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Quantitative Methods, Economics, and
Corporate Issuers

Level I Book 1

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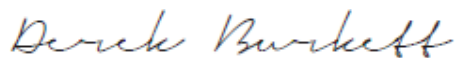
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Book 1: Quantitative Methods, Economics, and Corporate Issuers

SchweserNotes™ 2026

Level I CFA®

KAPLAN  **SCHWESER**

SCHWESERNOTES™ 2026 LEVEL I CFA® BOOK 1: QUANTITATIVE METHODS, ECONOMICS, AND CORPORATE ISSUERS

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Learning Outcome Statements (LOS)

1. Rates and Returns

The candidate should be able to:

- a. interpret interest rates as required rates of return, discount rates, or opportunity costs and explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk.
- b. calculate and interpret different approaches to return measurement over time and describe their appropriate uses.
- c. compare the money-weighted and time-weighted rates of return and evaluate the performance of portfolios based on these measures.
- d. calculate and interpret annualized return measures and continuously compounded returns, and describe their appropriate uses.
- e. calculate and interpret major return measures and describe their appropriate uses.

2. Time Value of Money in Finance

The candidate should be able to:

- a. calculate and interpret the present value (PV) of fixed-income and equity instruments based on expected future cash flows.
- b. calculate and interpret the implied return of fixed-income instruments and required return and implied growth of equity instruments given the present value (PV) and cash flows.
- c. explain the cash flow additivity principle, its importance for the no-arbitrage condition, and its use in calculating implied forward interest rates, forward exchange rates, and option values.

3. Statistical Measures of Asset Returns

The candidate should be able to:

- a. calculate, interpret, and evaluate measures of central tendency and location to address an investment problem.
- b. calculate, interpret, and evaluate measures of dispersion to address an investment problem.
- c. interpret and evaluate measures of skewness and kurtosis to address an investment problem.
- d. interpret correlation between two variables to address an investment problem.

4. Probability Trees and Conditional Expectations

The candidate should be able to:

- a. calculate expected values, variances, and standard deviations and demonstrate their application to investment problems.
- b. formulate an investment problem as a probability tree and explain the use of conditional expectations in investment application.
- c. calculate and interpret an updated probability in an investment setting using Bayes' formula.

5. Portfolio Mathematics

The candidate should be able to:

- a. calculate and interpret the expected value, variance, standard deviation, covariances, and correlations of portfolio returns.
- b. calculate and interpret the covariance and correlation of portfolio returns using a joint probability function for returns.
- c. define shortfall risk, calculate the safety-first ratio, and identify an optimal portfolio using Roy's safety-first criterion.

6. Simulation Methods

The candidate should be able to:

- a. explain the relationship between normal and lognormal distributions and why the lognormal distribution is used to model asset prices when using continuously compounded asset returns.
- b. describe Monte Carlo simulation and explain how it can be used in investment applications.

- c. describe the use of bootstrap resampling in conducting a simulation based on observed data in investment applications.

7. Estimation and Inference

The candidate should be able to:

- a. compare and contrast simple random, stratified random, cluster, convenience, and judgmental sampling and their implications for sampling error in an investment problem.
- b. explain the central limit theorem and its importance for the distribution and standard error of the sample mean.
- c. describe the use of resampling (bootstrap, jackknife) to estimate the sampling distribution of a statistic.

8. Hypothesis Testing

The candidate should be able to:

- a. explain hypothesis testing and its components, including statistical significance, Type I and Type II errors, and the power of a test.
- b. construct hypothesis tests and determine their statistical significance, the associated Type I and Type II errors, and power of the test given a significance level.
- c. compare and contrast parametric and nonparametric tests, and describe situations where each is the more appropriate type of test.

9. Parametric and Non-Parametric Tests of Independence

The candidate should be able to:

- a. explain parametric and nonparametric tests of the hypothesis that the population correlation coefficient equals zero, and determine whether the hypothesis is rejected at a given level of significance.
- b. explain tests of independence based on contingency table data.

10. Simple Linear Regression

The candidate should be able to:

- a. describe a simple linear regression model, how the least squares criterion is used to estimate regression coefficients, and the interpretation of these coefficients.
- b. explain the assumptions underlying the simple linear regression model, and describe how residuals and residual plots indicate if these assumptions may have been violated.
- c. calculate and interpret measures of fit and formulate and evaluate tests of fit and of regression coefficients in a simple linear regression.
- d. describe the use of analysis of variance (ANOVA) in regression analysis, interpret ANOVA results, and calculate and interpret the standard error of estimate in a simple linear regression.
- e. calculate and interpret the predicted value for the dependent variable, and a prediction interval for it, given an estimated linear regression model and a value for the independent variable.
- f. describe different functional forms of simple linear regressions.

11. Introduction to Big Data Techniques

The candidate should be able to:

- a. describe aspects of “fintech” that are directly relevant for the gathering and analyzing of financial data.
- b. describe Big Data, artificial intelligence, and machine learning.
- c. describe applications of Big Data and Data Science to investment management.

12. The Firm and Market Structures

The candidate should be able to:

- a. determine and interpret breakeven and shutdown points of production, as well as how economies and diseconomies of scale affect costs under perfect and imperfect competition.
- b. describe characteristics of perfect competition, monopolistic competition, oligopoly, and pure monopoly.
- c. explain supply and demand relationships under monopolistic competition, including the optimal price and output for firms as well as pricing strategy.
- d. explain supply and demand relationships under oligopoly, including the optimal price and output for firms as well as pricing strategy.

- e. identify the type of market structure within which a firm operates and describe the use and limitations of concentration measures.

13. Understanding Business Cycles

The candidate should be able to:

- a. describe the business cycle and its phases.
- b. describe credit cycles.
- c. describe how resource use, consumer and business activity, housing sector activity, and external trade sector activity vary over the business cycle and describe their measurement using economic indicators.

14. Fiscal Policy

The candidate should be able to:

- a. compare monetary and fiscal policy.
- b. describe roles and objectives of fiscal policy as well as arguments as to whether the size of a national debt relative to GDP matters.
- c. describe tools of fiscal policy, including their advantages and disadvantages.
- d. explain the implementation of fiscal policy and difficulties of implementation as well as whether a fiscal policy is expansionary or contractionary.

15. Monetary Policy

The candidate should be able to:

- a. describe the roles and objectives of central banks.
- b. describe tools used to implement monetary policy tools and the monetary transmission mechanism, and explain the relationships between monetary policy and economic growth, inflation, interest, and exchange rates.
- c. describe qualities of effective central banks; contrast their use of inflation, interest rate, and exchange rate targeting in expansionary or contractionary monetary policy; and describe the limitations of monetary policy.
- d. explain the interaction of monetary and fiscal policy.

16. Introduction to Geopolitics

The candidate should be able to:

- a. describe geopolitics from a cooperation versus competition perspective.
- b. describe geopolitics and its relationship with globalization.
- c. describe functions and objectives of the international organizations that facilitate trade, including the World Bank, the International Monetary Fund, and the World Trade Organization.
- d. describe geopolitical risk.
- e. describe tools of geopolitics and their impact on regions and economies.
- f. describe the impact of geopolitical risk on investments.

17. International Trade

The candidate should be able to:

- a. describe the benefits and costs of international trade.
- b. compare types of trade restrictions, such as tariffs, quotas, and export subsidies, and their economic implications.
- c. explain motivations for and advantages of trading blocs, common markets, and economic unions.

18. Capital Flows and the FX Market

The candidate should be able to:

- a. describe the foreign exchange market, including its functions and participants, distinguish between nominal and real exchange rates, and calculate and interpret the percentage change in a currency relative to another currency.
- b. describe exchange rate regimes and explain the effects of exchange rates on countries' international trade and capital flows.
- c. describe common objectives of capital restrictions imposed by governments.

19. Exchange Rate Calculations

The candidate should be able to:

- a. calculate and interpret currency cross-rates.
- b. explain the arbitrage relationship between spot and forward exchange rates and interest rates, calculate a forward rate using points or in percentage terms, and interpret a forward discount or premium.

20. Organizational Forms, Corporate Issuer Features, and Ownership

The candidate should be able to:

- a. compare the organizational forms of businesses.
- b. describe key features of corporate issuers.
- c. compare publicly and privately owned corporate issuers.

21. Investors and Other Stakeholders

The candidate should be able to:

- a. compare the financial claims and motivations of lenders and shareholders.
- b. describe a company's stakeholder groups and compare their interests.
- c. describe environmental, social, and governance factors of corporate issuers considered by investors.

22. Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits

The candidate should be able to:

- a. describe the principal-agent relationship and conflicts that may arise between stakeholder groups.
- b. describe corporate governance and mechanisms to manage stakeholder relationships and mitigate associated risks.
- c. describe potential risks of poor corporate governance and stakeholder management and benefits of effective corporate governance and stakeholder management.

23. Working Capital and Liquidity

The candidate should be able to:

- a. explain the cash conversion cycle and compare issuers' cash conversion cycles.
- b. explain liquidity and compare issuers' liquidity levels.
- c. describe issuers' objectives and compare methods for managing working capital and liquidity.

24. Capital Investments and Capital Allocation

The candidate should be able to:

- a. describe types of capital investments.
- b. describe the capital allocation process, calculate net present value (NPV), internal rate of return (IRR), and return on invested capital (ROIC), and contrast their use in capital allocation.
- c. describe principles of capital allocation and common capital allocation pitfalls.
- d. describe types of real options relevant to capital investments.

25. Capital Structure

The candidate should be able to:

- a. calculate and interpret the weighted-average cost of capital for a company.
- b. explain factors affecting capital structure and the weighted-average cost of capital.
- c. explain the Modigliani–Miller propositions regarding capital structure.
- d. describe optimal and target capital structures.

26. Business Models

The candidate should be able to:

- a. describe key features of business models.
- b. describe various types of business models.

READING 1

RATES AND RETURNS

MODULE 1.1: INTEREST RATES AND RETURN MEASUREMENT

LOS 1.a: Interpret interest rates as required rates of return, discount rates, or opportunity costs and explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk.

Interest rates measure the time value of money, although risk differences in financial securities lead to differences in their equilibrium interest rates. Equilibrium interest rates are the **required rate of return** for a particular investment, in the sense that the market rate of return is the return that investors and savers require to get them to willingly lend their funds. Interest rates are also referred to as **discount rates** and, in fact, the terms are often used interchangeably. If an individual can borrow funds at an interest rate of 10%, then that individual should discount payments to be made in the future at that rate to get their equivalent value in current dollars or other currencies. Finally, we can also view interest rates as the **opportunity cost** of current consumption. If the market rate of interest on 1-year securities is 5%, earning an additional 5% is the opportunity forgone when current consumption is chosen rather than saving (postponing consumption).

The **real risk-free rate** of interest is a theoretical rate on a single-period loan that contains no expectation of inflation and zero probability of default. What the real risk-free rate represents in economic terms is **time preference**, the degree to which current consumption is preferred to equal future consumption.

When we speak of a real rate of return, we are referring to an investor's increase in purchasing power (after adjusting for inflation). Because expected inflation in future periods is not zero, the rates we observe on U.S. Treasury bills (T-bills), for example, are essentially risk-free rates, but not real rates of return. T-bill rates are nominal risk-free rates because they contain an **inflation premium**. This is the relation:

$$(1 + \text{nominal risk-free rate}) = (1 + \text{real risk-free rate})(1 + \text{expected inflation rate})$$

Often, including in many parts of the CFA curriculum, this relation is approximated as follows:

$$\text{nominal risk-free rate} \approx \text{real risk-free rate} + \text{expected inflation rate}$$

Securities may have one or more types of risk, and each added risk increases the required rate of return. These types of risks are as follows:

- **Default risk.** This is the risk that a borrower will not make the promised payments in a timely manner.
- **Liquidity risk.** This is the risk of receiving less than fair value for an investment if it must be sold quickly for cash.
- **Maturity risk.** As we will see in the Fixed Income topic area, the prices of longer-term bonds are more volatile than those of shorter-term bonds. Longer-maturity bonds have more maturity risk than shorter-term bonds and require a maturity risk premium.

Each of these risk factors is associated with a risk premium that we add to the nominal risk-free rate to adjust for greater default risk, less liquidity, and longer maturity relative to a liquid, short-term, default risk-free rate such as that on T-bills. We can write the following:

$$\begin{aligned}
 \text{nominal rate of interest} &= \text{real risk-free rate} \\
 &+ \text{inflation premium} \\
 &+ \text{default risk premium} \\
 &+ \text{liquidity premium} \\
 &+ \text{maturity premium}
 \end{aligned}$$

LOS 1.b: Calculate and interpret different approaches to return measurement over time and describe their appropriate uses.

Holding period return (HPR) is simply the percentage increase in the value of an investment over a given period:

$$\text{holding period return} = \frac{\text{end-of-period value}}{\text{beginning-of-period value}} - 1$$

For example, a stock that pays a dividend during a holding period has an HPR for that period equal to:

$$\frac{P_t + \text{Div}_t}{P_0} - 1, \text{ or } \frac{P_t - P_0 + \text{Div}_t}{P_0}$$

If a stock is valued at €20 at the beginning of the period, pays €1 in dividends over the period, and at the end of the period is valued at €22, the HPR is:

$$\text{HPR} = (22 + 1) / 20 - 1 = 0.15 = 15\%$$

Returns over multiple periods reflect compounding. For example, given HPRs for Years 1, 2, and 3, the HPR for the entire three-year period is:

$$\text{HPR} = (1 + \text{HPR}_{\text{Year 1}})(1 + \text{HPR}_{\text{Year 2}})(1 + \text{HPR}_{\text{Year 3}}) - 1$$

Later in this reading, we will see that a return over multiple years is typically stated as an *annualized return* rather than an HPR.

Average Returns

The **arithmetic mean return** is the simple average of a series of periodic returns. It has the statistical property of being an unbiased estimator of the true mean of the underlying distribution of returns:

$$\text{arithmetic mean return} = \frac{(R_1 + R_2 + R_3 + \dots + R_n)}{n}$$

The **geometric mean return** is a compound rate. When periodic rates of return vary from period to period, the geometric mean return will have a value less than the arithmetic mean return:

$$\text{geometric mean return} = \sqrt[n]{(1 + R_1) \times (1 + R_2) \times (1 + R_3) \times \dots \times (1 + R_n)} - 1$$

For example, for returns R_t over three annual periods, the geometric mean return is calculated as the following example shows.

EXAMPLE: Geometric mean return

For the last three years, the returns for Acme Corporation common stock have been -9.34%, 23.45%, and 8.92%. Calculate the compound annual rate of return over the three-year period.

Answer:

$$\begin{aligned} R_G &= \sqrt[3]{(1 - 0.0934) \times (1 + 0.2345) \times (1 + 0.0892)} - 1 \\ &= \sqrt[3]{0.9066 \times 1.2345 \times 1.0892} - 1 \\ &= \sqrt[3]{1.21903} - 1 \\ R_G &= 1.06825 - 1 = 6.825\% \end{aligned}$$

Solve this type of problem with your calculator as follows:

- On the TI, enter 1.21903 [y^x] 3 [$1/x$] [=]
- On the HP, enter 1.21903 [ENTER] 3 [$1/x$] [y^x]

In the previous example, the geometric mean results in an annual rate of return because the holding periods were years. If the holding periods are other than years, the geometric mean is not the same as the annual return. The root for the geometric mean is the number of *periods*, while the root for the annual return is the number of *years*.

EXAMPLE: Geometric mean vs. annual return

For the last four semiannual periods, the 6-month holding period returns on an investment were 2.0%, 0.5%, -1.0%, and 1.5%. Calculate the geometric mean and the annual rate of return.

Answer:

$$\begin{aligned} \text{Geometric mean} &= \sqrt[4]{(1 + 0.02)(1 + 0.005)(1 - 0.01)(1 + 0.015)} - 1 \\ &= 0.007435 = 0.7435\%. \end{aligned}$$

This is the geometric mean of the 6-month holding period returns.

$$\begin{aligned}\text{Annual return} &= \sqrt[2]{(1 + 0.02)(1 + 0.005)(1 - 0.01)(1 + 0.015)} - 1 \\ &= 0.0149 = 1.49\%.\end{aligned}$$

The four semiannual periods equal two years, so to get an annual return we use 2 as the root.



PROFESSOR'S NOTE

The geometric mean is always less than or equal to the arithmetic mean, and the difference increases as the dispersion of the observations increases. The only time the arithmetic and geometric means are equal is when there is no variability in the observations (i.e., all observations are equal).

A **harmonic mean** is used for certain computations, such as the average cost of shares

purchased over time. The harmonic mean is calculated as $\frac{N}{\sum_{i=1}^N \frac{1}{X_i}}$ where there are N values of X_i .

EXAMPLE: Calculating average cost with the harmonic mean

An investor purchases \$1,000 of mutual fund shares each month, and over the last three months, the prices paid per share were \$8, \$9, and \$10. What is the average cost per share?

Answer:

$$\bar{X}_H = \frac{3}{\frac{1}{8} + \frac{1}{9} + \frac{1}{10}} = \$8.926 \text{ per share}$$

To check this result, calculate the total shares purchased as follows:

$$\frac{1,000}{8} + \frac{1,000}{9} + \frac{1,000}{10} = 336.11 \text{ shares}$$

The average price is $\frac{\$3,000}{336.11} = \8.926 per share.

The previous example illustrates the interpretation of the harmonic mean in its most common application. Note that the average price paid per share (\$8.93) is less than the arithmetic average of the share prices, which is $\frac{8 + 9 + 10}{3} = 9$.

We can only calculate a harmonic mean of positive numbers. For a set of returns that includes negative numbers, we can treat them the same way we did with geometric means, using $(1 + \text{return})$ for each period, then subtracting 1 from the result.

EXAMPLE: Harmonic mean with negative returns

For four periods, the returns on an investment were 2.0%, 0.5%, -1.0%, and 1.5%. Calculate the harmonic mean of these returns.

Answer:

$$\begin{aligned}\text{Harmonic mean} &= \frac{4}{\frac{1}{(1 + 0.02)} + \frac{1}{(1 + 0.005)} + \frac{1}{(1 - 0.01)} + \frac{1}{(1 + 0.015)}} - 1 \\ &= 0.007369 = 0.7369\%\end{aligned}$$

The relationship among arithmetic, geometric, and harmonic means can be stated as follows:

$$\text{arithmetic mean} \times \text{harmonic mean} = (\text{geometric mean})^2$$



PROFESSOR'S NOTE

The proof of this is beyond the scope of the Level I exam.

For values that are not all equal, harmonic mean < geometric mean < arithmetic mean. This mathematical fact is the basis for the claimed benefit of purchasing the same money amount of mutual fund shares each month or each week. Some refer to this practice as **cost averaging**.

Measures of average return can be affected by outliers, which are unusual observations in a dataset. Two of the methods for dealing with outliers are a *trimmed mean* and a *winsorized mean*. We will examine these in our reading on Statistical Measures of Asset Returns.

Appropriate uses for the various return measures are as follows:

- **Arithmetic mean.** Include all values, including outliers.
- **Geometric mean.** Compound the rate of returns over multiple periods.
- **Harmonic mean.** Calculate the average share cost from periodic purchases in a fixed money amount.
- **Trimmed or winsorized mean.** Decrease the effect of outliers.



MODULE QUIZ 1.1

1. An interest rate is *best* interpreted as a:
 - A. discount rate or a measure of risk.
 - B. measure of risk or a required rate of return.
 - C. required rate of return or the opportunity cost of consumption.
2. An interest rate from which the inflation premium has been subtracted is known as a:
 - A. real interest rate.
 - B. risk-free interest rate.
 - C. real risk-free interest rate.
3. The harmonic mean of 3, 4, and 5 is:
 - A. 3.74.
 - B. 3.83.

C. 4.12.

4. XYZ Corp. Annual Stock Returns

Year	20X1	20X2	20X3	20X4	20X5	20X6
Return	22%	5%	-7%	11%	2%	11%

The mean annual return on XYZ stock is *most appropriately* calculated using the:

- A. harmonic mean.
- B. arithmetic mean.
- C. geometric mean.

MODULE 1.2: TIME-WEIGHTED AND MONEY-WEIGHTED RETURNS

LOS 1.c: Compare the money-weighted and time-weighted rates of return and evaluate the performance of portfolios based on these measures.

The **money-weighted return** applies the concept of the **internal rate of return (IRR)** to investment portfolios. An IRR is the interest rate at which a series of cash inflows and outflows sum to zero when discounted to their present value. That is, they have a **net present value (NPV)** of zero. The IRR and NPV are built-in functions on financial calculators that CFA Institute permits candidates to use for the exam.



PROFESSOR'S NOTE

We have provided an online video in the Resource Library on how to use the TI calculator. You can view it by logging in to your account at www.schweser.com.

The **money-weighted rate of return** is defined as the IRR on a portfolio, taking into account all cash inflows and outflows. The beginning value of the account is an inflow, as are all deposits into the account. All withdrawals from the account are outflows, as is the ending value.

EXAMPLE: Money-weighted rate of return

Assume an investor buys a share of stock for \$100 at $t = 0$, and at the end of the year ($t = 1$), she buys an additional share for \$120. At the end of Year 2, the investor sells both shares for \$130 each. At the end of each year in the holding period, the stock paid a \$2 per share dividend. What is the money-weighted rate of return?

Step 1: Determine the timing of each cash flow and whether the cash flow is an inflow (+), into the account, or an outflow (-), available from the account.

$t = 0$: purchase of first share	= +\$100.00	inflow to account
$t = 1$: purchase of second share	= +\$120.00	
dividends from first share	= -\$2.00	
subtotal, $t = 1$	+ \$118.00	inflow to account
$t = 2$: dividend from two shares	= -\$4.00	
proceeds from selling shares	= -\$260.00	
subtotal, $t = 2$	- \$264.00	outflow from account

Step 2: Net the cash flows for each period and set the PV of cash inflows equal to the PV of cash outflows.

$$PV_{\text{inflows}} = PV_{\text{outflows}}$$

$$\$100 + \frac{\$118}{(1+r)} = \frac{\$264}{(1+r)^2}$$

Step 3: Solve for r to find the money-weighted rate of return. This can be done using trial and error or by using the IRR function on a financial calculator or spreadsheet.

The intuition here is that we deposited \$100 into the account at $t = 0$, then added \$118 to the account at $t = 1$ (which, with the \$2 dividend, funded the purchase of one more share at \$120), and ended with a total value of \$264.

To compute this value with a financial calculator, use these net cash flows and follow the procedure(s) described to calculate the IRR:

$$\text{net cash flows: } CF_0 = +100; CF_1 = +120 - 2 = +118;$$

$$CF_2 = -260 + -4 = -264$$

Calculating money-weighted return with the TI Business Analyst II Plus®

Note the values for F01, F02, and so on, are all equal to 1.

Keystrokes	Explanation	Display
[CF] [2 nd][CLR WORK]	Clear cash flow registers	CF0 = 0.00000
100 [ENTER]	Initial cash outlay	CF0 = +100.00000
[↓] 118 [ENTER]	Period 1 cash flow	C01 = +118.00000
[↓] [↓] 264 [+/-] [ENTER]	Period 2 cash flow	C02 = -264.00000
[IRR] [CPT]	Calculate IRR	IRR = 13.86122

The money-weighted rate of return for this problem is 13.86%.



PROFESSOR'S NOTE

In the preceding example, we entered the flows into the account as a positive and the ending value as a negative (the investor could withdraw this amount from the account). Note that there is no difference in the solution if we enter the cash flows into the account as negative values (out of the investor's pocket) and the ending value as a positive value (into the investor's pocket).