Applications of the Normal Distribution

> **Note:**
> In this section, there are normal distribution problems that cannot be solved entirely by hand or using a calculator. You may need to refer to the Appendices at the end of CFA Curriculum Volume I for the respective tables.
>
> For exam, you are expected to know the common values for a standard normal distribution (e.g., 95%, 97.5%, and 99%). However, if a question requires more specific information, the relevant portion of the table will be provided with the question.

The *normal distribution* can be completely described by two parameters – mean (μ) and variance (σ^2). Modern portfolio theory (MPT), which is based on **mean-variance analysis**, generally assumes that returns are normally distributed. Traditionally, mean-variance analysis focuses on **symmetric risk**. Downside risk, such as safety first, is an alternative.

If returns are normally distributed, investors can focus on the probability the portfolio return, R_P, falls below a specified minimum level, R_L. The risk of suffering returns below this threshold can be minimized by maximizing the *safety-first ratio (SFRatio)*, also known as *Roy's safety-first ratio*.

$$\text{SFRatio} = \frac{E(R_P) - R_L}{\sigma_P}$$

The probability of R_P falling below R_L is:

$$P(R_P < R_L) = N(-\text{SFRatio})$$

Note that the SFRatio is equivalent to the **Sharpe ratio**, but with R_L substituted for R_F.

Risk at financial institutions is often measured with *stress testing* and *value at risk (VaR)*. Stress testing uses very unfavorable events or scenarios to manage the risk. VaR measures the minimum losses expected over a specified time period at a given probability.

Example: Safety-First Calculation

Your portfolio is currently worth $400,000. At the end of one year, you want to have at least $420,000. The expected return is 10%, and the standard deviation is 14% annually.

Calculate the probability the portfolio will not grow to at least $420,000.

> **Solution**
>
> To have **at least $420,000** at the end of one year, the required return of the portfolio is:
>
> $$R_L = \frac{20{,}000}{400{,}000} = 5\%$$
>
> So, the SFRatio is:
>
> $$\text{SFRatio} = \frac{10\% - 5\%}{14\%} = 0.36$$
>
> Therefore,
>
> $$\begin{aligned} P(R_P < R_L) &= N(-\text{SFRatio}) \\ &= N(-0.36) \\ &= 1 - N(0.36) \\ &= 1 - 0.641 \\ &= \mathbf{35.94\%} \end{aligned}$$