LM01 Returns of Financial Assets and Instruments

Financial Returns

Total Return

Financial assets normally generate returns in two ways: capital appreciation and capital distribution.

Capital appreciation return: Also called 'price return', it is the change in price between two different points of time.

$$r_{price} = \frac{P_1 - P_0}{P_0}$$

Capital distribution return: It is the income earned through dividends, coupon payments, etc.

$$r_{distribution} = \frac{Inc}{P_0}$$

The total return takes into account both sources of returns.

$$r = r_{price} + r_{distribution}$$

$$r = \frac{P_1 - P_0}{P_0} + \frac{Inc}{P_0} = \frac{P_1 - P_0 + Inc}{P_0}$$

Example:
(*This is based on the case study from the curriculum.*)

The following information is provided for a stock.

Date	Price per share	Dividend per share
31 December 2016	267.60	-
31 December 2017	338.70	6.00
31 December 2018	435.70	9.00
31 December 2019	689.00	9.75
31 December 2020	1,243.50	10.50
31 December 2021	835.20	11.50
31 December 2022	631.30	12.50
31 December 2023	374.30	13.50

Calculate the price return, capital distribution return, and total return for this stock in 2020.

Solution:

$$r_{price} = \frac{P_1 - P_0}{P_0} = \frac{1{,}243.50 - 689.00}{689.00} = 80.48\%$$

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$$r_{distribution} = \frac{Inc}{P_0} = \frac{10.50}{689.00} = 1.52\%$$

$$r = r_{price} + r_{distribution} = 80.48\% + 1.52\% = 82.00\%$$

Holding period return is the return that an investor earns over a specified holding period. The holding period can range from days to years. In the above example, the holding period was one year, 2020.

Expected returns vs. actual returns

Expected returns (or ex ante returns) are the anticipated returns on an investment over a specified period.

Actual returns (or ex post returns) are the returns on an investment after the investment period has ended.

Unrealized returns vs. realized returns

Unrealized returns ("paper" gains/losses) are potential profits or losses on investments that you currently hold but have not yet sold. These values fluctuate based on market prices.

Realized returns are actual, locked-in profits or losses from selling an asset. In most cases, taxes are applied only on realized returns.

Arithmetic and Geometric Holding Period Return

An arithmetic mean return is the simple average of all holding period returns.

$$\bar{r}_i = \frac{r_{i1} + r_{i2} + \cdots + r_{i,T}}{T}$$

A geometric mean return represents the average rate at which an asset's value grows over time, accounting for the compounding of returns.

$$\bar{r}_{Gi} = [(1 + r_{i1}) \times (1 + r_{i2}) \times \ldots \times (1 + r_{iT})]^{\frac{1}{T}} - 1$$

Example:
(*This is based on the case study from the curriculum.*)

The following information is provided for a stock.

Date	Price per share	Dividend per share	Total return (%)
31 December 2016	267.60	-	-
31 December 2017	338.70	6.00	28.81%
31 December 2018	435.70	9.00	31.30%
31 December 2019	689.00	9.75	60.37%
31 December 2020	1,243.50	10.50	82.00%
31 December 2021	835.20	11.50	−31.91%
31 December 2022	631.30	12.50	−22.92%

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31 December 2023	374.30	13.50	–38.57%

Calculate the arithmetic and geometric average annual total return for this stock for 2017-2023.

Solution:

The arithmetic mean return can be calculated as:

$$\bar{r}_i = \frac{28.81\% + 31.30\% + 60.37\% + 82.00\% - 31.91\% - 22.92\% - 38.57\%}{7} = 15.58\%$$

The geometric mean return can be calculated as:

$$\bar{r}_{Gi} = [(1 + 0.2881) \times (1 + 0.3130) \times (1 + 0.6037) \times (1 + 0.8200) \times (1 - 0.3191) \times (1 - 0.2292) \times (1 - 0.3857)]^{\frac{1}{7}} - 1 = 6.86\%$$

The geometric mean is a more accurate measure of investment performance, whereas the arithmetic mean can be misleading. If the annual returns on an investment are constant, the geometric mean will be equal to the arithmetic mean. If the annual returns on an investment are volatile, the geometric mean will always be lower than the arithmetic mean (6.86% vs. 15.58% in our example.)

Compounding: Daily, Monthly, Quarterly, and Annual Returns

To compare the performance of investments with different holding periods, we can annualize all available returns. The general equation to annualize returns is:

$$r_{annualized} = (1 + r_{period})^c - 1$$

A major limitation of annualizing returns is the implicit assumption that money can be reinvested repeatedly while earning the same return.

Example:
(*This is based on the Knowledge Check example from the curriculum.*)

An investor wants to evaluate the returns of three recently formed ETFs. He gathers the following information.

ETF	Time since Inception	Return since Inception (%)
1	146 days	4.61
2	5 weeks	1.10
3	15 months	14.35

Assuming that a year has 365 days, or 52 weeks, or 12 months, which ETF has the highest annualized rate of return?

Solution:

The annualized return for ETF 1 is:

$1.0461^{365/146} - 1 = 11.93\%$

The annualized return for ETF 2 is:

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$1.0110^{52/5} - 1 = 12.05\%$

The annualized return for ETF 3 is:

$1.1435^{12/15} - 1 = 11.32\%$

Therefore, ETF 2 has the highest annualized rate of return.

Continuous Compounding

The continuously compounded return associated with a holding period return can be calculated as:

- Natural logarithm of one plus that holding period return, or
- Natural logarithm of the ending price over the beginning price

$$R_c = \ln(1 + R_t) = \ln(P_t/P_0)$$

A property of the continuously compounded rates of return is that they are additive.

$$r_{0,T} = r_{0,1} + r_{2,3} + \cdots + r_{T-1,T}$$

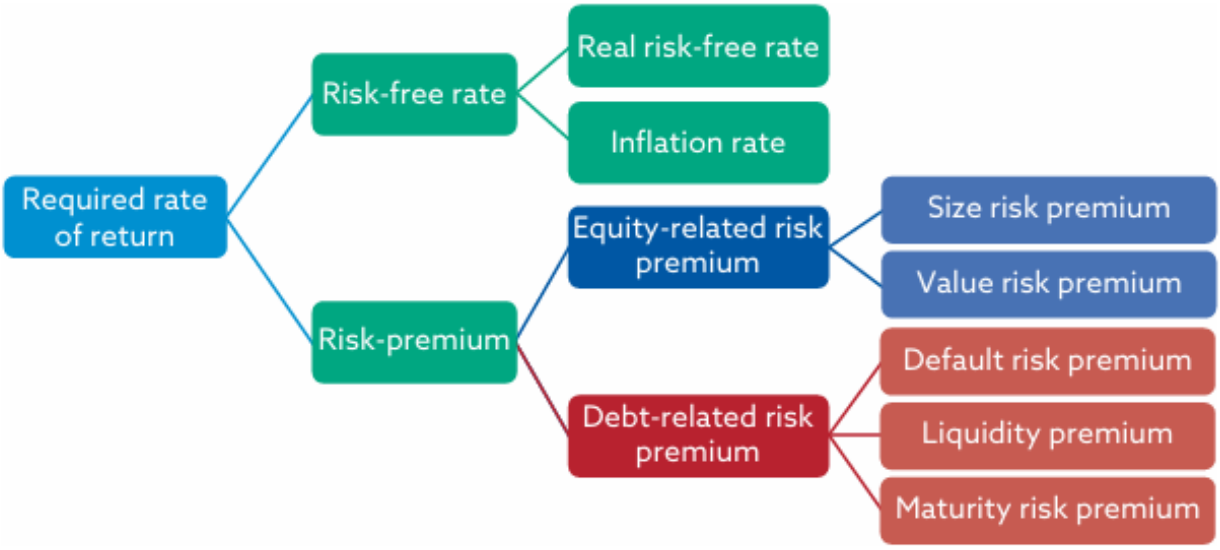
Example:

1. If the one-week holding period return is 4%, then the equivalent continuously compounded return is ln (1 + 0.04) = 3.922%.
2. A stock was purchased at t = 0 for $30. One period later at t = 1, the stock is valued at $34.50. The continuously compounded return over the period can be calculated as ln (34.50/30) = 13.976%.

Common Return Measures

The Required Rate of Return

The required rate of return is the minimum rate of return that an investor must receive to accept an investment. We can break the required rate of return into different components as shown in Exhibit 13 of the curriculum.



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Risk-free rate: Typically, the returns on short-term government bonds are considered as the risk-free rate. This is a nominal rate which combines the real risk-free rate and the inflation premium.

The relationship between the nominal risk-free rate, r_f, the real risk-free rate, r_{rf}, and inflation, infl is:

$$(1 + r_f) = (1 + r_{rf})(1 + \text{infl})$$

We can rearrange this equation and express it in terms of the real risk-free rate as:

$$r_{rf} = \frac{1 + r_f}{1 + \text{infl}} - 1$$

The above formula is used for precise calculations. We can also use a quick approximation to estimate the real risk-free rate:

$$r_{rf} = r_f - \text{infl}$$

Risk premiums: Risk premiums are additional returns that investors demand for investing in risky assets. The total return of any asset can be expressed as:

$$r_i = r_{rf} + \text{infl} + \Sigma\text{risk premia}$$

The risk premia differ for equity and debt. Examples of risk premia for stocks include:

- Size risk premium: Extra return for investing in small companies vs. large companies.
- Value risk premium: Extra return for investing in value stocks vs. growth stocks.

Examples of risk premia for bonds include:

- Default risk premium: Extra return for investing in a riskier corporate bond vs. a less risky government bond.
- Liquidity premium: Extra return for holding a bond that may be harder to sell quickly without impacting the price.
- Maturity risk premium: Extra return for holding longer maturity bonds, which are more sensitive to interest rates as compared to shorter maturity bonds

Example:
(*This is based on the knowledge check question from the curriculum.)*

An analyst observes the historic geometric average returns shown below:

Asset Class	**Geometric Average Return (%)**
Equities	8.0
Corporate bonds	6.5
Treasury bills	2.5

The average inflation rate was 2.1% during that time.

1. Calculate the real rate of return for equities.

Solution:

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The precise calculation of the real rate of return for equities is:

$$r_{rf} = \frac{1 + 0.080}{1 + 0.0210} - 1 = 5.78\%$$

The approximate real rate of return for equities is:

$$r_{rf} = 8.0\% - 2.1\% = 5.9\%$$

2. Calculate the real rate of return for corporate bonds.

Solution:

The precise calculation of the real rate of return for corporate bonds is:

$$r_{rf} = \frac{1 + 0.065}{1 + 0.0210} - 1 = 4.31\%$$

The approximate real rate of return for corporate bonds is:

$$r_{rf} = 6.5\% - 2.1\% = 4.4\%$$

3. Calculate the risk premium for equities.

Solution:

The total return for equities can be expressed as:

$$r_i = r_{rf} + \text{infl} + \Sigma\text{risk premia}$$

The approximate sum of the real risk-free rate and the inflation rate is the nominal risk-free rate, which is the return on treasury bills. i.e.

$$r_{rf} + \text{infl} = 2.5\%$$

Therefore:

$$8.0\% = 2.5\% + \Sigma\text{risk premia}$$

$$\Sigma\text{risk premia} = 8.0\% - 2.5\% = 5.5\%$$

Excess Returns: Excess return is the difference between an asset's return and the return on some reference rate or benchmark.

Real Returns: The real rate of return is the annual percentage profit earned on an investment after adjusting for inflation to show true purchasing power. The real return of an asset can be calculated as:

$$r_{real} = \frac{1 + r_i}{1 + \text{infl}} - 1$$

Gross and Net Return:

Gross return is the return earned by an asset manager prior to deducting management fees and taxes. It measures investment skill.

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Net return accounts for all managerial and administrative expenses. It is the return an investor is concerned with.

Exhibit 16 from the curriculum shows the relationship between total return, gross return, and net return.



Mathematically, the relationship can be expressed as:

Gross return = Total return – Transaction costs

Net return = Gross return – Management fees – Administrative expenses

Example:
(*This is based on the knowledge check example from the curriculum.*)

An investment fund, specialized in environmentally sustainable and socially responsible investments, reported a total annual return of −5% before costs. The fund's total assets were valued at CHF500 million. The fund incurs transaction costs of 0.05%, a management fee of 1.00%, and administrative expenses amounting to 0.50%—all as a percentage of its total asset value. The management fee is assessed on the assets at the end of the year.

1. Calculate the fund's gross return.

Solution:

Gross return = Total return – Transaction costs

Gross return = -5% - 0.05% = -5.05%

2. Calculate the fund's net return.

Solution:

Net return = Gross return – Management fees – Administrative expenses

Net return = -5.05% - 1.00% - 0.50% = -6.55%

Pre-Tax and After-Tax Nominal Return

Pre-tax nominal return is the return before accounting for inflation and taxes; this is the default, unless otherwise stated.

After-tax nominal return is the return after accounting for taxes.

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$$r_{net-tax} = r_{price} \times (1 - tax_{capital\ gains}) + r_{capital} \times (1 - tax_{capital\ distributions})$$

Minimizing overall tax liabilities is an important portfolio management strategy.

Example:
(*This is based on the case study from the curriculum.*)

In 2020, an investor purchased 1,000 shares of a stock at a price of 689.00 per share. By the end of the year, the price had risen to 1,243.50 per share. Additionally, the investor received a dividend of 10.50 per share during the year. The investor sold all their shares at the end of the year. The capital gains tax rate is 42%. The capital distributions, such as dividends, are taxed at the income tax rate of 27%.

1. Calculate the investor's realized capital gain for tax purposes.

Solution:

The realized capital gain per share is: 1,243.50 – 689.00 = 544.50.

Total capital gain on the 1,000 shares is: 544.50 × 1,000 = 554,500.

2. Calculate the capital gains tax that the investor must pay.

Solution:

Capital gains tax = Total capital gain × Capital gains tax rate

Capital gains tax = 554,500 × 42% = 232,980

3. Calculate the total after-tax return on the investment.

Solution:

The price return can be calculated as:

$$r_{price} = \frac{P_1 - P_0}{P_0} = \frac{1{,}243.50 - 689.00}{689.00} = 80.48\%$$

The capital distribution return can be calculated as:

$$r_{capital} = \frac{Inc}{P_0} = \frac{10.50}{689.00} = 1.52\%$$

The total after-tax return can be calculated as:

$$r_{net-tax} = r_{price} \times (1 - tax_{capital\ gains}) + r_{capital} \times (1 - tax_{capital\ distributions})$$

$$r_{net-tax} = 80.48\%(1 - 42\%) + 1.52\%(1 - 27\%) = 46.68\% + 1.11\% = 47.79\%$$

Leveraged Returns

Leveraged return is the return earned by the investor on his money after accounting for the interest paid on borrowed money. The levered return can be calculated as:

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$$r_{LP} = r_p + \frac{V_D}{V_E}(r_p - r_d)$$

Leverage boosts returns as long as the investment return exceeds the borrowing cost.

Example:
(*This example is taken from the curriculum.*)

A EUR10 million portfolio generates an 8% total investment return over one year and is financed 30% with debt at 5%. Calculate the leveraged return for the portfolio.

Solution:

$$r_{LP} = r_p + \frac{V_D}{V_E}(r_p - r_d)$$

$$r_{LP} = 8\% + \frac{\text{EUR 3 million}}{\text{EUR 7 million}}(8\% - 5\%) = 8\% + 0.43 \times 3\% = 9.29\%$$

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