Economics

Learning Module 1
Currency Exchange Rates: Understanding Equilibrium Value

LOS: Calculate and interpret the bid-offer spread on a spot or forward currency quotation and describe the factors that affect the bid-offer spread.

LOS: Identify a triangular arbitrage opportunity and calculate its profit, given the bid-offer quotations for three currencies.

LOS: Explain spot and forward rates and calculate the forward premium/discount for a given currency.

LOS: Calculate the mark-to-market value of a forward contract.

LOS: Explain international parity conditions (covered and uncovered interest rate parity, forward rate parity, purchasing power parity, and the international Fisher effect).

LOS: Describe relations among the international parity conditions.

LOS: Evaluate the use of the current spot rate, the forward rate, purchasing power parity, and uncovered interest parity to forecast future spot exchange rates.

LOS: Explain approaches to assessing the long-run fair value of an exchange rate.

LOS: Describe the carry trade and its relation to uncovered interest rate parity and calculate the profit from a carry trade.

LOS: Explain how flows in the balance of payment accounts affect currency exchange rates.

LOS: Explain the potential effects of monetary and fiscal policy on exchange rates.

LOS: Describe objectives of central bank or government intervention and capital controls and describe the effectiveness of intervention and capital controls.

LOS: Describe warning signs of a currency crisis.

Foreign Exchange Market Concepts

LOS: Calculate and interpret the bid-offer spread on a spot or forward currency quotation and describe the factors that affect the bid-offer spread.

An exchange rate represents the price of one currency in terms of another currency. It is stated as the number of units of a particular currency (the price currency) required to purchase one unit of another currency (the base currency).

CFA curriculum uses the convention *P/B*: the number of units of the price (*P*) currency needed to purchase one unit of the base (*B*) currency. For example, suppose the USD/GBP exchange rate is currently 1.5125. From this exchange rate quote, we can infer the following:

- The GBP is the base currency, and the USD is the price currency.
- 1 GBP will buy 1.5125 USD.
- It will take 1.5125 USD to purchase 1 GBP, or 1 GBP costs 1.5125 USD.
- A decrease in the exchange rate (eg, from 1.5125 to 1.5120) means that 1 GBP will be able to purchase fewer USD.
    - Fewer USD will now be required to purchase 1 GBP (ie, the cost of 1 GBP has fallen).
    - This decrease in the exchange rate means that the GBP has depreciated (ie, lost value) against the USD, or equivalently, that the USD has appreciated (ie, gained value) against the GBP.

Just like the price of any product, the price reflected in an exchange rate is the amount of the numerator currency that can be purchased with one unit of the denominator currency.

Spot exchange rates (*S*) are quotes for transactions that call for immediate delivery. For most currencies, immediate delivery means "*T* + 2" (ie, the transaction is settled 2 days after the trade is agreed upon by the parties).

In professional FX markets, an exchange rate is usually quoted as a two-sided price. Dealers typically quote both a bid price (ie, the price at which they are willing to buy) and an offer price (ie, the price at which they are willing to sell). Bid-offer prices are always quoted in terms of buying and selling the base currency.

Bid-offer quotes in foreign exchange have two main points:

- The offer price is higher than the bid price, which creates the bid-offer spread, a compensation for providing foreign exchange.
- Requesting a two-sided quote from the dealer allows a choice between whether the base currency will be bought (ie, paying the offer) or sold (ie, hitting the bid). This choice provides flexibility in transactions.

In FX, dealers have two pricing levels: one level for clients and another for the interbank market. Dealers engage in currency transactions among themselves in the interbank market in order to adjust their inventories and risk positions, distribute foreign currencies to clients, and transfer FX rate risk to willing market participants. This global network handles large transactions, typically over 1 million units of the base currency; nonbank entities like institutional asset managers and hedge funds can also access the network.

The bid-offer spread that dealers provide to clients is typically wider than what is observed in the interbank market.

The bid-offer spread is sometimes measured in points, or pips, which are scaled to the last digit in the spot exchange rate quote. Exchange rates for most currency pairs (except those involving the Japanese yen) are quoted to four decimal places. For example, the bid-offer spread in the interbank market for USD/EUR might be 1.2500–1.2504. This is a difference of 0.0004, or 4 pips, while a dealer’s spread for the same currency pair may be 0.0006, or 6 pips.

The bid-offer spread in the FX market, as quoted to dealers’ clients, can vary widely among different exchange rates and can change over time, even for a single exchange rate. The spread size is primarily influenced by the bid-offer spread in the interbank market, transaction size, and the relationship between the dealer and the client. A client’s creditworthiness can also be a factor, although, given the short settlement cycle in the spot FX market, credit risk is not the primary determinant of bid-offer spreads.

The spread size in the interbank market depends on liquidity, which is influenced by the following:

- **Currency pair:** The liquidity and market participation levels differ between currency pairs. Major pairs like USD/EUR and JPY/USD usually have high liquidity and narrower spreads due to widespread market activity. Conversely, more obscure currency pairs (eg, MXN/CHF) have thinner market participation, resulting in wider spreads.
- **Time of day:** Liquidity in the interbank FX market is highest when major trading centers like London and New York overlap, typically from 8:00 a.m. to 11:00 a.m. (New York time). During this period, most currency pairs are more liquid. The Asian session, starting around 7:00 p.m. (New York time) is less liquid for most pairs.
- **Market volatility:** Greater uncertainty in the market—caused by events like geopolitical conflicts, market crashes, or major data releases—leads to wider bid-offer spreads. During such times, market participants seek to reduce their risk exposure or charge higher prices for assuming risk.

Arbitrage Constraints on Spot Exchange Rate Quotes

LOS: Identify a triangular arbitrage opportunity and calculate its profit, given the bid-offer quotations for three currencies.

Bid-offer quotes provided by dealers in the interbank market must adhere to two arbitrage constraints:

- The bid quoted by a dealer cannot be higher than the current interbank offer, and the offer from another dealer cannot be lower than the current interbank bid. Otherwise, other participants in the interbank market would be able to earn riskless arbitrage profits by purchasing low and selling high. To illustrate, assume that the current USD/EUR exchange rate in the interbank market is 1.3802–1.3806.
  - If a dealer quoted a (misaligned) exchange rate of 1.3807–1.3811, other market participants would buy the EUR in the interbank market (at 1.3806 USD/EUR) and sell EUR to the dealer (at 1.3807 USD/EUR) to make a profit of 1 pip.
  - If a dealer quoted a (misaligned) exchange rate of 1.3797–1.3801, other market participants would buy the EUR from the dealer (at 1.3801 USD/EUR) and sell EUR in the interbank market (at 1.3802 USD/EUR) to make a profit of 1 pip.
- The cross-rate bid quoted by a dealer cannot be higher than the implied cross rate offered in the interbank market, while the cross rate offer quoted by another dealer cannot be lower than the implied cross rate bids in the interbank market.
  - The implied A/C cross rate is derived from exchange rate quotes for two currency pairs, A/B and C/B. This cross rate must align with the A/B and C/B rates to ensure consistency and prevent arbitrage opportunities.
  - In FX cross rates, there are two trading methods: (1) using the direct cross rate A/C or (2) using the A/B and C/B rates. Inconsistent rates prompt arbitrageurs to buy the undervalued currency and sell the overvalued one, restoring balance.

To illustrate **triangular arbitrage** involving three currencies, assume the following exchange rate quotes from a European bank: USD/EUR = 1.3802–1.3806 and GBP/EUR = 0.8593–0.8599.

When working with bid-offer quotes to determine whether to use the bid or offer rate in an exchange rate quote for a specific transaction:

- first identify the base currency of the exchange rate quote, and then
- determine whether the client is buying or selling the base currency.

In this example, the bid and offer quotes for EUR/USD (with the USD as the base currency) are calculated as:

$$\frac{EUR}{USD_{bid}} = \frac{1}{\frac{USD}{EUR_{offer}}} = \frac{1}{1.3806} = 0.7243$$

$$\frac{EUR}{USD_{offer}} = \frac{1}{\frac{USD}{EUR_{bid}}} = \frac{1}{1.3802} = 0.7245$$

Therefore, EUR/USD bid/ask rates = 0.7243−0.7245.

To calculate the market-implied bid-offer quote for the GBP/USD cross rate, first examine the transactions needed to exchange GBP for USD, which involve the EUR/USD and GBP/EUR currency pairs. The transactions are to:

- sell GBP and buy EUR, and then
- sell EUR and buy USD.

This transaction can be represented by the equation:

$$\frac{GBP}{EUR} \times \frac{EUR}{USD} = \frac{GBP}{USD}$$

On the right-hand side of the equals sign, "sell GBP, buy USD" is given in the GBP/USD price quote. Since the aim is to buy the currency in the denominator (USD), the offer rate calculated is for GBP/USD.

On the left-hand side, this breaks down as follows:

- "Sell GBP, buy EUR"—the offer rate for GBP/EUR
- "Sell EUR, buy USD"—the offer rate for EUR/USD

Then, a GBP/USD offer rate is:

$$\left(\frac{GBP}{USD}\right)_{offer} = \left(\frac{EUR}{USD}\right)_{offer}\left(\frac{GBP}{EUR}\right)_{offer} = 0.7245 \times 0.8599 = 0.6230$$

Calculating the implied GBP/USD bid rate follows the same process, but with "buy GBP, sell USD," which results in:

$$\left(\frac{GBP}{USD}\right)_{bid} = \left(\frac{EUR}{USD}\right)_{bid}\left(\frac{GBP}{EUR}\right)_{bid} = 0.7243 \times 0.8593 = 0.6224$$

Therefore, the market-implied bid-offer quote for GBP/USD is 0.6224–0.6230.

To prevent arbitrage opportunities, the implied bid rate must be lower than the implied offer rate. The constraints on implied cross rates are like those for spot rates but involve currency pairs. In practice, any violations of these constraints quickly disappear since traders and algorithms exploit the pricing inefficiencies.

Forward Markets

LOS: Explain spot and forward rates and calculate the forward premium/discount for a given currency.

Forward contracts involve exchanging one currency for another on a future date at a pre-agreed exchange rate. Any currency exchange transaction with a settlement date beyond T + 2 qualifies as a forward contract.

When determining forward exchange rates, an arbitrage relationship is applied to ensure the return on two equivalent investments is the same. Bid-offer spreads on exchange rates and money market instruments should be ignored in the explanation of this relationship in order to keep it simple and clear.

In addition, assuming the domestic currency is the base currency in the exchange rate quote, the exchange rate notation can be shifted from price/base currency (*P/B*) to foreign/domestic currency (*f/d*). This notation helps to illustrate an investor's decision-making process when choosing between domestic and foreign investments, and it shows how arbitrage relationships ensure equalized returns when the investments' risk profiles are similar.

An investor with 1 unit of domestic currency for a year has two options:

- Invest in domestic cash at the domestic risk-free rate (i_d), resulting in $(1 + i_d)$ at year-end.
- Convert domestic currency to foreign currency at the spot rate ($S_{f/d}$), invest for a year at the foreign risk-free rate (i_f), and end up with $S_{f/d}(1 + i_f)$ units of foreign currency. These funds need to be converted back to domestic currency. By using a one-year forward contract with a forward rate denoted as $F_{f/d}$, the investor eliminates foreign exchange risk, as the future exchange rate is set at the start of the period. The investment's worth in domestic currency at the end of the year equals $S_{f/d}(1 + i_f)(1 / F_{f/d})$.

These two risk-free investment alternatives must provide the same return to avoid creating an opportunity for riskless arbitrage.

$$(1 + i_d) = S_{f/d}(1 + i_f)\left(\frac{1}{F_{f/d}}\right)$$

This explanation is based on a one-year horizon, but it holds for any investment timeline. In this arbitrage relationship, risk-free assets, typically represented by bank deposits, are quoted using the appropriate **market reference rate** (MRR) for each currency. Day count conventions, such as actual/360 or actual/365, are used to calculate interest. For simplicity, this discussion consistently employs the actual/360-day count convention, except for GBP, which uses the actual/365 convention. Integrating this day count convention into the arbitrage formula leads to:

$$\left(1 + i_d\left[\frac{\text{Actual}}{360}\right]\right) = S_{f/d}\left(1 + i_f\left[\frac{\text{Actual}}{360}\right]\right)\left(\frac{1}{F_{f/d}}\right)$$

The equation can be rearranged to isolate the forward rate:

$$F_{f/d} = S_{f/d}\left(\frac{1 + i_f\left[\frac{\text{Actual}}{360}\right]}{1 + i_d\left[\frac{\text{Actual}}{360}\right]}\right)$$

This equation represents **covered interest rate parity**, which relies on an arbitrage relationship involving risk-free interest rates, spot exchange rates, and forward exchange rates. This relationship between investment options asserts that the covered (ie, currency-hedged) interest rate differential between two markets is equal to zero.

The forward premium or discount can be calculated using the rearranged covered interest rate parity equation.

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{\left[\frac{\text{Actual}}{360}\right]}{1 + i_d\left[\frac{\text{Actual}}{360}\right]}\right)(i_f - i_d)$$

The domestic currency trades at a forward premium ($F_{f/d} > S_{f/d}$) only when the foreign risk-free interest rate is greater than the domestic risk-free interest rate ($i_f > i_d$). If it is possible to earn a higher interest rate in the foreign market than in the domestic market, the forward discount on the foreign currency will offset the higher foreign interest rate. If not, covered interest rate parity would not be maintained, and arbitrage opportunities would arise.

Conversely, when the foreign currency is at a higher rate in the forward contract than the spot rate, it is trading at a forward premium. In this scenario, the foreign risk-free interest rate is less than the domestic risk-free interest rate.

Furthermore, the premium or discount is influenced by the spot exchange rate ($S_{f/d}$)—the interest rate differential ($i_f - I_d$) between the markets—and is roughly proportional to the time to maturity (actual/360).

The covered interest rate parity equation can be expressed using price and base currencies(P/B), a more standard exchange rate quoting convention.

$$F_{P/B} = S_{P/B}\left(\frac{1 + i_P\left[\frac{\text{Actual}}{360}\right]}{1 + i_B\left[\frac{\text{Actual}}{360}\right]}\right)$$

The forward premium or discount equation can be expressed in the same way:

$$F_{P/B} - S_{P/B} = S_{P/B}\left(\frac{\left[\frac{\text{Actual}}{360}\right]}{1 + i_B\left[\frac{\text{Actual}}{360}\right]}\right)(i_P - i_B)$$

Example 1 Calculating the forward premium (discount)

An analyst gathered the following information:

- Spot USD/EUR = 1.4562
- 180-day risk-free rate (USD) = 1.05%
- 180-day risk-free rate (EUR) = 2.38%

Calculate the forward premium (discount) for a 180-day forward contract for USD/EUR.

Solution

$$F_{P/B} - S_{P/B} = S_{P/B}\left(\frac{\left[\frac{\text{Actual}}{360}\right]}{1 + i_B\left[\frac{\text{Actual}}{360}\right]}\right)(i_P - i_B) = 1.4562\left(\frac{\left[\frac{180}{360}\right]}{1 + 0.238\left[\frac{180}{360}\right]}\right)(0.0105 - 0.0238) = -0.00865$$

Therefore, the forward discount for a 180-day forward contract for USD/EUR is approximately 0.865%.

In professional foreign exchange (FX) markets, FX rates are quoted in points (pips), indicating the difference between the forward and spot rates (forward premium or discount). The pips are scaled to correspond with the last digit in the spot quote, typically the fourth decimal place.

Exhibit 1 Sample spot and forward quotes (bid offer)

Maturity	Spot rate or forward points
Spot USD/EUR	1.3802 / 1.3806
One month	-5.4 / -4.9
Three months	-15.8 / -15.2
Six months	-36.9 / -36.2
12 months	-93.9 / -91.4

It is important to note that:

- The bid rate is always lower than the offer rate.
- In this scenario, negative forward points indicate that the EUR (base currency) is trading at a forward discount, while the USD (price currency) is trading at a forward premium.
- The absolute value of forward points increases with the time to maturity.

- Quoted forward points are scaled to each maturity and are not annualized, so no adjustment is needed before adding them to the spot rate to calculate the forward rate.
- To convert forward point quotes into an actual forward exchange rate, divide the number of pips by 10,000 (scaling them down to the fourth decimal place), and then add the result to the quoted spot exchange rate:

Spreads in the forward market are influenced by the term of maturity of the contract. Spreads tend to widen with longer terms due to the:

- reduced liquidity of longer-term contracts,
- greater credit risk in longer-term contracts, and
- greater interest rate risk in forward contracts. Forward rates are based on interest rate differentials. Longer maturities result in greater duration or higher sensitivity to changes in interest rates.

The Mark-to-Market Value of a Forward Contract

LOS: Calculate the mark-to-market value of a forward contract.

A forward contract is priced to have zero value to either party at contract initiation. In the case of currency forwards, this no-arbitrage forward price is determined based on interest rate parity. However, once the counterparties have entered the contract, changes in the forward price (due to changes in the spot exchange rate or interest rate changes in either of the two currencies) will alter the contract's mark-to-market value. The contract then holds positive value for one counterparty and an equivalent negative value for the other.

Example 2 Valuing a forward contract prior to expiration

An investor purchased GBP 10 million for delivery against AUD in 6 months (t = 180) at an all-in forward rate of 1.5920 AUD/GBP. Four months later (t = 120), the investor wants to close out their position.

Spot exchange rate and forward points at t = 120

Maturity	Spot rate or forward points
Spot AUD/GBP	1.6110 / 1.6115
One month	5.1 / 5.2
Two months	10.3 / 10.5
Three months	15.9 / 16.2
Four months	26.4 / 26.8

Assume that the 60-day risk-free rate at t = 120 is 4.20%.

- What position would the investor take, and on which contract, to effectively close out their forward position at t = 120?
- Calculate the gain or loss the investor would incur when closing out the forward position at t = 120.

The all-in forward rate is simply the sum of the spot rate and the forward points, appropriately scaled to size.

Solutions

- The investor initially held a long GBP position in a 6-month contract (t = 180). Four months into this contract (at t = 120), to close out the forward position, the investor takes an opposite short position in a GBP 10 million offsetting forward contract that expires in another 2 months (at t = 180).

  To sell GBP forward, the relevant exchange rate is the all-in AUD/GBP bid. The appropriate all-in two-month forward exchange rate is calculated based on the spot rate at t = 120 and the forward points on the two-month forward bid exchange rate:

  1.6110 + (10.3 / 10,000) = 1.61203 AUD/GBP

  This means that the investor initially purchased 10 million GBP (for delivery at t = 180) at 1.5920 AUD/GBP and then (at t = 120) sold 10 million GBP (for delivery at t = 180) at 1.61203 AUD/GBP. The GBP amounts will net to zero at settlement, but the AUD amounts will not, as the forward rate has changed over the four months. At contract expiration, the investor stands to make a profit (loss) of:

  Profit (loss) = (1.61203 − 1.5920) AUD/GBP × 10,000,000 GBP = AUD 200,300

  The investor profits since there was a long position on the GBP and the forward rate increased (GBP appreciated) during the four-month period from t = 0 to t = 120. It is important to note that the investor would realize this profit at t = 180 when both forward contracts settle.

  To calculate the mark-to-market value of the investor's position at t = 120 (when the forward position is effectively closed out through the offsetting contract), it is necessary to discount the settlement payment for two months (the time remaining until contract expiration) at the two-month discount rate. Given that the 60-day risk-free rate at t = 120 is 4.20%, the mark-to-market value of the original long GBP 10 million six-month forward contract two months before settlement is calculated as follows:

$$\text{AUD } 200{,}300 \times \frac{1}{\left[1 + \left(0.042 \times \frac{60}{360}\right)\right]} = \text{AUD } 198{,}907.65$$

The steps for marking to market a currency forward position are outlined below:

- Create an equal offsetting forward position to the initial forward contract. Ensure that the settlement dates and notional amounts in both contracts are the same.
- Determine the all-in forward rate for the offsetting forward contract. If the base currency in the exchange rate quote should be sold in the offsetting contract, use the bid side of the quote. If it should be purchased, use the offer side.
- Calculate the profit or loss on the net position as of the settlement date.
  - A profit occurs if the currency the investor was long on in the initial forward contract has appreciated; a loss occurs if that currency has depreciated.
  - A loss occurs if the currency the investor was short on in the initial forward contract has appreciated; a profit occurs if that currency has depreciated.
- Calculate the present value of the profit or loss (as of the date of initiation of the offsetting contract). Use the appropriate LIBOR rate and adjust it if necessary.

The International Parity Conditions

LOS: Explain international parity conditions (covered and uncovered interest rate parity, forward rate parity, purchasing power parity, and the international Fisher effect).

Before discussing international parity relations, it is essential to grasp the following concepts:

- **Long run versus short run:** Parity relations provide estimates of exchange rates in the long run, and they are typically poor predictors of exchange rates in the short run. Long-term equilibrium values act as an anchor for exchange rate movements, and short-term exchange rates fluctuate around those values.
- **Expected versus unexpected changes:** Expected changes are generally reflected in current prices (including exchange rates), while unexpected changes introduce risk as they can lead to more significant price movements. Consequently, investors demand a premium for bearing the risk associated with unpredictable outcomes.
- **Relative movements:** Exchange rates are influenced by relative changes in economic factors across countries, not absolute or isolated changes. Since an exchange rate represents the price of one currency in terms of another, it is essential to evaluate the inflation rate in one country relative to the inflation rate in the other country when determining the impact on the exchange rate between their currencies.

There is no simple formula, model, or theory that can enable investors to accurately forecast exchange rates. However, the theories that will be discussed offer a framework for developing a perspective on exchange rates and for understanding some of the forces that influence them.

International Parity Conditions

International parity conditions serve as the foundational principles for most exchange rate models, and include:

- Covered interest rate parity
- Uncovered interest rate parity
- Forward rate parity
- Purchasing power parity
- The international Fisher effect

These conditions aim to establish connections between expected inflation differentials, interest rate disparities, forward and spot exchange rates, and anticipated future spot exchange rates, under idealized circumstances. It is important to note that the conditions often depend on simplifying assumptions such as negligible transaction costs, universally available information, risk neutrality, and freely adjustable market prices.

In practice, empirical studies have shown that these parity conditions are rarely met in the short term. However, they play a crucial role in forming a comprehensive, long-term perspective on exchange rates and associated risk exposures. The exception to this rule is covered interest rate parity, which is the only condition actively enforced by arbitrage opportunities.

Covered and Uncovered Interest Rate Parity and Forward Rate Parity

LOS: Explain international parity conditions (covered and uncovered interest rate parity, forward rate parity, purchasing power parity, and the international Fisher effect).

LOS: Describe relations among the international parity conditions.

LOS: Evaluate the use of the current spot rate, the forward rate, purchasing power parity, and uncovered interest parity to forecast future spot exchange rates.

Covered interest rate parity describes a no-arbitrage condition in which the covered or currency-hedged interest rate differential between two currencies equals zero. This means that there is a no-arbitrage relationship among risk-free interest rates, spot exchange rates, and forward exchange rates.

- If the risk-free rate on one currency is higher than on another currency, the currency with the higher risk-free rate will trade at a forward discount relative to the other currency. The benefit of the higher interest rate is offset by a decline in the currency's value.
- If the risk-free rate of the price currency is greater than that of the base currency, the base currency will trade at a forward premium (ie, the forward exchange rate will be higher than the spot exchange rate). This implies that the price currency will trade at a forward discount and is expected to depreciate in the future.
- Conversely, the currency with the lower risk-free rate will trade at a forward premium relative to the other currency. The benefit of the expected appreciation of the currency is offset by the lower interest rate.

For covered interest rate parity to hold, zero transaction costs and free capital mobility must be assumed. It is also assumed that the underlying money market instruments are identical in terms of liquidity, maturity, and default risk. Covered interest rate differentials are generally close to zero under normal market conditions, indicating that covered interest parity tends to hold.

Uncovered Interest Rate Parity

Uncovered interest rate parity (UIP) states that the expected return on an uncovered or unhedged foreign currency (FC) investment should equal that of a comparable domestic currency (DC) investment. UIP asserts that an investor's expected return from the following investment options should be equal.

Option 1: Invest the funds at the domestic nominal risk-free rate (i_{DC}) for a particular period of time.

- If 1 unit of DC is invested at i_{DC} for 1 year, the value after 1 year would be $(1 + i_{DC})$.

Option 2: Convert funds into a foreign currency (at the current spot rate, $S_{FC/DC}$), invest them at the foreign nominal risk-free rate (i_{FC}) for the same period as in Option 1, and then convert them back into the domestic currency after 1 year at the expected spot exchange rate 1 year from today ($S^e_{FC/DC}$).

- Convert 1 unit of DC into FC today: receive 1 DC × $S_{FC/DC}$ = S_{FC} units of FC.
- Invest S_{FC} units of FC at the foreign risk-free rate (i_{FC}). After one year, receive $S_{FC} \times (1 + i_{FC})$.
- Convert this amount back into DC at the expected spot exchange rate 1 year from today ($S^e_{FC/DC}$). After 1 year, the value of the investment (in DC terms) equals:

$$\frac{S_{FC} \times (1 + i_{FC})}{S^e_{FC/DC}}$$

Therefore, the uncovered interest rate parity equation (assuming a time horizon of 1 year) is given as:

$$(1 + i_{DC}) = (1 + i_{FC}) \times \frac{S_{FC/DC}}{S^e_{FC/DC}}$$

This equation can be rearranged to derive the formula for the expected future spot exchange rate:

$$S^e_{FC/DC} = S_{FC/DC} \times \frac{(1 + i_{FC})}{(1 + i_{DC})}$$

The expected percentage change in the spot exchange rate can be calculated as:

$$\%\Delta S^e_{FC/DC} = \frac{S^e_{FC/DC} - S_{FC/DC}}{S_{FC/DC}}$$

The expected percentage change in the spot exchange rate can be estimated as:

$$\%\Delta S^e_{FC/DC} \approx i_{FC} - i_{DC}$$

Notice that the numerator-denominator rule also applies here. In an A/B exchange rate quote, Country A's interest rate is in the numerator and Country B's interest rate is in the denominator of the uncovered interest rate parity equation.

In covered interest rate parity, the investor locks in the forward exchange rate today and therefore is not exposed to currency risk.

- If covered interest rate parity holds, the forward premium/discount offsets the yield differential.

In uncovered interest rate parity, the investor leaves the foreign exchange position uncovered (ie, unhedged) and expects to convert foreign currency holdings back into the domestic currency at the expected future spot rate.

- If uncovered interest rate parity holds, the expected appreciation/depreciation of the currency offsets the yield differential.

Example 3 Covered versus uncovered interest rate parity

Consider the following information:

- Risk-free rate on the USD = i_{USD} = 4%
- Risk-free rate on the GBP = i_{GBP} = 5%
- Current spot USD/GBP exchange rate = $S_{USD/GBP}$ = 1.5025
- Assume that the USD is the foreign currency, and the GBP is the domestic currency.

Compute the 1-year USD/GBP forward rate and the forward premium (discount), assuming that interest rate parity holds.

Solution

Covered interest rate parity states that the holding period return on an investment in a domestic money-market instrument and an investment in a fully currency-hedged foreign money-market instrument must be equal.

If covered interest rate parity holds, we can compute the forward exchange rate today as:

$$F_{USD/GBP} = S_{USD/GBP}\left(\frac{1 + i_{USD}}{1 + i_{GBP}}\right) = 1.5025 \times \left(\frac{1 + 0.04}{1 + 0.05}\right) = 1.4882 \text{ USD/GBP}$$

Then, the forward premium (discount) on the GBP can be computed as:

$$\frac{F_{USD/GBP} - S_{USD/GBP}}{S_{USD/GBP}} = \frac{1.4882 - 1.5025}{1.5025} \approx -0.009524 \approx -0.9524\%$$

According to covered interest rate parity, the GBP would experience a forward discount of approximately 1% against the USD. This forward discount can be attributed to its relatively high interest rate compared with the USD (5% versus 4%). Conversely, the USD would exhibit a forward premium of around 1% against the GBP due to its lower interest rate.

Now, compute the expected USD/GBP spot rate in 1 year and the expected change in the spot exchange rate over the year, assuming that uncovered interest parity is expected to hold.

Solution

Uncovered interest rate parity states that the expected holding period return on an investment in a domestic money-market instrument and an unhedged investment in a foreign money-market instrument (against currency risk) would be the same. If uncovered interest rate parity holds, the expected future spot rate can be computed as follows:

$$S^e_{USD/GBP} = S_{USD/GBP} \times \left(\frac{1 + i_{USD}}{1 + i_{GBP}}\right) = 1.5025 \times \left(\frac{1 + 0.04}{1 + 0.05}\right) = 1.4882 \text{ USD/GBP}$$

The change in the spot exchange rate for the GBP against the USD over the following year is expected to be:

$$\frac{S^e_{USD/GBP} - S_{USD/GBP}}{S_{USD/GBP}} = \frac{1.4882 - 1.5025}{1.5025} = -0.952\%$$

According to uncovered interest rate parity, the GBP is expected to depreciate by about 1% over the following year, with the spot exchange rate falling from 1.5025 USD/GBP to 1.4882 USD/GBP. This depreciation is expected as the GBP has a higher interest rate relative to the USD (5% versus 4%). Simultaneously, the USD is expected to appreciate by approximately 1% over the year against the GBP.

It is important to note that:

- Under uncovered interest rate parity, the predicted change in spot rates goes against intuition. Typically, an increase in interest rates would lead to a currency's appreciation, but uncovered interest rate parity suggests the opposite. In this example, although the GBP has a higher interest rate, if uncovered interest rate parity holds, it implies that the GBP is expected to depreciate against the USD.
- Uncovered interest rate parity asserts that the expected return on the unhedged foreign investment is the same as the return on the domestic investment. Nevertheless, the distribution of potential return outcomes differs. The domestic currency return is known with certainty, whereas the unhedged foreign investment return could:
  - Equal the domestic currency return: The percentage appreciation of the USD matches the interest rate differential (1%), as in the second question in this example.
  - Be less than the domestic currency return: The percentage appreciation of the USD is less than the interest rate differential (1%).
  - Be greater than the domestic currency return: The percentage appreciation of the USD is greater than the interest rate differential (1%).

Due to the uncertainty associated with the future spot exchange rate, uncovered interest rate parity is often violated. Investors, who are generally not risk neutral, demand a risk premium to compensate for the exchange rate risk inherent in leaving their positions unhedged. Consequently, future spot exchange rates typically do not equal the forward exchange rate. Forward rates, which are based purely on interest rate differentials to prevent covered interest arbitrage, are therefore poor predictors of future spot exchange rates.

The uncovered interest parity equation is quite similar to the covered interest parity equation, except that the expected future spot exchange rate replaces the forward rate. In conclusion:

- Covered interest rate parity is a no-arbitrage condition that uses the forward exchange rate.
- Uncovered interest rate parity is a theory regarding expected future spot rates.

Empirical evidence suggests that:

- Uncovered interest rate parity is not valid in the short and medium terms but tends to work better in the long term. In the short and medium terms, interest rate differentials fail to explain changes in exchange rates; this makes forward rates (which are computed based on these differentials) poor predictors of future exchange rates. In contrast, over the long term, uncovered interest rate parity and rate parity have more empirical support.
- Current spot exchange rates are also unreliable predictors of future spot exchange rates due to the high volatility in exchange rate movements. This indicates that exchange rates do not adhere to a random walk.

Forward Rate Parity

Forward rate parity is based on both covered and uncovered interest rate parity. When covered interest rate parity holds (which is generally the case since it is a no-arbitrage condition), the forward premium or discount roughly matches the interest rate differential.

$$\text{Forward premium (discount) as a \%} \simeq F_{/DC} - S_{FC/DC} \simeq i_{FC} - i_{DC}$$

If uncovered interest rate parity holds, the expected future spot rate equals the forward rate ($S^e_{FC/DC} = F_{FC/DC}$), and the expected change in the spot exchange rate roughly equals the interest rate differential.

$$\text{Expected \% change in spot exchange rate} \simeq \%\Delta S^e_{FC/DC} \simeq i_{FC} - i_{DC}$$

Therefore, if both covered and uncovered interest rate parity hold, the forward premium (discount) will roughly equal the interest rate differential, and the forward rate will roughly equal the expected spot exchange rate. This condition, in which the forward rate equals the expected spot rate, is known as forward rate parity.

$$F_{FC/DC} - S_{FC/DC} = S^e_{FC/DC} - S_{FC/DC} \simeq i_{FC} - i_{DC} \rightarrow F_{FC/DC} = S^e_{FC/DC}$$

In this condition, the forward rate is an unbiased forecast of the future spot exchange rate.

Purchasing Power Parity

LOS: Explain international parity conditions (covered and uncovered interest rate parity, forward rate parity, purchasing power parity, and the international Fisher effect).

LOS: Describe relations among the international parity conditions.

LOS: Evaluate the use of the current spot rate, the forward rate, purchasing power parity, and uncovered interest parity to forecast future spot exchange rates.

Purchasing power parity (PPP) is based on the law of one price, which states that identical goods should have the same price across countries when valued in a common currency. For example, if a pen in the US costs USD 2, and an identical pen in Europe is EUR 3, assuming no transaction costs or trade restrictions, the USD/EUR exchange rate must be 0.667, as shown in the equation below.

$$\text{Price of pen in USD} = \text{Price of pen in EUR} \times \frac{\text{USD}}{\text{EUR}} \text{ exchange rate} = \text{USD2} = \text{EUR3} \times \frac{\text{USD}}{\text{EUR}} (0.667)$$

Therefore, the law of one price can be expressed as follows:

$$\text{Law of one price: } P^x_f = P^x_d \times S_{f/d}$$

If the price of these pens rises in Europe, there will be a flow of pens from the US to Europe, increasing the supply of EUR (and demand for USD) to purchase pens. Eventually, the USD/EUR exchange rate will fall until the price differential is eliminated.

Thus, according to the law of one price, a relative increase (decrease) in prices in one country will result in depreciation (appreciation) of that country's currency, ensuring that exchange rate-adjusted prices remain constant across countries.

Absolute purchasing power parity (PPP) extends the law of one price to a broad range of goods and services consumed in different countries. Instead of focusing on just one individual good, absolute PPP states that one country's general price level (P_d) should equal the currency-adjusted general price level (P_f) in the other country. Absolute PPP can be expressed as:

$$P_f = P_d \times S_{f/d}$$

Absolute PPP assumes that all goods are tradeable, and that price indexes (used to determine the general price level) in both countries include the same goods and services with identical weights. The equations above can be rearranged to solve for the nominal exchange rate ($S_{f/d}$):

$$S_{f/d} = \frac{P_f}{P_d}$$

Absolute PPP asserts that the equilibrium exchange rate between two countries is determined by the ratio of their respective national price levels. However, absolute PPP generally does not hold, due to differences in product mixes and consumption baskets across countries, as well as the transaction costs and trade restrictions involved in international trade.

Instead of assuming that there are no transaction costs and other trade impediments (as is the case with absolute PPP), **relative PPP** assumes that these factors are constant over time. Relative PPP claims that changes in exchange rates are linked to relative changes in national price levels, even if the relation between exchange rate levels and price levels does not hold.

Given an f/d exchange rate quote, the foreign inflation rate will be in the numerator and the domestic inflation rate will be in the denominator. Relative PPP can be expressed as follows:

$$S_{f/d}^{T} = S_{f/d}^{0} \times \left(\frac{1 + \pi_f}{1 + \pi_d}\right)^T$$

According to relative PPP, changes in the spot exchange rate can be approximated as:

$$\%\Delta S_{f/d} \simeq \pi_f - \pi_d$$

Relative PPP suggests that the percentage change in the spot exchange rate ($\%\Delta S_{f/d}$) is entirely determined by the difference between foreign and domestic inflation. For example, if US inflation is 5% and Eurozone inflation is 8%, then the USD/EUR exchange rate should fall by 3% and the USD (a low-inflation currency) should appreciate, while the EUR (a high-inflation currency) should depreciate. If relative PPP is maintained across countries, currencies of countries with higher (lower) inflation rates depreciate (appreciate). This is in keeping with the basic economic principle that inflation devalues currency as a medium of exchange.

The **ex ante version of PPP** is based on relative PPP. While relative PPP asserts that actual changes in the exchange rate are driven by actual relative changes in inflation, ex ante PPP suggests that expected changes in spot exchange rates are entirely driven by expected differences in national inflation rates. According to ex ante PPP, countries expecting persistently high (low) inflation rates should also expect currency depreciation (appreciation) over time.

$$\%\Delta S_{f/d}^{e} \simeq \pi_f^{e} - \pi_d^{e}$$

Historically, it has been observed that:

- In the short run, nominal exchange rates often deviate from the path predicted by PPP.
- Over the long run, nominal exchange rates tend to move toward their long-run PPP equilibrium values, although sometimes this process is very slow. Thus, PPP does provide a valid framework for assessing the long-run fair value of a currency.

The Fisher Effect, Real Interest Rate Parity, and International Parity Conditions

LOS: Explain international parity conditions (covered and uncovered interest rate parity, forward rate parity, purchasing power parity, and the international Fisher effect).

LOS: Describe relations among the international parity conditions.

LOS: Evaluate the use of the current spot rate, the forward rate, purchasing power parity, and uncovered interest parity to forecast future spot exchange rates.

The Fisher effect asserts that the nominal interest rate (i) in a country is the sum of its real interest rate (r) and the expected inflation rate (π^e) and can be expressed as:

$$i = r + \pi^e$$

Therefore, the expressions for the domestic and foreign nominal interest rates are given as:

$$i_d = r_d + \pi_d^e$$
$$i_f = r_f + \pi_f^e$$

Exhibit 2 International Fisher effect: real interest rate parity

Considerations	Mathematical expression
Fisher effect in one currency	$i = r + \pi^e$
Fisher effect applied to domestic (d) and foreign (f) currency	$i_d = r_d + \pi_d^e$ $i_f = r_f + \pi_f^e$
Nominal interest rate spread	$i_f - i_d = (r_f - r_d) + (\pi_f^e - \pi_d^e)$
Rearrange equation	$r_f - r_d = (i_f - i_d) - (\pi_f^e - \pi_d^e)$
International Fisher effect	$i_f - i_d = \pi_f^e - \pi_d^e$
Implication	$r_f - r_d = 0$ or $r_f = r_d$

r = Real interest rate $\quad$ i = Nominal interest rate
π^e = Expected inflation $\quad$ d = Domestic
f = Foreign

Real interest rate parity can be seen as the international application of the law of one price for securities (as real interest rates represent the real prices of securities).

- If uncovered interest rate parity holds, the nominal interest rate spread ($i_f - i_d$) roughly equals the expected change in the exchange rate ($\%\Delta S^e_{f/d}$).
- If ex ante PPP holds, the difference in expected inflation rates ($\pi^e_f - \pi^e_d$) approximately equals the expected change in the spot rate ($\%\Delta S^e_{f/d}$).

If both uncovered interest rate parity and ex ante PPP are assumed to hold, the real yield spread between foreign and domestic countries ($r_f - r_d$) will equal zero. This proposition, holding that real interest rates converge to the same level across different countries as real yield spreads equal zero, is known as the real interest rate parity condition.

Further, if the real yield spread ($r_f - r_d$) equals zero in all markets, it follows that the foreign-domestic nominal yield spread will be determined by the foreign-domestic expected inflation rate differential. This is known as the international Fisher effect.

The international Fisher effect and real interest rate parity assume that currency risk is uniform worldwide. In reality, however, countries with relatively high debt levels may carry greater currency risk, increasing the likelihood of currency depreciation. In such cases, subtracting the expected inflation rate from the nominal interest rate will yield a calculated real interest rate higher than in other countries, as the nominal interest rate also includes a currency risk premium. Consequently, elevated risk may lead to a country's nominal and real risk-free rates being higher than expected under the international Fisher effect and real interest parity conditions.

International Parity Conditions: Tying All the Pieces Together

Covered interest rate parity: Arbitrage ensures that differences in nominal interest rates equal the forward premium (discount).

- Currencies with higher nominal interest rates trade at a forward discount, while those with lower rates trade at a premium.

Uncovered interest rate parity: The expected change in the spot rate equals the nominal interest rate spread.

- Currencies with higher nominal interest rates are expected to depreciate, and those with lower rates are expected to appreciate.

If both covered and uncovered interest rate parity hold, the nominal interest rate spread equals the forward premium (discount) and the expected appreciation (depreciation) in the exchange rate. Therefore, the forward rate serves as an unbiased predictor of the future spot exchange rate.

Ex ante PPP: Differences in expected inflation rates lead to future changes in spot rates.

- Currencies with higher expected inflation rates are expected to depreciate, and those with lower expected inflation rates are expected to appreciate.

International Fisher effect: Under the assumption that the Fisher effect holds in each market and real interest rate parity holds, the difference between domestic and foreign nominal interest rates equals the difference between domestic and foreign expected inflation rates.

If ex ante PPP and the Fisher effect hold, the expected inflation differential equals both the expected change in the spot exchange rate and the nominal interest rate differential. This implies that uncovered interest rate parity holds.

If all international parity relations held, global investors would be unable to consistently profit from currency movements, and the expected percentage change in the spot rate would be equal to:

- the forward premium or discount (expressed as a percentage),
- the nominal yield spread between countries, and
- the difference in expected inflation rates across countries.