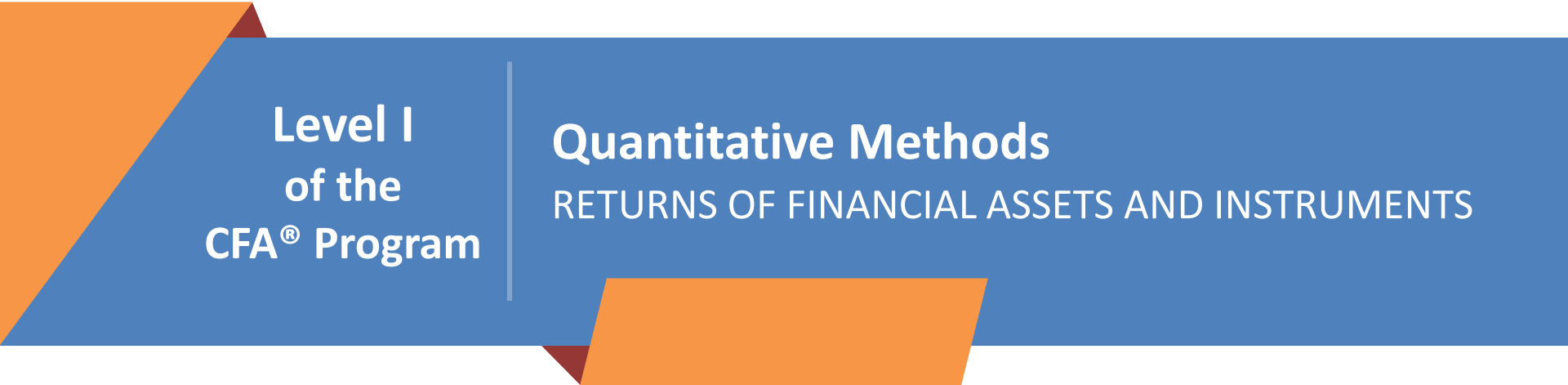




**Level I
of the
CFA® Program**

Quantitative Methods

RETURNS OF FINANCIAL ASSETS AND INSTRUMENTS

Learning Outcome Statements



LOS: Describe, compare, and interpret **returns**.

LOS: Describe, compare, and interpret **required rates of return, risk-free rates, risk premia, and inflation**.

LOS: Describe, compare, and interpret returns

Analyzing Sources and Uses of Cash

Returns are generated in two ways:

1. **Capital Appreciation:** Change in price relative to initial price.

$$r_{\text{Price}} = \frac{P_1 - P_0}{P_0}$$

2. **Capital Distribution:** Income from dividends or interest.

For equities: dividend yield; for bonds: current yield.

$$r_{\text{Distribution}} = \frac{\text{Inc}}{P_0}$$

The total return is the sum of both components.

$$r = \frac{P_1 - P_0 + \text{Inc}}{P_0}$$

LOS: Describe, compare, and interpret returns

Other Type Returns



Expected (Ex Ante) Return: Forecast based on analysis – not guaranteed.



Actual (Ex Post) Return: Realized after the investment period.



Realized vs. Unrealized: Gains/losses are unrealized until the asset is sold or income received; only realized gains are typically taxed.

LOS: Describe, compare, and interpret returns

Multi-Period Returns

1. Arithmetic return

- Simple **average** of the **holding period returns**.
- Calculated by adding all the returns in each period and then dividing this figure by the total number of returns.
- Denoted by $\bar{R}_i$ for asset i , and is given by:

$$\bar{R}_i = \frac{R_{i,1} + R_{i,2} + \cdots + R_{i,T-1} + R_{iT}}{T} = \frac{1}{T} \sum_{t=1}^T R_{it}$$

Where:

R_{it} = Return of asset i in period t .

T = Total number of periods.

LOS: Describe, compare, and interpret returns

Example: Arithmetic return

If a share has returned **25%, 10%, 12%** and **-3%** over the last **four years** consecutively, then the arithmetic mean can be calculated as:

$$\bar{R}_i = \frac{1}{T} \sum_{t=1}^T R_{it}$$

$$\bar{R}_i = \frac{25\% + 10\% + 12\% + (-3\%)}{4} = 11\%$$

LOS: Describe, compare, and interpret returns

2. Geometric Mean Return

- The **geometric mean return** gives the **average rate** per period on an investment that is compounded over multiple periods.
- For returns with any variability, geometric mean < arithmetic mean. The greater the volatility, the larger the gap.
- It is denoted by $\bar{R}_{Gi}$ and is given by the following equation:

$$\bar{R}_{Gi} = \sqrt[T]{(1 + R_{i,1}) \times (1 + R_{i2}) \times \cdots \times (1 + R_{i,T-1}) \times (1 + R_{iT})} - 1$$
$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^T (1 + R_t)} - 1$$

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Example: Geometric Mean Return

If a share has returned **25%, 10%, 12%** and **-3%** over the last **four years**, then the geometric mean can be calculated as:

$$\begin{aligned}\bar{R}_{Gi} &= \sqrt[t]{(1 + R_{i,1}) \times (1 + R_{i,2}) \times \cdots \times (1 + R_{i,T-1}) \times (1 + R_{iT})} - 1 \\ &= (1.25 \times 1.10 \times 1.12 \times 0.97)^{\frac{1}{4}} - 1 = 10.55\%\end{aligned}$$

LOS: Describe, compare, and interpret returns

Annualized Return

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{\text{period}})^c - 1$$

Where:

$\text{Return}_{\text{period}}$ = Quoted return for the period.

c = Number of periods in a year.

Example #1

LOS: Describe, compare, and interpret returns

Annualized Return

Example:

- If the monthly return is 0.7%, then the compound annual return is:

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{\text{monthly}})^{12} - 1 = (1.007)^{12} - 1 = 0.0873 = 8.73\%$$

- For a period of more than one year, for example, a 15-month return of 16% can be annualized as:

$$\text{Return}_{\text{annual}} = (1 + \text{Return}_{15 \text{ month}})^{\frac{12}{15}} - 1 = (1.16)^{\frac{4}{5}} - 1 = 12.61\%$$

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Example: Annualized Return

An investor is evaluating the returns of two recently formed bonds. Selected return information on the bonds is presented below:

Bond	Time Since Issuance	Return Since Issuance (%)
A	120 days	2.50
B	8 months	6.00

Which bond has the highest annualized rate of return?

LOS: Describe, compare, and interpret returns

Solution

Bond A

Time = 120 days

Return = 2.50% = 0.0250

$$\begin{aligned}\text{Annualized Return} &= (1 + 0.0250)^{\frac{365}{120}} - 1 \\ &= \mathbf{0.078 \text{ or } 7.8\%}\end{aligned}$$

Bond B

Time = 8 months = 240 days

Return = 6.00% = 0.0600

$$\begin{aligned}\text{Annualized Return} &= (1 + 0.0600)^{\frac{365}{240}} - 1 \\ &= \mathbf{0.093 \text{ or } 9.3\%}\end{aligned}$$

So, Bond B has the highest annualized rate of return at 9.3%.

LOS: Describe, compare, and interpret returns

Continuously Compounded Return

- Defined as the **natural logarithm** of **one plus a holding period return** of a period.

Example

- If a month's holding period return is 1.2%, the continuously compounded return is $\ln(1.012) = 0.01193$
- Generally, continuously compounded from t to $t + 1$ is given by:

$$r_{t,t+1} = \ln\left(\frac{P_{t+1}}{P_t}\right) = \ln(1 + R_{t,t+1})$$

LOS: Describe, compare, and interpret returns

- ◆ Assume investment horizon is from time $t = 0$ to time $t = T$:

$$r_{0,T} = \ln\left(\frac{P_T}{P_0}\right)$$

- ◆ Apply the exponential function on both sides of the above equation:

$$P_T = P_0 e^{r_{0,T}}$$

- ◆ Note that $\frac{P_T}{P_0}$ can be written as:

$$\begin{aligned}\frac{P_T}{P_0} &= \left(\frac{P_T}{P_{T-1}}\right) \left(\frac{P_{T-1}}{P_{T-2}}\right) \cdots \left(\frac{P_1}{P_0}\right) \\ \Rightarrow r_{0,T} &= r_{T-1,T} + r_{T-2,T} + \cdots + r_{0,T}\end{aligned}$$

- ◆ Taking logs both sides of the above equation:
- Continuously compounded return to time T is equivalent to the **sum of one-period continuously compounded returns**.

LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Required Rate of Return

- **Required Rate of Return** is the minimum return an investor expects to earn to commit capital. It represents the investor's opportunity cost.

- It can be decomposed into:

Required Return

$$\approx \text{Real Risk-Free Rate} + \text{Inflation Premium} + \sum \text{Risk Premia}$$

- **Real risk-free interest rate: single-period** interest rate for a completely **risk-free security** if no inflation was expected.
- **Inflation premium:** compensates investors for **expected inflation**.
 - Reflects the **average inflation** rate expected over the **maturity** of the debt.

LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Equity-Related Risk Premium

Size risk premium.

Value risk premium.

Debt-Related Risk Premium

Default risk premium.

Liquidity risk premium.

Maturity risk premium.

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Nominal Risk-Free Interest Rate

▶ Defined as the **sum** of the **real risk-free interest rate** and the **inflation premium**.

- Can be seen as the combination of the real risk-free rate plus an inflation premium:

$$(1 + \text{Nominal risk-free rate}) = (1 + \text{Real risk-free rate}) (1 + \text{Inflation premium})$$

▶ The above equation is generally approximated as follows:

- **Nominal risk-free rate = Real risk-free rate + Inflation premium**

LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Real return

- Recall the relationship between the nominal rate and the real return:

$$(1 + \text{Nominal Risk free rate}) = (1 + \text{Real risk free rate})(1 + \text{Inflation premium})$$

- The equation above can be extended to show the relationship between the nominal and the real returns so that:

$$\text{Real return} = \frac{1 + r_i}{1 + \text{Infl}} - 1$$



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Real return

- Real returns are especially helpful when comparing returns across different time periods and between countries
 - When returns are expressed in local currencies rather than a constant investor currency.
 - Inflation rates differ between the countries.
- After-tax real returns** are the amount the investor receives as payment for delaying consumption and taking on risk after paying taxes on investment.



LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Example: Real Rate of Return

- A portfolio manager has noted the historical geometric returns for different asset classes as follows: **Equities** have yielded a return of **9.0%**, **municipal bonds 5.0%**, and **cash equivalents 1.5%**. The rate of **inflation** over the same period has been **2.3%**.
- The real rate of return for equities is *closest* to:



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Example: Real Rate of Return

- A portfolio manager has noted the historical geometric returns for different asset classes as follows: **Equities** have yielded a return of **9.0%**, **municipal bonds 5.0%**, and **cash equivalents 1.5%**. The rate of **inflation** over the same period has been **2.3%**.
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Solution

Recall:

$$\begin{aligned}\Rightarrow \text{Real return} &= \frac{1 + r_i}{1 + \text{Infl}} - 1 \\ &= \frac{1.09}{1.023} - 1 = \mathbf{6.55\%}\end{aligned}$$

LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Gross and net returns

- **Gross return** is earned **prior** to the deduction of fees (management fees, custodial fees, etc.).
- A **net return** is the return **post**-deduction of fees.
- An investor has a gross return of \$120,000. If management fees is 5% of an investor's gross return, the investor's net return would be \$114,000.

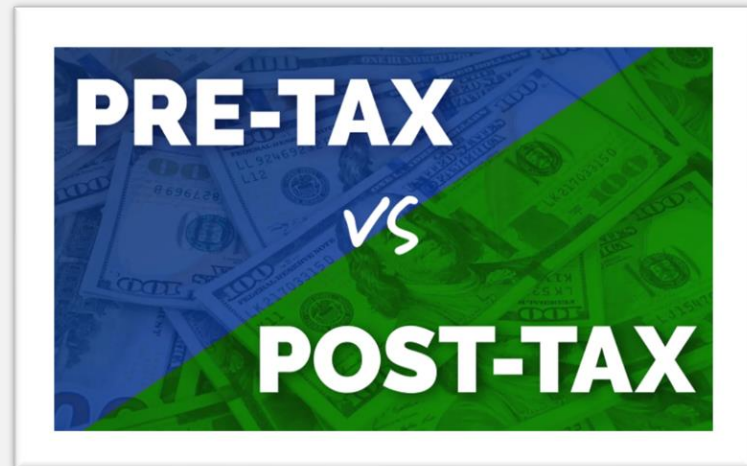
$$120,000 \times (1 - 5\%) = \$114,000$$



LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Pre-tax and after-tax return

- All **returns are pre-tax nominal returns** unless stated otherwise. Capital gains and income may be taxed differently in different jurisdictions.
- The **after-tax nominal return** is computed as the total return minus any allowance for taxes on dividends, interest and realized gains.
- An investor that earns a pre-tax return of 15% on their investments. Applying a 25% tax rate, their after-tax return would be
$$= 15\% \times (1 - 25\%) = \mathbf{11.25\%}$$



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Leveraged Return

- If an investor uses **derivative instruments** within a portfolio or **borrow money** to invest, then leverage is introduced into the portfolio.
 - Amplifies returns both upwards and downwards
- Leverage return considers actual return on the investment and the cost of the borrowed money.

$$R_L = \frac{\text{Portfolio return}}{\text{Portfolio equity}} = \frac{[R_P \times (V_E + V_B) - (V_B \times r_D)]}{V_E} = R_P + \frac{V_B}{V_E} (R_P - r_D)$$

Where:

R_L = Return earned on the leveraged portfolio.

R_P = Total investment return earned on the leveraged portfolio.

V_B = Value of debt in the portfolio.

V_E = Value of equity in the portfolio.

r_D = Borrowing cost on debt.

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Example: Leveraged Return

A **\$7,500,000** equity portfolio is **35% financed by debt** at a cost of **5% per annum**. If the equity portfolio generates a **9% annual total investment return**, the leveraged return is *closest* to:

LOS: Describe, compare, and interpret required rates of return, risk-free rates, risk premia, and inflation

Example: Leveraged Return

A **\$7,500,000** equity portfolio is **35% financed by debt** at a cost of **5% per annum**. If the equity portfolio generates a **9% annual total investment return**, the leveraged return is *closest* to:

Solution

$$R_L = R_P + \frac{V_B}{V_E} (R_P - r_D)$$

$$R_L = 9\% + \frac{\$2,625,000}{\$4,875,000} (9\% - 5\%) = \mathbf{11.15\%}$$

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