Question 1 of 15

P1.T2.710.1

The following probability matrix contains the joint probabilities for random variables $X = \{2, 7, 12\}$ and $Y = \{1, 3, 5\}$:

		Y			
		1.0	3.0	5.0	
X:	2.0	5.00%	15.00%	5.00%	25.0%
	7.0	10.00%	30.00%	10.00%	50.0%
	12.0	5.00%	15.00%	5.00%	25.0%
		20.0%	60.0%	20.0%	100.0%

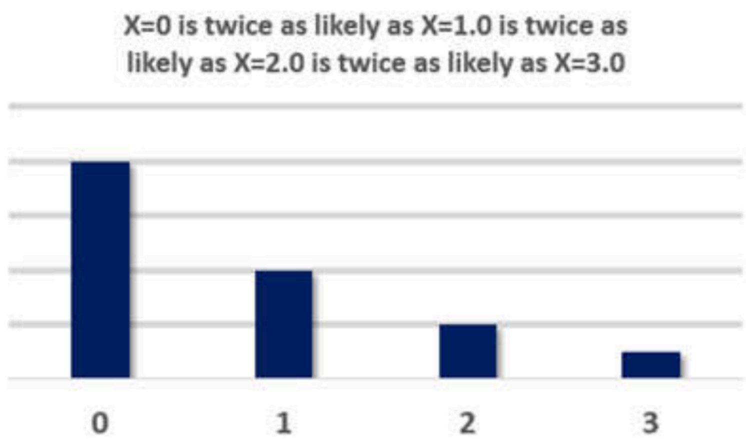
We are informed that (X) and (Y) are independent. What is the expected value of the product of X and Y, $E(XY)$?

- **A.** 15.0
- **B.** 21.0
- **C.** 30.5
- **D.** 35.0

Question 2 of 15

P1.T2.710.2

Peter is running the first draft of a Monte Carlo simulation and he wants a simple random variable to capture the loss frequency per week (i.e., the number of loss events) associated with an operational process. As he is just experimenting with his model, he does not have a probability density function in mind. Instead, he has a simple rule-based idea that he wants to express in a probability function. The number of losses per week will be either zero, one, two or three; $X = \{0, 1, 2, 3\}$. An outcome of $X = 3.0$ is the least likely; an outcome of $X = 2.0$ is twice as likely as $X = 3.0$; an outcome of $X = 1.0$ is twice as likely as $X = 2.0$, and finally an outcome of $X = 0$ is twice as likely as $X = 1.0$. This is illustrated below:



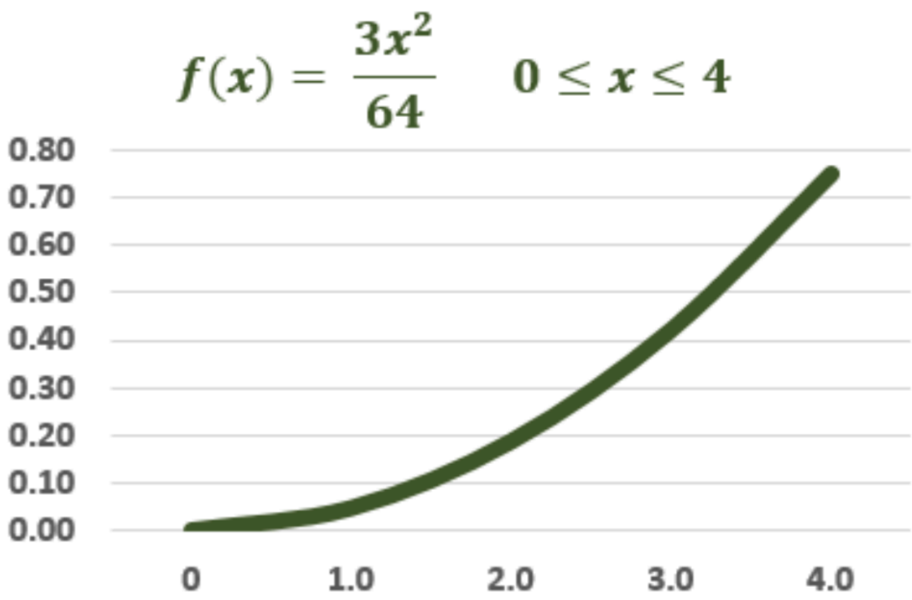
Assuming this qualifies as a probability distribution, which is **NEAREST** to the standard deviation of this random variable?

- **A.** Zero
- **B.** 0.625
- **C.** 0.750
- **D.** 0.930

Question 3 of 15

P1.T2.710.3

Consider a random variable that represents the loss severity of a risky asset and is given by the continuous probability distribution $f(x) = 3x^2/64$ on the domain from zero to four:

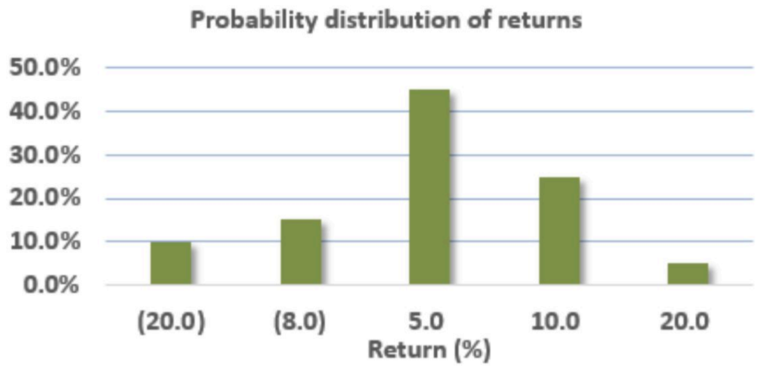


What is this variable's mean and variance?

- **A.** $\mu = 1.0$ and $\sigma^2 = 3.50$
- **B.** $\mu = 2.0$ and $\sigma^2 = 2.75$
- **C.** $\mu = 3.0$ and $\sigma^2 = 0.60$
- **D.** $\mu = 3.6$ and $\sigma^2 = 0.25$

P1.T2.712.1

Consider the following discrete probability distribution of asset returns:



			Central moments ("about the mean")		
X	f(X)	X*f(X)	2nd	3rd	4th
			$(X - \mu_X)^2 \cdot f(X)$	$(X - \mu_X)^3 \cdot f(X)$	$(X - \mu_X)^4 \cdot f(X)$
(20.0)	10.0%	(2.00)	50.9	(1,146.7)	25,857.5
(8.0)	15.0%	(1.20)	16.7	(176.1)	1,858.2
5.0	45.0%	2.25	2.7	6.6	16.2
10.0	25.0%	2.50	13.9	103.4	770.1
20.0	5.0%	1.00	15.2	265.7	4,636.1
Sum:		2.55	99.3	(947.1)	33,138.2

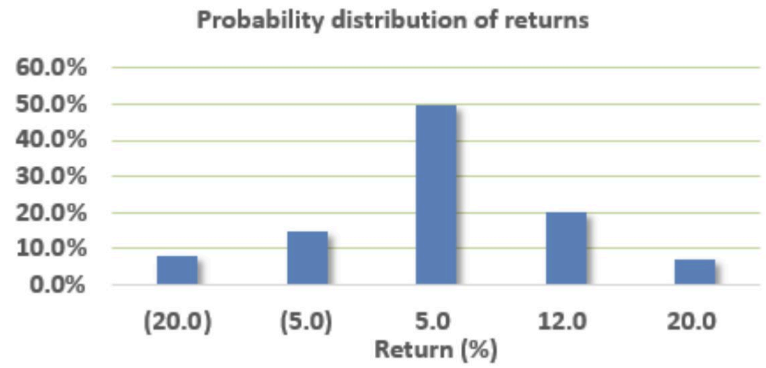
As shown, this asset's expected return is +2.55%. Which is **NEAREST** to this variable's skew; aka, standardized third central moment?

- **A.** -2.25
- **B.** -0.96
- **C.** +0.33
- **D.** +1.06

Question 5 of 15

P1.T2.712.2

Consider the following discrete probability distribution of asset returns:



			Central moments ("about the mean")		
X	f(X)	X*f(X)	2nd	3rd	4th
			$(X - \mu_X)^2 \cdot f(X)$	$(X - \mu_X)^3 \cdot f(X)$	$(X - \mu_X)^4 \cdot f(X)$
(20.0)	8.0%	(1.60)	45.9	(1,099.0)	26,321.6
(5.0)	15.0%	(0.75)	12.0	(107.5)	962.5
5.0	50.0%	2.50	0.6	0.6	0.6
12.0	20.0%	2.40	13.0	104.3	839.9
20.0	7.0%	1.40	18.0	289.4	4,645.1
Sum:		3.95	89.4	(812.2)	32,769.7

As shown, this asset's expected return is +3.95%. Which is **NEAREST** to this variable's kurtosis; aka, standardized fourth central moment?

- **A.** -1.47
- **B.** +2.81
- **C.** +4.10
- **D.** +7.50

Question 6 of 15

P1.T2.712.3

Portfolio manager Peter manages a large portfolio with 100 component positions. He is interested in analyzing the non-trivial cross moments in the portfolio (trivial cross-moments are the position's coskew/cokurtosis with itself, which is simply the position's standard skew or kurtosis, so these are analogous to the diagonal of a covariance matrix which is mere variances. Each of the following statements is true **EXCEPT** which is inaccurate?

- **A.** Between any two ($n = 2$) positions in the portfolio, the number of non-trivial coskew moments between them is two
- **B.** Between any two ($n = 2$) positions in the portfolio, the number of non-trivial cokurtosis moments between them is three
- **C.** Given a sub-portfolio consisting of any two positions, lower coskew values (i.e., where positives are gains and negatives are losses) imply greater risk for the sub-portfolio
- **D.** Although it is easy to estimate this portfolio's set of higher-order cross moments, standard skew and kurtosis are preferred because they are BLUE and the informational utility of coskew and cokurtosis is negligible

Question 7 of 15

P1.T2.303.1

Assume a continuous probability density function (pdf) is given by $f(x) = ax$ such that $0 \leq x \leq 12$, where (a) is a constant (we can retrieve this constant, knowing this is a probability density function):

$$f(x) = ax \quad \text{s.t.} \quad 0 \leq x \leq 12$$

What is the mean of (x)?

- **A.** 9.3
- **B.** 5.5
- **C.** 6.0
- **D.** 8.0

Question 8 of 15

P1.T2.303.2

Assume a continuous probability density function (pdf) be given by $f(x) = ax^2$ such that $0 \leq x \leq 3$, where (a) is a constant (that we can find).

$$f(x) = ax^2 \quad \text{s.t.} \quad 0 \leq x \leq 3$$

Let us arbitrarily define the unexpected loss (UL) as the difference between this distribution's mean and its 5.0% quantile function, i.e., $UL(X) = \text{mean}(X) - \text{inverse CDF}_{5\%}(X)$. We could call this a 95% relative VaR since it is relative to the mean. What is this UL?

- **A.** 0.62
- **B.** 1.14
- **C.** 2.05
- **D.** 3.37

Question 9 of 15

P1.T2.20.3.1

Assume a continuous random variable over the domain $\{6 < X < 10\}$ is given by $f(x) = ax$ where (a) is a constant, i.e., $\{f(x) = ax \mid x \in (6, 10)\}$. What is the $\Pr(X \leq 8)$?

Bonus: what is the variable's expected value?

- **A.** 19.53%
- **B.** 43.75%
- **C.** 50.00%
- **D.** 56.25%

Question 10 of 15

P1.T2.20.3.2

A discrete random variable is characterized by the probability mass function (pmf) as given by $f(x) = xa$, and its domain is the set of integers {6, 7, 8, 9, and 10}. What is the variable's expected value?

- **A.** 6.67
- **B.** 8.00
- **C.** 8.25
- **D.** 9.33

Question 11 of 15

P1.T2.20.4.3

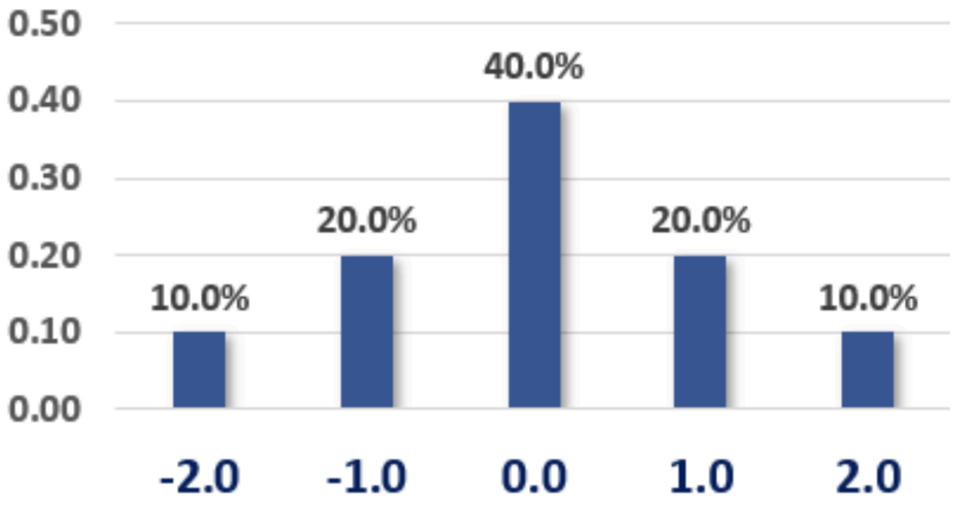
Let Z be a random variable that is a linear function of random variables X and Y, where $Z = 3X + 7Y$? If the standard deviation of X and Y, respectively, are $\sigma(X) = 4.0$ and $\sigma(Y) = 5.0$ and the correlation between X and Y is $\rho(X, Y) = 0.50$, then what is the standard deviation of Z, $\sigma(Z)$?

- **A.** 6.40
- **B.** 7.81
- **C.** 37.00
- **D.** 42.30

Question 12 of 15

P1.T2.20.4.1

Consider the probability mass function (pmf) below. For example, $\Pr(X = -1.0) = 20.0\%$.



As we can see, this distribution is symmetrical, so we know that its skewness is zero without performing any calculations. We are told the variance is 1.20 (although we can calculate the variance). What is this distribution's kurtosis?

- **A.** Zero
- **B.** 2.50
- **C.** 3.60
- **D.** 4.40

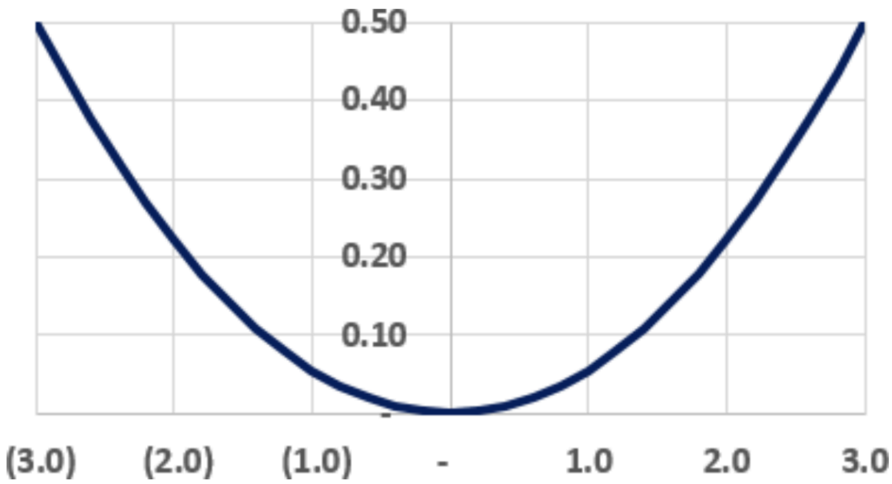
Question 13 of 15

P1.T2.20.4.2

Peter decides that the payoff of his option straddle strategy can be approximated by the probability density function (pdf) illustrated below. The function is: $f(x) = ax^2$ over the real domain $\{-3 < X < +3\}$; i.e., $\{f(x) = ax^2 \mid x \in (-3.0, 3.0)\}$. He used x-squared because he wants the shape of a parabola, but how does he determine the value of the constant, a? It cannot be anything! Because this is a probability distribution, it must be true that the integral, $ax^3/3$, when evaluated over $[3, -3]$ equals one: the area under the pdf curve must equal one, and the integral of the pdf is the cumulative distribution function (CDF).

Given that $a3^3/3 - a(-3)^3/3 = 1.0$, we must have $9a - (-9a) = 1.0$ such that $a = 1/18$. Therefore, a is $1/18$ and his probability density function is $f(x) = x^2/18$. But is $x^3/54$ the integral? He notices that if $F(x) = x^3/54$ then $F(3) = 27/54 = 0.5$ and $F(-3) = -0.5$, but the CDF requires that $F(3) = 1.0$ and $F(-3.0) = 0$. Seeing this, he realizes that the indefinite integral includes a constant and is given by $ax^3 + c$, or specifically, $x^3/54 + c$. Now it is possible for $3^3/54 + c = 1.0$, if $c = 1.0 - 0.5 = 0.5$. Finally, he can specify the exact pdf, $f(x) = x^2/18$, as the derivative of the correct CDF, $F(x) = x^3/54 + 0.5$, so that $F(3) = 1.0$ and $F(-3) = 0$.

Value-at-risk (VaR) is the distributional quantile associated with a specified probability. The inverse CDF (aka, quantile function) returns the quantile implied by a probability, $q = F^{-1}(p)$, whereas the CDF returns the probability associated with a quantile: $p = F(q)$, or in this case, $p = F(X)$. In this distribution, losses are on the left (negative payoffs), so the 95.0% VaR is the quantile implied by a 5.0% probability: $X = F^{-1}(p) = F^{-1}(0.050)$. For this distribution, what is the 95.0% VaR, in this case, the 0.050 quantile?



- A. -2.90
- B. -2.70
- C. -1.50
- D. +0.33

Question 14 of 15

P1.T2.20.3.3

Ralph is an analyst who wants to characterize a random variable with a discrete probability distribution where the only outcomes are {0, 1, 2, 3, and 4}. The Poisson distribution with a mean of 1.0 is a surprisingly good fit as it gives $\Pr(X = 0) = \Pr(X = 1) = 36.8\%$. The problem with the Poisson is that it has a long tail such that the $\Pr(X \geq 5) = 0.366\%$; for example, there is a very tiny possibility (1.0E-08) that the outcome could exceed ten given the Poisson's support is the entire set of natural numbers. To enforce a true probability distribution that is bounded at four, Ralph simply rounds the pmf densities and, due to sheer luck, when rounded (to two digits) the first five outcomes (including zero) sum exactly to 100.0%. The ensuing probability distribution is the following, which might be dubbed a "truncated Poisson" with $\lambda = 1.0$:

X	f(X)
0	37.0%
1	37.0%
2	18.0%
3	6.0%
4	2.0%
	100.0%

The exact mean (aka, expected value or weighted average) of this variable is 0.990, but let's assume its average is a round 1.0. What is the skewness of this variable?

- **A.** -0.774
- **B.** Zero
- **C.** +0.869
- **D.** +2.440

Question 15 of 15

P1.T2.303.3

Assume the following probability density function (pdf) for a random variable X:

$$f(x) = \frac{x}{18} \quad s.t. \quad 0 \leq \mathrm{x} \leq 6$$

What is the variance of X?

- **A.** 2.0
- **B.** 3.3
- **C.** 4.1
- **D.** 5.7