500 × 42% = 232,980

3. Calculate the total after-tax return on the investment.

Solution:

The price return can be calculated as:

$$r_{price} = \frac{P_1 - P_0}{P_0} = \frac{1{,}243.50 - 689.00}{689.00} = 80.48\%$$

The capital distribution return can be calculated as:

$$r_{capital} = \frac{Inc}{P_0} = \frac{10.50}{689.00} = 1.52\%$$

The total after-tax return can be calculated as:

$r_{net-tax} = r_{price} \times (1 - tax_{capital\ gains}) + r_{capital} \times (1 - tax_{capital\ distributions})$

$r_{net-tax} = 80.48\%(1 - 42\%) + 1.52\%(1 - 27\%) = 46.68\% + 1.11\% = 47.79\%$

Investors are more concerned with the after-tax real return than the pre-tax nominal return. Suppose an investor realizes a 10% nominal return in one year and pays 30% taxes on the realized return; then the after-tax realized return is:

$r_i \times (1 - tax) = 10\% \times (1 - 30\%) = 7\%$.

Further, suppose that inflation is 5%; then the after-tax realized return is:

$$r_{real} = \frac{1 + r_i}{1 + infl} - 1 = \frac{1.07}{1.05} - 1 = 1.9\%$$

Although the investment resulted in a 10% nominal return, the actual increase in purchasing power was only 1.9%. Over time, the combined eroding impacts of inflation and taxes slow the speed of investment growth.

The after-tax real return is the most relevant benchmark for investors. However, most asset managers do not calculate it because it is difficult to estimate a general tax component that applies to all investors. The tax component depends on:

- The investor's specific income sources and marginal tax rate.
- The time period the investor holds the investment (long-term vs. short-term)
- The type of account in which the investor holds the asset (tax-exempt, tax-deferred, or taxable)
- The different tax systems that apply based on residence (local, municipal, county, state, regional, and national taxes)

Minimizing overall tax liabilities is an important portfolio management strategy.

3.6 Leveraged Returns

In cases where an investor borrows money to invest in assets like bonds or real estate, the leveraged return is the return earned by the investor on his money after accounting for interest paid on borrowed money.

The levered return can be calculated as:

$$r_{LP} = \frac{r_p \times (V_E + V_D) - V_D \times r_d}{V_E}$$

where:
r_{LP}= returns on a levered portfolio
r_p = total investment return
r_d = borrowing cost on debt
V_E = unleveraged amount of equity of the portfolio

V_D = amount of debt or borrowed funds

Through some algebraic manipulation, we can express this formula as:

$$r_{LP} = r_p + \frac{V_D}{V_E}(r_p - r_d)$$

Example:
(*This example is taken from the curriculum.*)

A EUR10 million portfolio generates an 8% total investment return over one year and is financed 30% with debt at 5%. Calculate the leveraged return for the portfolio.

Solution:

$$r_{LP} = r_p + \frac{V_D}{V_E}(r_p - r_d)$$

$$r_{LP} = 8\% + \frac{\text{EUR 3 million}}{\text{EUR 7 million}}(8\% - 5\%) = 8\% + 0.43 \times 3\% = 9.29\%$$

Leverage boosts returns as long as the investment return exceeds the borrowing cost. However, when the investment return is less than the borrowing cost, leverage results in a greater loss compared to an unleveraged investment.

Leverage can also be achieved through the use of derivatives such as options. An option buyer can pay a relatively small premium to gain market exposure to a large quantity of the underlying. This allows an investor to potentially realize large percentage gains from relatively small, favorable movements in the underlying asset's price. For example, if you anticipate that a stock priced at $100 will rise, purchasing a $3 call option (which represents 100 shares) would cost $300. This investment enables you to control $10,000 worth of stock for only $300.

Instructor's Note: You will understand 'options' better in the 'Derivatives' section of the curriculum.

Summary

LO: Describe, compare, and interpret returns.

The total return generated by a financial asset has two components: capital appreciation and capital distributions.

$$r = \frac{P_1 - P_0}{P_0} + \frac{Inc}{P_0}$$

An arithmetic mean return is the simple average of all holding period returns.

$$\bar{r}_i = \frac{r_{i1} + r_{i2} + \cdots + r_{i,T}}{T}$$

A geometric mean return represents the average rate at which an asset's value grows over time, accounting for the compounding of returns.

$$\bar{r}_{Gi} = [(1 + r_{i1}) \times (1 + r_{i2}) \times \ldots \times (1 + r_{iT})]^{\frac{1}{T}} - 1$$

To compare the performance of investments with different holding periods, we can annualize all available returns.

$$r_{annualized} = (1 + r_{period})^c - 1$$

The continuously compounded return associated with a holding period return can be calculated as:

$$R_c = \ln(1 + R_t) = \ln(P_t/P_0)$$

LO: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation.

The required rate of return is the minimum rate of return an investor must receive to accept an investment.

The total return of any asset can be expressed as:

$$r_i = r_{rf} + infl + \Sigma risk\ premia$$

The risk premia differ for equity and debt. Risk premia for equity include: size risk premium and value risk premium. Risk premia for debt include: default risk premium, liquidity premium, and maturity risk premium.

The real return of an asset can be calculated as:

$$r_{real} = \frac{1 + r_i}{1 + infl} - 1$$

Gross return is the return earned by an asset manager prior to deducting management fees and taxes. It measures investment skill.

Net return accounts for all managerial and administrative expenses. It is the return an investor is concerned with.

Pre-tax nominal return is the return before accounting for inflation and taxes; this is the default, unless otherwise stated.

After-tax nominal return is the return after accounting for taxes. It can be calculated as:

$$r_{net-tax} = r_{price} \times \left(1 - tax_{capital\ gains}\right) + r_{capital} \times (1 - tax_{capital\ distributions})$$

Leveraged return is the return earned by the investor on his money after accounting for interest paid on borrowed money. The levered return can be calculated as:

$$r_{LP} = r_p + \frac{V_D}{V_E}(r_p - r_d)$$