UpperMark™
Study Handbook

CAIA® Level II

Volume 3

Topic 8: Volatility and Complex Strategies

Topic 9: Universal Investment Considerations

Topic 10: Emerging Topics

UpperMark™

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It is recommended that candidates use any exam preparation product together with the original CAIA curriculum readings.[1]

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Preface

Volume 3 of the UpperMark™ *Study Handbooks* provides a comprehensive and concise account of each learning objective (L.O.) in Topics 8-10 of the CAIA Level II curriculum. The *Study Handbook* is compiled using the reference materials recommended by the CAIA Association and, as in Volume 1, is organized as follows.

- Each Chapter in the curriculum is presented as a separate chapter, keywords are indicated in ***bold italics***, and learning objective sub-bullets are indicated by underlined, capitalized subheadings (e.g., ROLE OF INVESTMENT OBJECTIVES AND CONSTRAINTS).
- The lists of keywords and learning objectives are provided at the start of each chapter.
- Space is provided at the end of each chapter for you to record your Personal Study Notes.
- A set of practice exam questions is provided at the end of each chapter. A considerably larger set of practice questions (multiple-choice and constructed-response questions, and constructed-response question sets ["Essays"]) is in our TestBank software.

Supplementary information is included in footnotes.

The CAIA Equation Exception List is provided in Appendix 2. As in Volume 1, the first occurrence of each equation on the list is followed by a (***G***) to indicate that it will be *given* on the exam.

We wish you the best with your exam preparation.

Padideh Jalali, Ph.D., CAIA
President and CEO
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Topic 8

Volatility and Complex Strategies

Topic 8 is composed of five Readings on volatility and complex strategies.

1. Reading 8.1 discusses volatility as a factor exposure, volatility models, and implied volatility structures.
2. Reading 8.2 describes volatility, correlation, and dispersion products and strategies.
3. Reading 8.3 discusses the concept of complexity in investment products and describes several structured products.
4. Reading 8.4 reviews international real estate investing and its key risks and challenges.
5. Reading 8.5 reviews investment styles, strategies, and risk management practices implemented in the cryptocurrency space.

Reading 8.1

Volatility as a Factor Exposure

This Reading discusses measures of volatility and describes different volatility exposures, option Greeks (e.g., vega), modelling volatility processes, and implied volatility structures.

Learning Objectives

8.1.1 Demonstrate knowledge of measures of volatility.

i. Understand differences between implied volatility and realized volatility
ii. Identify limitations of realized volatility as a measure of dispersion
iii. State the properties of realized volatility

Keywords

1. Implied return volatility
2. Realized return volatility

8.1.2 Demonstrate knowledge of volatility and the vegas, gammas, and thetas of options.

i. Describe option vegas
ii. Interpret the scaling of the vega of an option
iii. Interpret and apply vega as an option for finite shifts
iv. Understand how vega shifts as underlying variables change
v. Interpret option gammas
vi. Understand the interrelationships between option vegas, gammas, and thetas

8.1.3 Demonstrate knowledge of exposures to volatility as a factor.

i. Contrast long volatility with short volatility
ii. Understand distinctions between positive vega and long volatility exposures
iii. Explain how volatility can be used to hedge risk
iv. Understand volatility as an unobservable but unique risk factor
v. Understand how long volatility carries a negative risk premium
vi. Explain how short volatility earns a positive risk premium

Keywords

1. Long volatility
2. Negative volatility risk premium
3. Short volatility
4. Volatility derivatives

8.1.4 Demonstrate knowledge of modeling volatility processes

i. Understand volatility processes with jump risk
ii. Construct volatility processes and regime changes
iii. Discuss reasons why volatility strategies recover
iv. Identify reasons why volatility mean reversion cannot be arbitraged

Keywords

1. Mixture model or a regime switching model
2. Regime change
3. Volatility clustering
4. Volatility diffusion risk
5. Volatility jump risk
6. Volatility risk

8.1.5 Demonstrate knowledge of implied volatility structures

i. Describe methods of computing implied volatility
ii. Identify structures regarding implied volatility and moneyness
iii. Identify an implied volatility surface
iv. Explain key reasons for implied volatility structures and surfaces
v. Discuss reasons for high implied volatility and out-of-the-money puts

Keywords

1. Implied volatility structure
2. Options volatility surface
3. Smile or a smirk
4. Volatility skew

L.O. 8.1.1 **DEMONSTRATE KNOWLEDGE OF MEASURES OF VOLATILITY.**

This L.O. describes implied and realized volatilities.

DIFFERENCES BETWEEN IMPLIED VOLATILITY AND REALIZED VOLATILITY

Implied return volatility is the volatility inferred from an option price under assumptions including risk-neutrality, the validity of an option pricing model, and the accuracy of the model's inputs (other than volatility). Implied volatility (IV) is typically estimated using market prices. The option pricing model used makes assumptions about the underlying asset's return process. For instance, in the Black-Scholes option pricing model, the option's underlying asset is assumed to follow a geometric Brownian motion (GBM) in which the asset's instantaneous returns have the following traits.

1. Constant variance over time (i.e., are homoscedastic)
2. Normally distributed
3. Uncorrelated over time

Realized return volatility is the actual variation in a given time period using a specific return measurement interval (e.g., weekly return granularity). It is typically measured as the standard deviation of returns. In practice, observed returns are discrete; they are not continuous and do not have the properties of a GBM process.

Since options are exposed to changes in volatility, IVs should differ from expected realized volatilities to reflect volatility-related risk premiums. Therefore, IV does not represent an accurate, unbiased consensus of realized volatility expected by market participants due to risk premiums.

LIMITATIONS OF REALIZED VOLATILITY AS A MEASURE OF DISPERSION

Realized volatility is not a complete description of dispersion; it is an estimate that describes the dispersion of a frequency return distribution of a sample. Realized volatility has three limitations.

1. It does not describe the shape of the return distribution.
2. Assets with identical realized volatilities may have markedly different underlying returns; i.e., they may be trending, mean reverting, or minimally autocorrelated.
3. It does not describe whether most of the dispersion occurred near a particular price of the underlying asset or during a particular time period in the sample.

For instance, in a set of assets with the same realized return volatility, one asset may have mostly moderate returns, another may have one extremely large return and many small returns, another may trend, and another may mean-revert. Despite having the same realized volatility, these assets will not have the same results for most volatility strategies. In addition, the granularity of the return interval can substantially alter its return characteristics.

PROPERTIES OF REALIZED VOLATILITY

Sinclair (2013) presents six observations regarding realized volatility, many of which are assumptions of volatility arbitrage strategies and risk management techniques.

1. Realized volatility slowly mean-reverts (thus is not constant) and clusters.
   - Thus, volatility is often modeled using generalized autoregressive conditional heteroscedasticity (GARCH) and regime switching models.
2. Realized volatility is typically low for a period of time until a market shock occurs, and then it increases and remains at the higher level for a period of time.
3. Short-term realized volatility can be high, but, in the long run, reverts to a long-term mean.
4. High volatility increases investors' risk aversion, which indicates that high volatility is generally negatively correlated with risky asset returns.
5. Equity market realized volatility is negatively correlated with stock prices (i.e., rises in bear markets; declines in bull markets).
6. Realized equity volatility increases faster in bear markets than in bull markets.

L.O. 8.1.2 **DEMONSTRATE KNOWLEDGE OF VOLATILITY AND THE VEGAS, GAMMAS, AND THETAS OF OPTIONS.**

This L.O. discusses vega, gamma, and theta; which reflect sensitivities of assets to changes in underlying factors.

OPTION VEGA

Vega indicates the sensitivity of an option value to a change in the volatility of the option's underlying asset. Some volatility strategies involve positions with predetermined levels of sensitivity to changes in implied volatility (i.e., with target exposures to vega), while minimizing exposure to changes in the underlying asset value (i.e., minimizing delta). Some traders may aim to earn a volatility risk premium using short vega positions.

The vega of a call or put option on an asset is the partial derivative of the option value with respect to its volatility (i.e., rate of change in the option value with respect to an infinitesimal shift in volatility). From the Black-Scholes option pricing model, the vega, v, of a call or put on an asset, S, may be expressed as:

$$v = \frac{\partial p}{\partial \sigma} = SN'(d)\sqrt{T}, \qquad \textbf{(G)} \qquad (1)$$

where p is the option value, σ is the underlying asset's volatility, $N'(d)$ is the (non-cumulative) probability density function of the normal distribution at d, and T is the option's time to expiration or tenor.

SCALING THE VEGA OF AN OPTION

In practice, vega is commonly scaled to reflect the risk of a one basis point change in volatility. As such, it is expressed as vega per basis point: $\frac{v}{100} = \frac{SN'(d)\sqrt{T}}{100}$.

- Vega in Equation (1) measures an option value's instantaneous rate of response to a change in one percentage point in volatility; e.g., a change from 20% to 21% (i.e., 0.20 to 0.21).
- Vega per basis point measures an option value's instantaneous rate of response to a change in one basis point in volatility; e.g. a change from 20% to 20.01% (i.e., 0.20 to 0.2001).

Practitioners and financial data providers typically report vega scaled per basis point.

Example 1

A put option on a non-dividend-paying stock valued at $50 has 0.25 years to expiration. If $N'(d)$ is 0.2, what is the vega and vega per basis point of this option?

Vega: $v = \frac{\partial p}{\partial \sigma} = SN'(d)\sqrt{T} = \$50 \times 0.2 \times \sqrt{0.25} = \$5$.

Vega per basis point = Vega/100 = $5/100 = $0.05.

The vega per basis point may be interpreted as the option value rising towards an increase of $0.05 as the option's implied volatility rises towards an increase of 0.01% (i.e., 0.0001).

The interpretation is not expressed as the option value increasing by $0.05 because the relationship between option value and volatility is non-linear and vega is only a precise measure when volatility makes infinitesimal shifts. For large shifts in volatility, higher-order derivatives are needed to determine accurate changes in option values.

VEGA AS AN APPROXIMATION FOR FINITE SHIFTS

The linear (first-order) approximate change in option value may be expressed as:

$$\Delta p \approx v\Delta\sigma.$$

Example 2

A call option on a non-dividend-paying stock has a vega per basis point of $0.005. What is the first-order approximate change in the call value for a decline in volatility from 0.27 to 0.24?

$\Delta p \approx v\Delta\sigma = \$0.005\Delta\sigma$

$\Delta\sigma$ (in basis points) = 0.27 – 0.24 = –0.03 = –3% = –300 basis points[1]

$\Rightarrow \Delta p \approx \$0.005\Delta\sigma = \$0.005 \times -300 = -\$1.50$

[1] 1 basis point = (1/100) of 1% [or 0.01%]; therefore, 1% = 100 bps.

HOW VEGA SHIFTS AS UNDERLYING VARIABLES CHANGE

Four observations may be made about vega given by Equation (1): $v = SN'(d)\sqrt{T}$.

1. Vega is always positive for a long call or put option.
   - This is because all three terms on the right side of Equation (1) are positive.
2. Vega is the same for a call and a put with the same underlying asset, strike price, time to expiration, and implied volatility are equal.
   - In put-call parity (i.e., Call – Put = Stock – Bond), IV does not appear on the right side of the equation. This indicates that the effects of IV on the left side (i.e., "Call – Put") must cancel out and, to do so, they must be equal. Thus, "Stock – Bond" on the right side may be considered a financed long stock position with a vega of zero and "Call – Put" has a net vega of zero.
3. Vega of an option approaches zero as the option's time to expiration approaches zero.
   - This is clear in Equation (1): as T approaches zero, v approaches zero.
4. Vega of an option approaches zero as the underlying asset value approaches zero or infinity.
   - Therefore, vega approaches zero for deep out-of-the-money (OTM) and in-the-money (ITM) options because $N'(d)$ in Equation (1) approaches zero when S is very small or very large (i.e., towards either tail of the normal distribution).

Figure 1 illustrates call option price for different IVs and/or tenors, where the differences in the call prices are driven primarily by the quantity $\sigma\sqrt{T}$. An up or down shift in volatility moves the option price curve higher or lower. As time passes, all else equal, option values move down to lower curves.

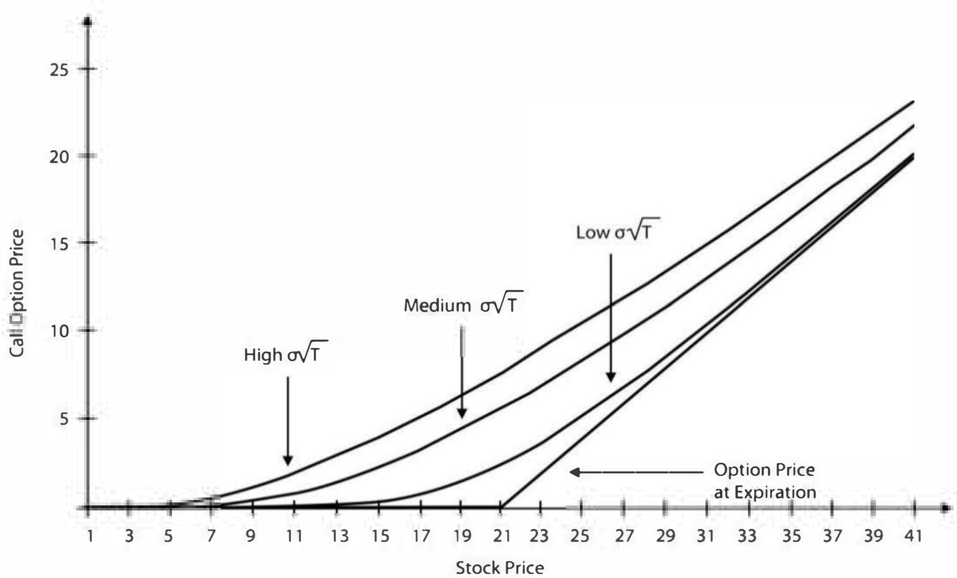

Figure 1: Call Option Values at Three Levels of Implied Volatility and/or Tenor

OPTION GAMMA

An option's gamma is the second-order partial derivative of its value with respect to the underlying asset value. Gamma may also be expressed as the first-order partial derivative of an option's delta with respect to the underlying asset value (since delta is the first-order partial derivative of an option's value with respect to its underlying asset value).

From the Black-Scholes model, the gamma of an option, γ, may be expressed as:

$$\gamma = \frac{N'(d)}{S\sigma\sqrt{T}} = \frac{v}{\sigma S^2 T}.$$

- Gamma is the same for a call and a put with the same underlying asset, strike price, and tenor.
- The terms in the gamma formula are all non-negative; therefore, gamma and vega are positive for simple long call or put options.
- Long option positions have long volatility and positive gamma, and short option positions are short volatility and have negative gamma.
- Graphically, an option's gamma depicts the degree of curvature in the relationship between option price and the underlying asset price. In Figure 1, the lowest curve has relatively sharp curvature when the option is near-the-money, which corresponds to a high value of gamma. The highest curve has less sharp curvature, corresponding to a low (but positive) gamma.
    - ➢ Gamma is highest for near-the-money options and smallest (or zero) for ITM or OTM options.

INTERRELATIONSHIPS BETWEEN OPTION GAMMAS, VEGAS, AND THETAS

Gamma provides long gamma call option holders with the desirable combination of increasing rates of gain as the option's underlying asset moves up and decreasing rates of loss as the underlying asset moves down. Holders of puts experience the desirable combination of increasing rates of gain as the underlying asset moves down and decreasing rates of loss as the underlying asset moves up.

- A portfolio with large positive gamma benefits greatly from large directional moves in the underlying asset in one direction but suffers relatively smaller losses from directional moves in the other direction.
- If options are competitively priced, there is no "free lunch". As such, positive gamma portfolios (that tend to be long options) have negative theta, which means that the option value declines as time passes, all else equal (i.e., changes in the underlying asset price and volatility are minimal).

  Vega reflects the sensitivity of options to volatility that is driven by gamma.
- Vega is small for options with near-zero gamma.
- An option's positive gamma makes anticipated higher volatility result in higher option values.

Therefore, gamma (which comes from asymmetric payoffs) is the primary reason that options sell for a premium above their intrinsic value (i.e., value at expiration). This positive time value to a call option is due to the call's virtually unlimited gain potential and relatively limited losses. It will decay over time (measured by option's theta) if anticipated volatility does not occur.

L.O. 8.1.3 **DEMONSTRATE KNOWLEDGE OF EXPOSURES TO VOLATILITY AS A FACTOR.**

This L.O. discusses the sign of an option's vega and whether the option is long or short volatility.

LONG VOLATILITY VS. SHORT VOLATILITY

The notion of an investment (e.g., option) being long or short volatility refers to the investment's empirically-observed correlation with volatility.

- When an investment's returns are negatively correlated with the volatility level of market returns, the position is ***short volatility*** (or short vol); i.e., short the volatility of the market.
- When an investment's returns are positively correlated with the volatility level of market returns, the position is ***long volatility*** (or long vol).

However, long option positions are not always long volatility.

- Consider that long at-the-money (ATM) equity options are generally long volatility with respect to the underlying assets' volatility, and short ATM options are short volatility. However, short-dated deeply in-the-money (DITM) call options on an equity index behave like the underlying index and, since an equity index has negative correlation with equity market volatility, a long position in the index is short volatility. Therefore, long DITM call options on an equity index are short volatility (similar to the index). The reason for this is that the effect of declining equity market prices on the value of a DITM equity call option (via its positive delta) generally dominates the value-increasing effect of the increased volatility on the value of the DITM option (via its positive vega).
- Long equity options are always long equity-market volatility (and short options are always short volatility) in delta-neutral portfolios, where the effects of directional moves in the equity market have been hedged.

Numerous securities or derivatives have been developed to be either long or short volatility. Thus, portfolios can be long or short volatility based on their direct option exposures and through exposures to products engineered to have volatility exposure.

POSITIVE VEGA VS. LONG VOLATILITY EXPOSURES

The concepts of positive vega and long volatility are related in that they both describe the value of an asset increasing when volatility increases. However, there are distinctions between the concepts. These are discussed below.

As the partial derivative of an option with respect to the implied volatility of the option's underlying asset, vega represents the option's response to changes in volatility, holding all other variables constant (e.g., value of the option's underlying asset).

- The vega of an asset is positive when the partial derivative of the asset price with respect to implied volatility is positive.
- The vega of a long simple option is positive and of a short simple option is negative.
- The vega of a simple long or short traditional asset position (e.g., in an equity index) is zero.

Volatility exposures typically focus on empirically-observed correlations between an asset's returns and volatility levels.

- An option is considered long volatility if its price tends to rise when realized volatility increases (e.g., long put or ATM call).
- Assets are short volatility if they tend to decline in volatile equity markets (e.g., short options).
- Assets are long volatility when their prices are positively correlated with volatility levels and are short volatility when their prices are negatively correlated with volatility.
- Options may be long or short volatility depending on whether they are puts or calls and their moneyness.

There are many distinctions between vega and volatility.

- A key distinction is that vega involves holding all variables other than volatility constant, while observed correlations in volatility do not hold other variables constant.
- As a practical distinction between long vega and long volatility, consider that long equity positions (that have zero vega) are often described as being short volatility since their returns are negatively correlated with volatility. All of the returns of an equity index are generated by its delta of one since its vega is zero. The notion that long equities are short volatility makes sense when short volatility is defined as an empirical observation that equity indexes tend to fall during periods of high equity volatility.
- DITM equity calls have positive vega, but tend to be negatively correlated with equity market volatility and thus are short volatility.

USING VOLATILITY DERIVATIVES TO HEDGE MARKET RISK

A key proposition regarding volatility derivatives is that they may be used to hedge long positions in traditional assets (e.g., equities), since these long positions are empirically observed to be short volatility. This proposition has important implications for evaluating expected returns and systematic risks of volatility products.

Some investors in traditional assets who are long market risk (and, presumably, short volatility) hedge their positions with long positions in volatility derivatives, since gains from long volatility positions tend to offset losses from traditional portfolios during financial market crises and periods of rising market volatility.

VOLATILITY IS AN UNOBSERVABLE, BUT UNIQUE RISK FACTOR

Volatility is a key macro factor exposure used in many multi-factor asset pricing models as a driver of asset returns. As an unobservable risk factor, a volatility factor is typically estimated using implied volatility (which is inferred from a current liquid option price and a specific option pricing model) and realized volatility (which is observed in returns).

Volatility is not directly traded using traditional assets; investment products are created so that it can be traded. Implied volatility (IV) can be traded using options.

Legitimate risk factors should have unique risk premiums that cannot be explained by other risk factors; i.e., risk factors should not be highly correlated with each other (such as value and momentum). Rennison and Pedersen (2012) show that volatility is a unique factor. Specifically, they found a low return correlation (i.e., less than 30%) between the following.

1. Selling straddles and the underlying market returns
2. Commodity volatility and commodity futures
3. Currency volatility and carry strategies
4. Interest rate swaptions and long credit investments

They also found a low return correlation (of 50%) between equity volatility and long stock investments and low correlations (of 17%-32%) for volatility strategies in the commodity, equity, interest rate, and currency markets. This evidence also supports the notion that assets directly related to volatility are a unique asset class (since investment products that provide investors the risk premium attached to the volatility factor are considered a separate asset class).

Volatility is an important factor that affects several asset classes. For instance, volatility is a factor exposure to option returns. In recent years, the market has witnessed the emergence of several ***volatility derivatives***, which are pure plays on volatility with returns that are driven directly by exposure to the volatility factor.

LONG VOLATILITY CARRIES A NEGATIVE RISK PREMIUM

The risk premium on an asset class comprises a set of risk premiums that represent the asset class's various factor exposures. Each risky asset class offers higher return for any increased systematic risk exposure. For instance, an equity risk premium can be earned by holding long stock positions and a credit risk premium can be earned by holding investment-grade corporate bonds. Risk premiums can also be earned on products with values tied to volatility (i.e., to volatility as a risk factor).

Since factor exposures that consistently generate positive returns in all market conditions result in arbitrage opportunities, long factor exposures should generate gains in some markets (e.g., bull markets) and especially losses in bear markets. Over several economic cycles, average returns to factor exposures with positive market beta are positive and with negative betas are negative (relative to the riskless rate). Therefore, risk premiums of traditional factor exposures are positive during normal periods, negative during periods of financial distress, and positive over the long term.

The level of return volatility has a negative correlation with market index returns, which has been characterized by equity markets tending to "take the escalator up and the elevator down." The negative correlation may be due to investor utility functions and behavioral factors (e.g., loss aversion). When equity markets suffer turmoil (e.g., huge price declines), they experience increased demand for put options, particularly by institutional investors. The negative correlation between equity market prices and demand for long puts is a reason given for the negative correlations between implied volatility and equity market returns.

Risk factors that perform poorly during poor economic conditions offer a positive risk premium as incentive for investors to hold a positive market beta asset expected to perform poorly when other risky assets perform poorly.

While positive risk premiums are typically considered as being attached to positive factor exposures, long volatility products are expected to perform well when other risky assets are performing poorly. Therefore, long volatility products have negative market beta and carry a negative risk premium. The more an asset has negative exposure to the volatility factor (i.e., is short volatility), the higher its positive risk premium. The positive expected return to being short volatility (which provides the counterparty with protection against equity market downside risk) is similar to the positive expected premiums that insurance companies seek by providing protection to insurance policy buyers.

- For instance, CBOE Volatility Index (VIX) futures contracts are positively correlated with equity market volatility and tend to decline in value due to a negative risk premium. Holders of long VIX contracts enjoy the hedging benefit of negative equity betas but pay for the protection via the negative risk premium. Thus, products that generate hedging benefits from returns that are positively correlated to volatility offer a ***negative volatility risk premium***, which means they tend to have expected returns less than the riskless rate.
- The sign of the volatility risk premium is determined by correlation, and the size of the premium is explained by supply and demand. Many natural buyers of tail risk protection seek to hedge their risky investments using volatility derivatives with long volatility positions. Given aversion to large losses, only a limited number of traders are willing to short volatility (and demand a premium for doing so). This is similar to Keynes' argument regarding normal backwardation of commodity futures.[2]
- The volatility risk premium must be large enough to balance the supply and demand. Since shorting volatility exposes traders to losses at the worst times (i.e., market distress), the volatility risk premium is expected to be substantially negative in equity markets if volatility and equity returns are negatively correlated.

SHORT VOLATILITY EARNS A POSITIVE RISK PREMIUM

If being short volatility earns a positive risk premium, then being long volatility should pay a risk premium, thus generating low or negative average returns (i.e., lower than the riskless rate). VIX futures contracts and VIX-related exchange-traded products (ETPs) offer a pure play on exposure to the U.S. equity market volatility factor.

- The VXX was an ETP that illustrated the long-term effect of positive volatility exposure (from long VIX futures; being long the VXX was a pure play on being long volatility. Its performance was an extreme demonstration of the negative volatility risk premium: the VXX declined 99.99% over its ten-year lifetime. The product matured in January 2019 and was replaced with series B of the VXX ETN.

Investors with short volatility positions should earn a consistent profit from exposure to the volatility factor. With options, the consistent profit to short option positions comes from implied volatility consistently exceeding realized volatility. In this case, options are frequently "overpriced" (relative to a risk-neutral world) and option writers consistently earn returns in excess of the riskless

[2] John Maynard Keynes argued that, under normal backwardation, producers of commodities are more likely to hedge their price risk than consumers of commodities. He theorized that commodity producers hedge their inherent long commodity positions by taking short commodity futures positions.

rate (because the implied volatility priced in the options they are selling consistently exceeds the realized volatility).

- Rennison and Pedersen (2012) estimate the volatility risk premium is about 10% of the level of implied volatility, which can be earned in options and volatility trading strategies in equities, interest rates, currencies, and commodities. In efficient markets, the excess return earned by option writers is compensation for bearing the risk of the volatility factor.
- Evidence indicates that the risk-return trade-off for strategies exposed to the volatility factor compare favorably to the risk premiums earned through exposure to equity and credit factors.
- Black and Szado (2016) show that, over the period 1990-2015, the implied volatility of S&P 500 options exceeded realized volatility in about 90% of all calendar quarters.
- Evidence also shows that products that are long S&P 500 volatility had consistently low or negative historical returns.

L.O. 8.1.4 **DEMONSTRATE KNOWLEDGE OF MODELING OF VOLATILITY PROCESSES.**

Theoretical models of volatility derivatives begin by specifying the underlying processes. While most valuation models for cash securities are based on continuous processes (i.e., process that assumes the underlying experiences infinitesimal changes with constant volatility), volatility derivative models are often based on changing volatility.

VOLATILITY PROCESSES WITH JUMP RISK

According to Nossman and Wilhelmsson (2009), there is a positive expected return to short positions in products tied to the implied volatility of options that compensates sellers for two volatility risk premiums: one for volatility diffusion and one for volatility jump.

- ***Volatility diffusion risk*** is the risk of volatility changes that represent the continuous accrual of small changes in an asset's volatility over time.
- ***Volatility jump risk*** refers to the risk of potentially large, periodic, and sudden increases in volatility.

A continuous-time diffusion process for returns on an equity index may be described as:

$$\frac{S_{t+\Delta} - S_t}{S_t} = \mu_t \Delta + \sigma_t \Delta \tilde{W},$$

where S_t is the equity price at time t, μ_t is the expected return, Δ is the length of time (e.g., one day), σ_t is the standard deviation of returns, and $\Delta\tilde{W}$ is a Wiener process. This diffusion process does not allow for discontinuities or jumps in values. Models for valuation of volatility products need to allow for volatility to change. A process for changes in volatility may be described as:

$$\sigma_{t+\Delta} - \sigma_t = \gamma\Delta + \delta\Delta\tilde{Y} + \phi\Delta\tilde{J}, \tag{1}$$

where ϕ is a random variable that determines the volatility of changes in volatility (permitting negative and positive jumps in volatility) and $\Delta\tilde{J}$ represents a source of uncertainty generating jumps in volatility.[3] An example of this volatility process is presented in Figure 2.

[3] γ is the expected change in volatility, δ determines the volatility of changes in volatility, and $\Delta\tilde{Y}$ represents the volatility diffusion. Examples of time-varying volatility models are the Heston and the Bates models, which were presented in Reading 5.3.

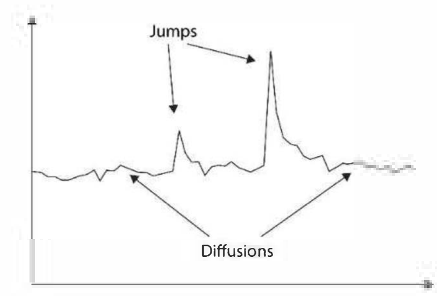

Figure 2: Volatility Process

VOLATILITY PROCESSES AND REGIME CHANGES

A ***regime change*** occurs when a financial series' observed behavior undergoes a significant change (e.g., new central bank policy target with tightening of monetary policy). Return volatilities and correlations periodically experience substantial regime changes.

Stock market volatility may be modeled using a ***mixture model*** or a ***regime-switching model***, which models volatility as a mixture of two return distributions.

- These models do not include a jump component. Instead, they are made up of two versions of Equation (1): one with a low δ and one with a higher δ; and they assume that volatility randomly switches between the two processes.
  - ➢ For instance, a low volatility market with a positive mean return may exist 75% of the time, and a high volatility market with a negative mean return may exist only 25% of the time.
- Because volatility clusters, staying in a volatility regime is more likely to occur than switching to a new regime in the next time period. This is because, with ***volatility clustering***, periods of large changes are likely to be followed by more large changes and periods of small changes are likely to be followed by more small changes.

REASONS WHY VOLATILITY STRATEGIES RECOVER QUICKLY

Historically, volatility strategies have recovered from drawdowns in less than one year, which contrasts with long-only stocks, credit, or commodity investments that can take 1-2 years to recover. There are two reasons for this.

1. Short high-volatility periods
   - Periods of high-volatility are short-lived due to the mean reversion in realized volatility (and, as the volatility declines, suppliers of long volatility products recover their losses).
   - When a major event (e.g., large corporate loss announcement) occurs, realized volatility spikes, and later tends to decline towards its normal level. Volatility spikes occur because major events tend to cluster (e.g., unexpected corporate loss announcement is often followed by other companies reacting to similar economic headwinds). If no additional events occur after a volatility spike, the market returns to more normal volatility.
2. Increased demand for long-volatility products post-crisis
   - There is increased demand for long-volatility products after a crisis by investors in traditional assets (which is met through higher expected volatility risk premiums demanded by short-volatility traders).

WHY VOLATILITY MEAN REVERSION CANNOT BE ARBITRAGED

Despite the strong mean reversion in realized volatility, mean reversion cannot necessarily be used as the basis for profitable volatility trading strategies. Competition generally prevents asset prices from exhibiting strong degrees of mean reversion in efficient markets (since tradable assets with mean-reverting prices can be exploited to generate substantial, almost risk-free returns). However, realized volatility is not directly traded. Other non-tradable market values (e.g., interest and inflation rates) can exhibit mean reversion since traders cannot directly exploit all patterns in rates. Similarly, while commodity prices can exhibit patterns of price changes and volatility relative to harvesting and heating seasons, these patterns cannot necessarily be arbitraged.

L.O. 8.1.5

DEMONSTRATE KNOWLEDGE OF IMPLIED VOLATILITY STRUCTURES.

An implied volatility (IV) can be derived for each option differentiated by the type of option (e.g., call or put) and the option's underlying asset, tenor, and moneyness. This L.O. discusses construction and analysis of ***implied volatility structures***, which are a representation of various IVs of a set of options relative to their type, tenor, or moneyness.

METHODS OF COMPUTING IMPLIED VOLATILITY

Option pricing models used to determine IV are based on assumptions and values of model inputs. Therefore, each estimated IV is based on a specific model. For instance, the Black-Scholes model requires the underlying asset price and the risk-free rate (which are observable), but is based on the unrealistic assumption that the underlying asset has constant volatility. Other option pricing models may have values that are not observable.

STRUCTURES REGARDING IMPLIED VOLATILITY AND MONEYNESS

A standard IV structure is a ***volatility skew***, which illustrates that options with different moneyness have different IVs.[4] The graph of a volatility skew for equity index options (i.e., IVs plotted against different strike prices on a single expiration date) generally resembles a ***smirk*** or ***smile***, where OTM and ITM options have higher IVs than ATM options.[5]

- OTM puts have higher IVs than OTM calls with similar distances from the current market price.
- Smiles or smirks may result due to negatively skewed returns, institutional demand for downside risk protection offered by long equity puts, or the negative correlation between returns and changes in volatility.

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[4] In this discussion, moneyness is measured as the stock-to-strike price ratio.

[5] Generally, a volatility smile is a balanced curve, as depicted below; and a smirk is weighted to one side, as in Figure 3.

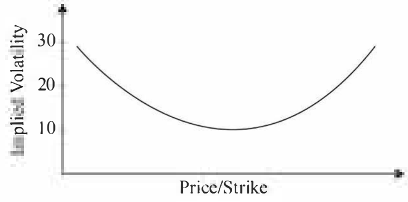

Figure 3 illustrates an example of a volatility skew of equity index puts on a given day, where the puts are OTM when price/strike is greater than 100%, ATM when price/strike is near 100%, and ITM when price/strike is less than 100%.[6]

Figure 3: Volatility Skew/Smirk on a Given Day

IMPLIED VOLATILITY SURFACE

An ***options volatility surface*** is a graphical representation of IV for several options with different expiration dates and strike prices. An example is presented in Figure 4.

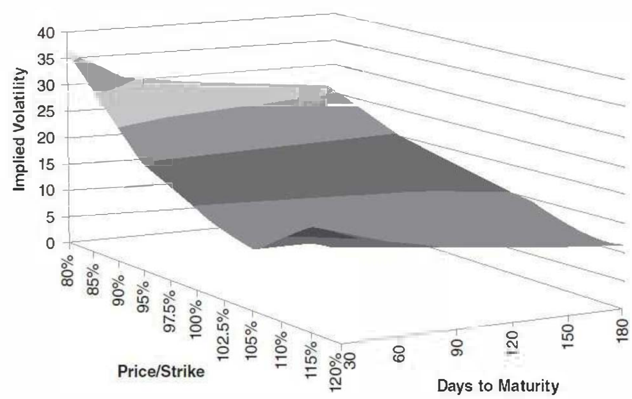

Figure 4: A Volatility Surface

[6] An OTM put has a stock price > strike. To see that this is the same as a stock-to-strike price ratio > 100%, take the inequality Stock > Strike and divide both sides by "Strike", and you get Stock/Strike > 1 (or 100%).

Note: if the horizontal axis of the volatility skew is Strike price (not Stock/Strike), then OTM puts fall on the left side of the curve.

KEY REASONS FOR IMPLIED VOLATILITY STRUCTURES AND SURFACES

Non-trivial IV structures and surfaces are driven by factors such as expectations of the underlying asset's future volatility. There are three key reasons that IVs differ for options with the same underlying asset but different strikes or tenors.

1. Skewness and kurtosis
   - IVs differ because standard deviation (or sigma) does not adequately describe non-normal returns (i.e., skewed or with non-zero kurtosis), even if an asset's expected volatilities over different time horizons are assumed to be equal. An asset's skewness and kurtosis over different time horizons may result in differences between expected realized volatilities and IVs due to the effects of skewness and kurtosis on expected option payoffs.
2. Time-varying volatility
   - Volatility structures may be driven by expected time-varying volatility in the asset's returns (e.g., expected seasonal effects such as earnings announcements, disclosure of revenues near holidays, or seasonal trading patterns such as year-end tax-related trading).
3. Autocorrelated returns
   - Autocorrelation in asset returns may cause different asset volatilities measured over different time periods (corresponding to option tenors) or with different return granularity (e.g., measuring daily or weekly returns).
   - The market index has degrees of positive autocorrelation (i.e., trending returns or momentum) and negative autocorrelation (i.e., mean-reversion) that differ depending on the lengths of the observation period and the granularity of returns.

REASONS FOR HIGH IMPLIED VOLATI IN OTM PUTS

Long deeply OTM (DOTM) put options on a broad equity index (e.g., S&P 500) provide protection against a portfolio's downside exposure. However, the IVs of these puts tend to be substantially higher than those of puts with different moneyness. There are two potential (not mutually exclusive) reasons for the high IVs.

1. Negatve skewness
   - The high IVs may reflect the negative skewness in the equity index's returns, which may be due to relatively high probability of market crashes and tendencies of the underlying asset to follow an asymmetric jump process.
2. Risk premium
   - The high IVs may be due to option writers requiring a large risk premium for writing the puts. The demand for the puts is due to portfolio managers or investors purchasing protection from the equity market's tail risk.
   - While, in theory, option payouts maybe replicated using dynamic portfolio strategies, the strategies may be difficult to implement (e.g., given market closures and discontinuous jumps). Thus, if the protection provided by these puts cannot be obtained in other ways, put prices need to include a premium to induce speculators to write the puts.
   - Evidence indicates that the relation of high IVs of OTM puts is not as strong for individual securities as for portfolios, likely because individual securities need less insurance.

Since broad equity markets have greater likelihood of huge declines than huge gains (i.e., are more likely to take the escalator up and take the elevator down), the higher IVs of DOTM puts appear to be driven at least in part by underlying asset's return skewness. Regarding the role of risk premiums in driving DOTM puts' high IVs, the evidence is mixed and inconclusive.

Personal Study Notes

Reading 8.1 – Practice Exam Questions

1. Which of the following characteristics applies to a long volatility product?

    I. Negative risk premium
    II. Strong performance when other risky assets perform well
    III. Positive market beta

    A. I only
    B. II only
    C. III only
    D. I and II only

2. A trader observes that periods of large changes in stock returns tend to be followed by large changes and periods of small changes in stock returns tend to be followed by small changes. This tendency is referred to as which of the following?

    A. volatility targeting
    B. volatility skew
    C. volatility diffusion
    D. volatility clustering

3. Which of the following statements is most accurate regarding the implied volatility of out-of-the-money puts as reflected in a volatility skew?

    A. It is lower than the implied volatility of similar out-of-the-money calls.
    B. It is about the same as the implied volatility of in-the-money puts.
    C. It is lower than the implied volatility of in-the-money puts.
    D. It is higher than the implied volatility of at-the-money puts.

Constructed-Response Question

4. Is the theta of a long option, positive gamma portfolio positive or negative?
(0.5pts)

Answers & Explanations

Question Number	Answer	Learning Objective	Explanation
1.	A	8.1.2	Long volatility products perform well when other risky assets perform poorly. Therefore, they have negative market betas and negative risk premiums.
2.	D	8.1.4	Volatility clustering is defined as the tendency of large changes (e.g., in stock returns) to be followed by large changes and periods of small changes to be followed by small changes. Other responses - • Volatility diffusion describes continuous accrual of small changes in an asset's volatility over time. If volatility diffusion is the key source of uncertainty, then volatility is low and has a continuous path. • Volatility skew describes the relationship between moneyness and implied volatilities (e.g., out-of-the-money puts have higher IVs than at-the-money puts). • Volatility targeting is a process used to determine the number of futures contracts to include in a managed futures portfolio. This was covered in the Level I curriculum.
3.	D	8.1.5	The volatility skew illustrates that the implied volatility of out-of-the-money puts is higher than that of other puts (i.e., at-the-money and in-the-money puts). Other response: OTM puts have higher IVs than similar OTM call options.

Constructed-Response Question

Question Number	Learning Objective	Answer
4.	8.1.2	Long option, positive gamma portfolios have negative thetas. Not needed for response: This means that, all else equal, the value of the portfolio declines as time passes.

Reading 8.2

Volatility, Correlation, and Dispersion Products and Strategies

This Reading discusses volatility-related derivative securities and strategies related to volatility, correlation, and dispersion among returns of securities.

Learning Objectives

8.2.1 Demonstrate knowledge of common option strategies and their volatility exposures.

i. Understand and apply theta as a measure of time decay in an option
ii. Describe writing option straddles and strangles as short volatility strategies
iii. Describe writing option butterflies and condors as short volatility strategies

Keywords

1. Iron butterfly
2. Iron condor
3. Short straddle
4. Short strangle

8.2.2 Demonstrate knowledge of volatility and delta-neutral portfolios with options and advanced option-based volatility strategies.

i. State the general performance drivers of delta-neutral portfolios with options
ii. Identify the key points that surround delta-neutral option portfolios
iii. Interpret delta normalization and exposure to volatility
iv. Describe vertical intra-asset option spreads
v. Create vertical spreads with delta hedging
vi. Understand horizontal intra-asset (skew) spreads
vii. Understand inter-asset option spreads

Keywords

1. Horizontal spread
2. Inter-asset option spread
3. Ratio spread
4. Vega normalization
5. Vertical spread

8.2.3 Demonstrate knowledge of variance-based and volatility-based derivative products.

i. Describe derivative strategies that create payoffs driven by realized variance
ii. Interpret implied volatility indices
iii. Understand how the CBOE Volatility Index is calculated
iv. Interpret futures contracts on the CBOE Volatility Index
v. Understand how to calculate the hypothetical price of an S&P VIX short-term futures contract
vi. Describe the process of engineering VIX-related financial derivatives
vii. Apply the VIX term structure to portfolio insurance

Keywords

1. CBOE Volatility Index (VIX)
2. S&P 500 Short-Term VIX Futures Index
3. VIX term structure

8.2.4 Demonstrate knowledge of correlation swaps and dispersion trades.

i. Understand and apply the mechanics of a correlation swap
ii. Model the relationship between correlations, security volatility, and portfolio volatility
iii. Recognize motivations to correlation trading
iv. Understand the basics of dispersion trades

Keyword

1. Correlation swap

8.2.5 Demonstrate knowledge of commonalities of volatility, correlation, and dispersion trading.

i. Understand the basics of volatility, correlation, and dispersion trading

L.O. 8.2.1 **DEMONSTRATE KNOWLEDGE OF COMMON OPTION STRATEGIES AND THEIR VOLATILITY EXPOSURES.**

In Reading 9.1, the relation between volatility and options was shown to be less definitive than the relation between vega and options. Vega, as a partial derivative, is positive for simple long option positions and negative for short option positions. However, volatility exposure, as an empirical tendency, is not always positive for long option positions, even for options on equity indices. This L.O. discusses common option strategies and their exposures to volatility.

In theory, an asset's expected return depends on its factor exposures. Investors can expect a positive volatility risk premium through equity-market option strategies that are short volatility. Short option positions are short vega and tend to be short volatility (with deep-in-the-money calls on the equity market being a possible exception). However, individual option positions (e.g., long or short a call or put) have exposures to factors other than volatility that determine their expected return. For instance, a long position in a call option on a market index has a positive market beta while a put option has a negative market beta.

THETA AS A MEASURE OF TIME DECAY IN AN OPTION

The theta of an option is the partial derivative of the option's price with respect to time, and depends on the tenor of the option. From the Black-Scholes option pricing model, the theta of a call or put option (assuming a zero riskless rate), $\theta_{r=0}$, may be expressed as:

$$\theta_{r=0} = \frac{-SN'(d)\sigma}{2\sqrt{T}},$$

where S is the underlying asset price, σ is the volatility of the underlying asset returns, $N'(d)$ is the (noncumulative) probability density function of the normal distribution at d, and T is the option's tenor or time to expiration.

- Theta is negative, since $N'(d)$, σ, and T are positive and the formula for theta has a negative sign.
- Theta is high when $N'(d)$ is high, which typically occurs for near-the-money options.
- Theta increases as T approaches zero (i.e., as the option nears its expiration date).

Theta is a measure of time decay: decline in an option value due to the passage of time (with other values such as asset price and anticipated volatility held stable). Since theta describes only one component of an option's return, it should not be viewed as causing long option positions to have a negative or low expected return. Expected returns depend on factor exposures in an informationally efficient market.

The formula for the value of an at-the-money (ATM) option (with zero riskless rate and constant volatility) illustrates time decay (since d is $\frac{\sigma}{2\sqrt{T}}$) and is proportional to $\sqrt{T}$. This is illustrated in Figure 5.

- For instance, for ATM options, all else equal, a 6-month option has $\sqrt{0.5}$ = 71% of the premium of a one-year option; and a 3-month option has $\sqrt{0.25}$ = 50% of the one-year option premium.
- Thus, a long position of a 1-year option held for nine months and a short position of a 3-month option held until expiration have offsetting time decay (holding all other variables constant).

In practice, an option's other components (e.g., underlying asset value and volatility) change over time. The long gamma of long option positions offer an asymmetrical combination of large profit potential and modest loss potential that (in an efficient market) offset the negative theta such that the option's expected return depends on its factor exposures, not its theta.