



SimpleSheets™+

CFA® Exam Formulas | Level 1

2025 Edition



SimpleSheets+

Formulas at Your Fingertips

Quantitative Methods - Prereadings

Common Probability Distributions

- Discrete Uniform Distribution

$F(x) = n \times p(x)$ for the n th observation.

- Binomial Distribution

$$P(X=x) = {}_n C_x (p)^x (1-p)^{n-x}$$

where:

p = probability of success

$1-p$ = probability of failure

${}_n C_x$ = number of possible combinations of having x successes in n trials. Stated differently, it is the number of ways to choose x from n when the order does not matter.

- Mean of a Binomial Random Variable

$$\overline{B(n,p)} = np$$

- Variance of a Binomial Random Variable

$$\sigma_x^2 = n \times p \times (1-p)$$

- The Continuous Uniform Distribution

$$P(X < a), P(X > b) = 0$$

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

- Confidence Intervals

For a random variable X that follows the normal distribution:

The 90% confidence interval is $\bar{x} - 1.65s$ to $\bar{x} + 1.65s$

The 95% confidence interval is $\bar{x} - 1.96s$ to $\bar{x} + 1.96s$

The 99% confidence interval is $\bar{x} - 2.58s$ to $\bar{x} + 2.58s$

The following probability statements can be made about normal distributions

- Approximately 50% of all observations lie in the interval $\mu \pm (2/3)\sigma$
- Approximately 68% of all observations lie in the interval $\mu \pm 1\sigma$
- Approximately 95% of all observations lie in the interval $\mu \pm 2\sigma$
- Approximately 99% of all observations lie in the interval $\mu \pm 3\sigma$

- z-Score

$$z = (\text{observed value} - \text{population mean}) / \text{standard deviation} = (x - \mu) / \sigma$$

- Continuously Compounded Returns

$$\text{EAR} = e^{r_{cc}} - 1 \quad r_{cc} = \text{continuously compounded annual rate}$$

$$r_{cc} = \ln(1 + \text{HPR})$$

$(1 + \text{HPR})$ simply equals (V_t/V_0) . Therefore, the continuously compounded rate of return can also be calculated as:

$$r_{cc} = \ln\left(\frac{V_t}{V_0}\right)$$

$$\text{HPR}_t = e^{r_{cc} \cdot t} - 1$$

Sampling and Estimation

- Sampling Error

$$\text{Sampling error of the mean} = \text{Sample mean} - \text{Population mean} = \bar{x} - \mu$$

- Standard Error of Sample Mean when Population Variance is known

$$\sigma_x = \frac{\sigma}{\sqrt{n}}$$

where:

σ_x = the standard error of the sample mean

σ = the population standard deviation

n = the sample size

- Standard Error of Sample Mean when Population Variance is not known

$$s_x = \frac{s}{\sqrt{n}}$$

where:

s_x = standard error of sample mean

s = sample standard deviation

- Confidence Intervals

Point estimate \pm (reliability factor \times standard error)

where:

Point estimate = value of the sample statistic that is used to estimate the population parameter

Reliability factor = a number based on the assumed distribution of the point estimate and the level of confidence for the interval $(1 - \alpha)$

Standard error = the standard error of the sample statistic (point estimate)

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where:

\bar{x} = The sample mean (point estimate of population mean)

$z_{\alpha/2}$ = The standard normal random variable for which the probability of an observation lying in either tail is $\sigma / 2$ (reliability factor)

$\frac{\sigma}{\sqrt{n}}$ = The standard error of the sample mean

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where:

\bar{x} = sample mean (the point estimate of the population mean)

$t_{\alpha/2}$ = the t-reliability factor

$\frac{s}{\sqrt{n}}$ = standard error of the sample mean

s = sample standard deviation

Rates and Returns

- Arithmetic Mean

$$\mu = \sum_{i=1}^N X_i / N \quad \text{for a population}$$

$$\bar{X} = \sum_{i=1}^n X_i / n \quad \text{for a sample}$$

- Weighted Mean

$$\bar{X}_w = \sum_{i=1}^n w_i X_i$$

- Geometric Mean

$$\bar{X}_G = \left[\prod_{i=1}^n (1 + X_i) \right]^{1/n} - 1$$

which can be alternatively written for portfolio returns as:

$$R_G = \sqrt[n]{(1 + R_1)(1 + R_2) \dots (1 + R_n)} - 1$$

- Holding Period Return

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}} = \text{Capital gain} + \text{Dividend yield}$$

$$= \frac{P_t + D_t}{P_0} - 1$$

where:

P_t = Price at the end of the period

P_{t-1} = Price at the beginning of the period

D_t = Dividend for the period

- Holding Period Returns for more than One Period

$$R = [(1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_n)] - 1$$

where:

R_1, R_2, \dots, R_n are sub-period returns

- Geometric Mean Return

$$R = [((1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_n))^{1/n}] - 1$$

- Time Weighted Rate of Return

If we have annual returns data, the annualized time-weighted return can be calculated as the geometric mean of N annual returns, as follows:

$$TW = [(1 + r_1) \times (1 + r_2) \times \dots \times (1 + r_n)]^{1/n} - 1$$

- Annualized Return

$$r_{\text{annual}} = (1 + r_{\text{period}})^n - 1$$

where:

r = Return on investment

n = Number of periods in a year

- Harmonic Return

$$\bar{X}_H = \frac{n}{\sum_{i=1}^n (1/X_i)} \quad \text{with } X_i > 0$$

The Time Value of Money in Finance

- Effective Annual Rates

$$EAR = (1 + \text{Periodic interest rate})^N - 1$$

- The Future Value of a Single Cash Flow

$$FV_N = PV (1 + r)^N$$

- The Present Value of a Single Cash Flow

$$PV = \frac{FV}{(1 + r)^N}$$

- The Present and Future Value of an Ordinary Annuity

PV_{Annuity} : # periods N ; % interest per period I/Y ; amount $PMT \rightarrow PV$

FV_{Annuity} : # periods N ; % interest per period I/Y ; amount $PMT \rightarrow FV$

- The Present and Future Value of an Annuity Due

$$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1 + r)$$

$$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1 + r)$$

- Present Value of a Perpetuity

$$PV_{\text{Perpetuity}} = \frac{PMT}{I/Y}$$

- Continuous Compounding and Future Values

$$FV_N = PV e^{r \times N}$$

- PV of a Coupon Bond (v1, p51, Eq 6)

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \frac{PMT_3}{(1+r)^3} + \dots + \frac{(PMT_N + FV_N)}{(1+r)^N}$$

- Periodic Payment for fully amortizing loan (v1, p54, Eq 8):

$$A = \frac{r \times \text{Principal}}{1 - (1+r)^{-N}}$$

- Equity with a constant dividend (v1, p56, Eq 10)

$$PV_t = \frac{D_t}{r}$$

- Equity with a dividend with constant growth (v1, p57, Eq 14)

$$PV_t = \frac{D_{t+1}}{(r-g)}$$

- Bond Implied Return (v1, p61, Eq 18)

$$r = \left(\frac{FV_t}{PV} \right)^{1/t} - 1$$

- Equity Implied Return (v1, p65, Eq 21)

$$r = \frac{D_{t+1}}{PV_t}$$

- Equity Implied Growth (v1, p65, Eq 22)

$$g = r - \frac{D_{t+1}}{PV_t}$$

Statistical Measures of Asset Returns

- Quantiles

$$L_y = (n+1) \frac{y}{100}$$

- Mean Absolute Deviation

$$MAD = \frac{\sum_{i=1}^n |X_i - \bar{X}|}{n}$$

- Sample Variance

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

$$S = \sqrt{S^2}$$

- Geometric Mean

$$X_G = \bar{X} - \frac{S^2}{2}$$

- Target Downside Deviation

$$S_{\text{target}} = \sqrt{\frac{\sum_{\text{for } X_i < B} (X_i - B)^2}{n-1}}$$

- Coefficient of Variation

$$CV = \frac{S}{\bar{X}}$$

- Skewness

$$\text{skewness} = \left(\frac{1}{n} \right) \frac{\sum_{i=1}^n (X_i - \bar{X})^3}{S^3}$$

- Kurtosis

$$K_E = \left[\frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^4}{S^4} \right] - 3$$

- Covariance

$$COV_{XY} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

- Correlation

$$\rho_{XY} = \frac{COV_{XY}}{S_X S_Y}, \text{ where } -1 \leq \rho_{XY} \leq +1$$

Probability Trees and Conditional Expectations

- Odds for an Event

Where the odds *for* are given as “a to b”, then:

$$P(E) = \frac{a}{(a + b)}$$

Where the odds *against* are given as “a to b”, then:

$$P(E) = \frac{b}{(a + b)}$$

- Conditional Probabilities

$$P(A|B) = \frac{P(AB)}{P(B)} \text{ given that } P(B) \neq 0$$

- Multiplication Rule for Probabilities

$$P(AB) = P(A|B) \times P(B)$$

- Addition Rule for Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

- For Independent Events

$$P(A|B) = P(A), \text{ or equivalently, } P(B|A) = P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

- The Total Probability Rule

$$P(A) = P(AS) + P(AS^c)$$

$$P(A) = P(A|S) \times P(S) + P(A|S^c) \times P(S^c)$$

- The Total Probability Rule for n Possible Scenarios

$$P(A) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + \dots + P(A|S_n) \times P(S_n)$$

where the set of events $\{S_1, S_2, \dots, S_n\}$ is mutually exclusive and exhaustive.

Portfolio Mathematics

- Expected Value

$$E(X) = P(X_1)X_1 + P(X_2)X_2 + \dots + P(X_n)X_n$$

$$E(X) = \sum_{i=1}^n P(X_i)X_i$$

where:

X_i = one of n possible outcomes.

- Variance and Standard Deviation

$$\sigma^2(X) = E[(X - E(X))^2]$$

$$\sigma^2(X) = \sum_{i=1}^n P(X_i) [X_i - E(X)]^2$$

- The Total Probability Rule for Expected Value

$$1. E(X) = E(X|S)P(S) + E(X|S^c)P(S^c)$$

$$2. E(X) = E(X|S_1) \times P(S_1) + E(X|S_2) \times P(S_2) + \dots + E(X|S_n) \times P(S_n)$$

where:

$E(X)$ = the unconditional expected value of X

$E(X|S_i)$ = the expected value of X given Scenario 1

$P(S_i)$ = the probability of Scenario 1 occurring

The set of events $\{S_1, S_2, \dots, S_n\}$ is mutually exclusive and exhaustive.

- Covariance

$$Cov(XY) = E[(X - E(X))(Y - E(Y))]$$

$$Cov(R_A, R_B) = E[(R_A - E(R_A))(R_B - E(R_B))]$$

- Correlation Coefficient

$$Corr(R_A, R_B) = \rho(R_A, R_B) = \frac{Cov(R_A, R_B)}{(\sigma_A)(\sigma_B)}$$

- Expected Return on a Portfolio

$$E(R_p) = \sum_{i=1}^N w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_N E(R_N)$$

where:

$$\text{Weight of asset } i = \frac{\text{Market value of investment } i}{\text{Market value of portfolio}}$$

- Portfolio Variance

$$\text{Var}(R_p) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j Cov(R_i, R_j)$$

- Variance of a 2 Asset Portfolio

$$\text{Var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B Cov(R_A, R_B)$$

$$\text{Var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B \rho(R_A, R_B) \sigma(R_A) \sigma(R_B)$$

- Variance of a 3 Asset Portfolio

$$\text{Var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + w_C^2 \sigma^2(R_C) + 2w_A w_B Cov(R_A, R_B) + 2w_B w_C Cov(R_B, R_C) + 2w_C w_A Cov(R_C, R_A)$$

- Bayes' Formula

$$P(\text{Event} | \text{Information}) = \frac{P(\text{Information} | \text{Event}) \times P(\text{Event})}{P(\text{Information})}$$

- Roy's Safety-First Criterion

$$\text{Minimize } P(R_p < R_T)$$

where:

R_p = portfolio return

R_T = target return

- Safety-First

$$\text{Safety-First ratio (SF Ratio)} = \frac{E(R_p) - R_T}{\sigma_p}$$

Hypothesis Testing

- Test Statistic

$$\text{Test statistic} = \frac{\text{Sample statistic} - \text{Hypothesized value}}{\text{Standard error of sample statistic}}$$

- Power of a Test

$$\text{Power of a test} = 1 - P(\text{Type II error})$$

- Decision Rules for Hypothesis Tests

Decision	H_0 is True	H_0 is False
Do not reject H_0	Correct decision	Incorrect decision Type II error
Reject H_0	Incorrect decision Type I error Significance level = P(Type I error)	Correct decision Power of the test = $1 - P(\text{Type II error})$

- Confidence Interval

$$\left[\left(\frac{\text{sample}}{\text{statistic}} \right) - \left(\frac{\text{critical}}{\text{value}} \right) \left(\frac{\text{standard}}{\text{error}} \right) \right] \leq \left(\frac{\text{population}}{\text{parameter}} \right) \leq \left[\left(\frac{\text{sample}}{\text{statistic}} \right) + \left(\frac{\text{critical}}{\text{value}} \right) \left(\frac{\text{standard}}{\text{error}} \right) \right]$$

$$\bar{x} - (z_{\alpha/2}) (s/\sqrt{n}) \leq \mu_0 \leq \bar{x} + (z_{\alpha/2}) (s/\sqrt{n})$$