

BOOK	CHAPTER	FORMULA		VARIABLES
<b>Book 1</b> <b>Foundation of Risk Management</b>	Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM)	CAPM formula (expected return on asset $i$ )	$E(R_i) = R_f + \beta_i(R_m - R_f)$	$\beta_i = \rho_{(im)} \frac{\sigma_i}{\sigma_m} = \frac{Cov(i, m)}{\sigma_m^2} = \frac{\sigma_{im}}{\sigma_m^2}$ $R_m = \text{expected market rate of return}$ $R_f = \text{risk-free rate}$ $E(R_i) = \text{expected return on asset } i$ $\rho_{im} = \frac{Cov(i, m)}{\sigma_i \sigma_m} = \frac{\sigma_{im}}{\sigma_i \sigma_m}$
		Capital market line	$E(R_p) = R_f + \frac{\sigma_p}{\sigma_m}(R_m - R_f)$	$R_m = \text{expected market rate of return}$ $R_f = \text{risk-free rate}$ $E(R_p) = \text{portfolio expected return}$ $\sigma_p = \text{portfolio standard deviation}$ $\sigma_m = \text{market standard deviation}$
		Capital allocation line	$E(R_p) = R_f + \frac{\sigma_p}{\sigma_i}(R_i - R_f)$	$R_i = \text{expected rate of return on asset } i$ $R_f = \text{risk-free rate}$ $E(R_p) = \text{portfolio expected return}$ $\sigma_p = \text{portfolio standard deviation}$ $\sigma_i = \text{standard deviation of asset}$

	Sharpe ratio	$\text{Sharpe ratio} = \frac{E(R_p) - R_f}{\sigma(R_p)}$	$\sigma(R_p)$ = portfolio standard deviation $E(R_p)$ = portfolio expected return $R_f$ = risk-free rate
	Treynor ratio	$\text{Treynor ratio} = \frac{E(R_p) - R_f}{\beta_p}$	$\beta_p$ = portfolio beta $E(R_p)$ = portfolio expected return $R_f$ = risk-free rate
	The tracking error (TE)	$TE = \sigma(R_p - R_B)$	$R_p$ = return on the portfolio $R_B$ = return on the benchmark portfolio $(R_p - R_B)$ = active returns $\sigma(R_p - R_B)$ = standard deviation of active returns
	Jensen's Alpha	$\alpha = R_p - [R_f + \beta_p(R_m - R_f)]$	$\alpha$ (alpha) = Jensen's alpha (the excess return) $R_p$ = Actual return of the portfolio $R_f$ = Risk-free rate $\beta_p$ = Beta of the portfolio (systematic risk) $R_m$ = Return of the market
	Sortino ratio (SR)	$SR = \frac{R_p - T}{\frac{1}{N} \sum_{t=1}^N \min(0, R_{pt} - T)^2}$	$T$ = target or required rate of return $\frac{1}{N} \sum_{t=1}^N \min(0, R_{pt} - T)^2$ = downside deviation, as measured by the standard deviation of negative returns

		Information ratio (IR)	$IR = \frac{E(R_P - R_B)}{\sqrt{Var(R_P - R_B)}}$	$R_P$ = return on the portfolio $R_{Benchmark}$ = return on the benchmark portfolio
		Sortino ratio (SR)	$SR = \frac{R_p - T}{\frac{1}{N} \sum_{t=1}^N \min(0, R_{pt} - T)^2}$	$T$ = target or required rate of return $\frac{1}{N} \sum_{t=1}^N (0, R_{pt} - T)^2$ = downside deviation, as measured by the standard deviation of returns below the target.
	The Arbitrage Pricing Theory and Multifactor Models of Risk and Return	Return on a security	$R_i = E(R_i) + \beta_{i1}[I_1 - E(I_1)] + \dots + \beta_{ik}[I_k - E(I_k)] + e_i$	$R_i$ = rate of return on security $i$ $I_1 - E(I_1)$ = difference between observed and expected values in factor $k$ $\beta_{ik}$ = coefficient measuring the effect of changes in a factor $I_k$ on the rate of return of security $i$ $e_i$ = noise factor (i.e., the idiosyncratic factor).
		Variance/Covariance for a factor model with $M$ factors	$M + \frac{M^2 - M}{M}$	$M$ = number of factors in the model
		Number of covariances required	$\frac{n^2 - n}{2}$	$n$ = number of variances
		The Fama-French Model (FFM)	$E(R_i) = R_f + \beta_{i,MKT}E(R_m - R_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML)$	$E(R_i)$ = expected return on stock $i$ $R_f$ = risk-free interest rate $SMB$ = size factor $\beta_{i,SMB}$ = factor-beta for the size factor

				$HML$ = value factor $\beta_{i,HML}$ = factor-beta for the value factor $E(R_m - R_f)$ = CAPM market factor $\beta_{i,MKT}$ = factor-beta for the market-factor
<b>Book 2</b> <b>Quantitative</b> <b>Analysis</b>	Fundamentals of Probability	Mutually exclusive events	$P(A \cap B) = P(A \text{ and } B) = 0$ $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$	$P(A \cap B)$ = probability A intersection B $P(A \cup B)$ = probability A union B
		Conditional probability	$P(A   B) = \frac{P(A \cap B)}{P(B)}$	$P(A   B)$ = probability of A given B $P(A \cap B) = P(A B)P(B)$
		Independent events	$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$	
			$P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)$	
	Conditional Probability	$P(A B) = \frac{P(A)P(B)}{P(B)} = P(A)$		
Bayes' Theorem	$P(A B) = \frac{P(B A)P(A)}{P(B)}$			
Univariate Random Variables	Binomial distribution	$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, x = 0,1,2, \dots, n$	$P(X = x)$ = probability mass function of X	

			$F_X(x) = \sum_{i=1}^{ x } \binom{n}{i} p^i (1-p)^{n-i}$	$F_X(x)$ = cumulative distribution function of X
			$E(X) = np$	$E(X)$ = expectation of X
			$V(X) = np(1-p)$	$V(X)$ = variance of X
	Bernoulli distribution		$P(X = x) = p^x (1-p)^{1-x}, x = 0, 1; 0 < p < 1$	$P(X = x)$ = probability mass function
			$F_X(x) = \begin{cases} 0, & y < 0 \\ 1-p, & 0 \leq y < 1 \\ 1, & y \geq 1 \end{cases}$	$F_X(x)$ = cumulative distribution function
			$E(X) = p$	$E(X)$ = expectation of X
			$V(X) = p(1-p)$	$V(X)$ = variance of X
	Uniform distribution		$P(X = x) = \frac{1}{n}, x = 1, 2, \dots, n$	$P(X = x)$ = probability mass function
			$E(X) = \frac{n+1}{2}$	$E(X)$ = expectation of X
			$V(X) = \frac{n^2-1}{12}$	$V(X)$ = variance of X
		Poisson distribution	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots; \lambda > 0$	$P(X = x)$ = probability mass function

			$F_X(x) = e^{-\lambda} \sum_{i=1}^{ x } \frac{\lambda^i}{i!}$	$F_X(x)$ = cumulative distribution function of X
			$E(X) = V(X) = \lambda$	$E(X)$ = expectation of X
	Continuous uniform distribution		$f_X(x) = \frac{1}{b-a}, a \leq X \leq b$	$f(x)$ = probability density function of X
			$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x \geq b \end{cases}$	$F_X(x)$ = cumulative distribution function
			$E[X] = \frac{a+b}{2}$	$E(X)$ = Expectation of X
			$V[X] = \frac{(b-a)^2}{12}$	$V(X)$ = Variance of X
	Normal distribution		$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < X < \infty$	$f(x)$ = probability density function of x
			$E(X) = \mu$	$E(X)$ = expectation of X
			$V(X) = \sigma^2$	$V(X)$ = variance of X
	Log-normal distribution		$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}$	$f(x)$ = probability density function of x

			$V(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$ $E[X] = e^{\mu + \frac{1}{2}\sigma^2}$	
			$F_X(x) = \Phi\left(\frac{\ln X - \mu}{\sigma}\right)$	
	Chi-square distribution		$f(x) = \frac{1}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}, \quad x > 0$	$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$
			$E(S) = k$	$S = \sum_{i=1}^k Z_i^2$
			$V(S) = 2k$	
	Student's t-distribution		$f(x) = \frac{\Gamma\left(k + \frac{1}{2}\right)}{\sqrt{k\pi} \Gamma\left(\frac{k}{2}\right)} \left(1 + \frac{x^2}{k}\right)^{-\frac{k+1}{2}}$	$X = \frac{Z}{\sqrt{\frac{U}{k}}}$ $Z = \text{standard normal variable,}$ $U = \text{a chi-square variable with } k \text{ degrees of freedom}$
			$E(X) = 0$ $V(X) = \frac{k}{k-2}$	
	F-distribution		$f(x) = \frac{\sqrt{\frac{(k_1 X)^{k_1} k_2^{k_2}}{(k_1 X + k_2)^{k_1 + k_2}}}}{xB\left(\frac{k_1}{2}, \frac{k_2}{2}\right)}$	$X = \frac{U_1/k_1}{U_2/k_2} \sim F(k_1, k_2)$ $U_i = \text{chi-squared distributions that are independent with } k_i \text{ degrees of freedom.}$