

2024 CFA®  
Exam Prep

**SchweserNotes™**  
Fixed Income, Derivatives, and  
Alternative Investments

LEVEL II BOOK 4

KAPLAN SCHWESER

# Book 4: Fixed Income, Derivatives, and Alternative Investments

**SchweserNotes™ 2024**

Level II CFA®

**KAPLAN**  **SCHWESER**

SCHWESERNOTES™ 2024 LEVEL II CFA® BOOK 4: FIXED INCOME, DERIVATIVES, AND ALTERNATIVE INVESTMENTS

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# Learning Outcome Statements (LOS)

## 25. The Term Structure and Interest Rate Dynamics

The candidate should be able to:

- a. describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve.
- b. describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping.
- c. describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management.
- d. describe the strategy of rolling down the yield curve.
- e. explain the swap rate curve and why and how market participants use it in valuation.
- f. calculate and interpret the swap spread for a given maturity.
- g. describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk.
- h. explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve.
- i. explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks.
- j. explain the maturity structure of yield volatilities and their effect on price volatility.
- k. explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes.

## 26. The Arbitrage-Free Valuation Framework

The candidate should be able to:

- a. explain what is meant by arbitrage-free valuation of a fixed-income instrument.
- b. calculate the arbitrage-free value of an option-free, fixed-rate coupon bond.
- c. describe a binomial interest rate tree framework.
- d. describe the process of calibrating a binomial interest rate tree to match a specific term structure.
- e. describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node.
- f. compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice.
- g. describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path.
- h. describe a Monte Carlo forward-rate simulation and its application.
- i. describe term structure models and how they are used.

## 27. Valuation and Analysis of Bonds with Embedded Options

The candidate should be able to:

- a. describe fixed-income securities with embedded options.
- b. explain the relationships between the values of a callable or puttable bond, the underlying option-free (straight) bond, and the embedded option.
- c. describe how the arbitrage-free framework can be used to value a bond with embedded options.
- d. explain how interest rate volatility affects the value of a callable or puttable bond.
- e. explain how changes in the level and shape of the yield curve affect the value of a callable or puttable bond.
- f. calculate the value of a callable or puttable bond from an interest rate tree.
- g. explain the calculation and use of option-adjusted spreads.
- h. explain how interest rate volatility affects option-adjusted spreads.
- i. calculate and interpret effective duration of a callable or puttable bond.
- j. compare effective durations of callable, puttable, and straight bonds.
- k. describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options.
- l. compare effective convexities of callable, puttable, and straight bonds.
- m. calculate the value of a capped or floored floating-rate bond.

- n. describe defining features of a convertible bond.
- o. calculate and interpret the components of a convertible bond's value.
- p. describe how a convertible bond is valued in an arbitrage-free framework.
- q. compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock.

## **28. Credit Analysis Models**

The candidate should be able to:

- a. explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment.
- b. explain credit scores and credit ratings.
- c. calculate the expected return on a bond given transition in its credit rating.
- d. explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses.
- e. calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters.
- f. interpret changes in a credit spread.
- g. explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads.
- h. compare the credit analysis required for securitized debt to the credit analysis of corporate debt.

## **29. Credit Default Swaps**

The candidate should be able to:

- a. describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product.
- b. describe credit events and settlement protocols with respect to CDS.
- c. explain the principles underlying and factors that influence the market's pricing of CDS.
- d. describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve.
- e. describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments.

## **30. Pricing and Valuation of Forward Commitments**

The candidate should be able to:

- a. describe how equity forwards and futures are priced, and calculate and interpret their no-arbitrage value.
- b. describe the carry arbitrage model without underlying cashflows and with underlying cashflows.
- c. describe how interest rate forwards and futures are priced, and calculate and interpret their no-arbitrage value.
- d. describe how fixed-income forwards and futures are priced, and calculate and interpret their no-arbitrage value.
- e. describe how interest rate swaps are priced, and calculate and interpret their no-arbitrage value.
- f. describe how currency swaps are priced, and calculate and interpret their no-arbitrage value.
- g. describe how equity swaps are priced, and calculate and interpret their no-arbitrage value.

## **31. Valuation of Contingent Claims**

The candidate should be able to:

- a. describe and interpret the binomial option valuation model and its component terms.
- b. describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration.
- c. identify an arbitrage opportunity involving options and describe the related arbitrage.
- d. calculate the no-arbitrage values of European and American options using a two-period binomial model.
- e. calculate and interpret the value of an interest rate option using a two-period binomial model.
- f. identify assumptions of the Black–Scholes–Merton option valuation model.
- g. interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying.

- h. describe how the Black–Scholes–Merton model is used to value European options on equities and currencies.
- i. describe how the Black model is used to value European options on futures.
- j. describe how the Black model is used to value European interest rate options and European swaptions.
- k. interpret each of the option Greeks.
- l. describe how a delta hedge is executed.
- m. describe the role of gamma risk in options trading.
- n. define implied volatility and explain how it is used in options trading.

### **32. Introduction to Commodities and Commodity Derivatives**

The candidate should be able to:

- a. compare characteristics of commodity sectors.
- b. compare the life cycle of commodity sectors from production through trading or consumption.
- c. contrast the valuation of commodities with the valuation of equities and bonds.
- d. describe types of participants in commodity futures markets.
- e. analyze the relationship between spot prices and futures prices in markets in contango and markets in backwardation.
- f. compare theories of commodity futures returns.
- g. describe, calculate, and interpret the components of total return for a fully collateralized commodity futures contract.
- h. contrast roll return in markets in contango and markets in backwardation.
- i. describe how commodity swaps are used to obtain or modify exposure to commodities.
- j. describe how the construction of commodity indexes affects index returns.

### **33. Real Estate Investments**

The candidate should be able to:

- a. compare the characteristics, classifications, principal risks, and basic forms of public and private real estate investments.
- b. explain portfolio roles and economic value determinants of real estate investments.
- c. discuss commercial property types, including their distinctive investment characteristics.
- d. explain the due diligence process for both private and public equity real estate investment.
- e. discuss real estate investment indexes, including their construction and potential biases.
- f. discuss types of publicly traded real estate securities.
- g. justify the use of net asset value per share (NAVPS) in valuation of publicly traded real estate securities and estimate NAVPS based on forecasted cash net operating income.
- h. describe the use of funds from operations (FFO) and adjusted funds from operations (AFFO) in REIT valuation.
- i. calculate and interpret the value of a REIT share using the net asset value, relative value (price-to-FFO and price-to-AFFO), and discounted cash flow approaches.
- j. explain advantages and disadvantages of investing in real estate through publicly traded securities compared to private vehicles.

### **34. Hedge Fund Strategies**

The candidate should be able to:

- a. discuss how hedge fund strategies may be classified.
- b. discuss investment characteristics, strategy implementation, and role in a portfolio of equity-related hedge fund strategies.
- c. discuss investment characteristics, strategy implementation, and role in a portfolio of event-driven hedge fund strategies.
- d. discuss investment characteristics, strategy implementation, and role in a portfolio of relative value hedge fund strategies.
- e. discuss investment characteristics, strategy implementation, and role in a portfolio of opportunistic hedge fund strategies.
- f. discuss investment characteristics, strategy implementation, and role in a portfolio of specialist hedge fund strategies.
- g. discuss investment characteristics, strategy implementation, and role in a portfolio of multi-manager hedge fund strategies.
- h. describe how factor models may be used to understand hedge fund risk exposures.
- i. evaluate the impact of an allocation to a hedge fund strategy in a traditional investment portfolio.

## READING 25

# THE TERM STRUCTURE AND INTEREST RATE DYNAMICS

### EXAM FOCUS

This topic review discusses the theories and implications of the term structure of interest rates. In addition to understanding the relationships between spot rates, forward rates, yield to maturity, and the shape of the yield curve, be sure you become familiar with concepts like the Z-spread, the TED spread and the MRR-OIS spread. Interpreting the shape of the yield curve in the context of the theories of the term structure of interest rates is always important for the exam. Also pay close attention to the concept of key rate duration.

### INTRODUCTION

The financial markets both impact and are controlled by interest rates. Understanding the term structure of interest rates (i.e., the graph of interest rates at different maturities) is one key to understanding the performance of an economy. In this reading, we explain how and why the term structure changes over time.

**Spot rates** are the annualized market interest rates for a single payment to be received in the future. Generally, we use spot rates for government securities (risk-free) to generate the spot rate curve. Spot rates can be interpreted as the yields on zero-coupon bonds, and for this reason we sometimes refer to spot rates as *zero-coupon rates*. A **forward rate** is an interest rate (agreed to today) for a loan to be made at some future date.



#### PROFESSOR'S NOTE

While most of the LOS in this topic review have *describe* or *explain* as the command words, we will still delve into numerous calculations, as it is difficult to really understand some of these concepts without getting in to the mathematics behind them.



Video covering this content is available online.

# MODULE 25.1: SPOT AND FORWARD RATES, PART 1

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**LOS 25.a: Describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve.**

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## SPOT RATES

The price today of \$1 par, zero-coupon bond is known as the discount factor, which we will call  $P_T$ . Because it is a zero-coupon bond, the spot interest rate is the yield to maturity of this payment, which we represent as  $S_T$ . The relationship between the discount factor  $P_T$  and the spot rate  $S_T$  for maturity  $T$  can be expressed as:

$$P_T = \frac{1}{(1 + S_T)^T}$$

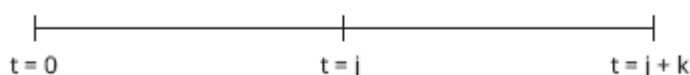
The term structure of spot rates—the graph of the spot rate  $S_T$  versus the maturity  $T$ —is known as the **spot yield curve** or **spot curve**. The shape and level of the spot curve changes continuously with the market prices of bonds.

## FORWARD RATES

The annualized interest rate on a loan to be initiated at a future period is called the **forward rate** for that period. The term structure of forward rates is called the **forward curve**. (Note that forward curves and spot curves are mathematically related—we can derive one from the other.)

We will use the following notation:

$f(j,k)$  = the annualized interest rate applicable on a  $k$ -year loan starting in  $j$  years.



$F_{(j,k)}$  = the forward price of a \$1 par zero-coupon bond maturing at time  $j+k$  delivered at time  $j$ .

$$F_{(j,k)} = \frac{1}{[1 + f(j,k)]^k}$$

## YIELD TO MATURITY

As we've discussed, the **yield to maturity (YTM)** or yield of a zero-coupon bond with maturity  $T$  is the spot interest rate for a maturity of  $T$ . However, for a coupon bond, if the spot rate curve is not flat, the YTM will not be the same as the spot rate.

### EXAMPLE: Spot rates and yield for a coupon bond

Compute the price and yield to maturity of a three-year, 4% annual-pay, \$1,000 face value bond given the following spot rate curve:  $S_1 = 5\%$ ,  $S_2 = 6\%$ , and  $S_3 = 7\%$ .

**Answer:**

1. Calculate the price of the bond using the spot rate curve:

$$\text{Price} = \frac{40}{(1.05)} + \frac{40}{(1.06)^2} + \frac{1040}{(1.07)^3} = \$922.64$$

2. Calculate the yield to maturity ( $y_3$ ):

$$N = 3; PV = -922.64; PMT = 40; FV = 1,000; \text{CPT I/Y} \rightarrow 6.94 \\ y_3 = 6.94\%$$

Note that the yield on a three-year bond is a weighted average of three spot rates, so in this case we would expect  $S_1 < y_3 < S_3$ . The yield to maturity  $y_3$  is closest to  $S_3$  because the par value dominates the value of the bond and therefore  $S_3$  has the highest weight.

## EXPECTED AND REALIZED RETURNS ON BONDS

Expected return is the ex-ante holding period return that a bond investor expects to earn.

The expected return will be equal to the bond's yield only when *all three* of the following are true:

- The bond is held to maturity.
- All payments (coupon and principal) are made on time and in full.
- All coupons are reinvested at the original YTM.

The second requirement implies that the bond is option-free and there is no default risk.

The last requirement, reinvesting coupons at the YTM, is the least realistic assumption. If the yield curve is not flat, the coupon payments will not be reinvested at the YTM and the expected return will differ from the yield.

Realized return on a bond refers to the actual return that the investor experiences over the investment's holding period. Realized return is based on actual reinvestment rates.

## THE FORWARD PRICING MODEL

The **forward pricing model** values forward contracts based on arbitrage-free pricing.

Consider two investors.

Investor A purchases a \$1 face value, zero-coupon bond maturing in  $j+k$  years at a price of  $P_{(j+k)}$ .

Investor B enters into a  $j$ -year forward contract to purchase a \$1 face value, zero-coupon bond maturing in  $k$  years at a price of  $F_{(j,k)}$ . Investor B's cost today is the present value of the cost:  $PV[F_{(j,k)}]$  or  $P_j F_{(j,k)}$ .

Because the \$1 cash flows at  $j+k$  are the same, these two investments should have the same price, which leads to the forward pricing model:

$$P_{(j+k)} = P_j F_{(j,k)}$$

Therefore:

$$F_{(j,k)} = \frac{P_{(j+k)}}{P_j}$$

### EXAMPLE: Forward pricing

Calculate the forward price two years from now for a \$1 par, zero-coupon, three-year bond given the following spot rates.

The two-year spot rate,  $S_2 = 4\%$ .

The five-year spot rate,  $S_5 = 6\%$ .

#### Answer:

Calculate discount factors  $P_j$  and  $P_{(j+k)}$ .

$$P_j = P_2 = 1 / (1 + 0.04)^2 = 0.9246$$

$$P_{(j+k)} = P_5 = 1 / (1 + 0.06)^5 = 0.7473$$

The forward price of a three-year bond in two years is represented as  $F_{(2,3)}$

$$F_{(j,k)} = P_{(j+k)} / P_j$$

$$F_{(2,3)} = 0.7473 / 0.9246 = 0.8082$$

In other words, \$0.8082 is the price agreed to today, to pay in two years, for a three-year bond that will pay \$1 at maturity.



#### PROFESSOR'S NOTE

In the Derivatives portion of the curriculum, the forward price is computed as the future value (for  $j$  periods) of  $P_{(j+k)}$ . It gives the same result and can be verified using the data in the previous example by computing the future value of  $P_5$  (i.e., compounding for two periods at  $S_2$ ).

$$FV = 0.7473(1.04)^2 = \$0.8082.$$

## The Forward Rate Model

The **forward rate model** relates forward and spot rates as follows:

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j [1 + f(j,k)]^k$$

or

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

This model is useful because it illustrates how forward rates and spot rates are interrelated.

This equation suggests that the forward rate  $f(2,3)$  should make investors indifferent between buying a five-year zero-coupon bond versus buying a two-year zero-coupon bond and at maturity reinvesting the principal for three additional years.

### EXAMPLE: Forward rates

Suppose that the two-year and five-year spot rates are  $S_2 = 4\%$  and  $S_5 = 6\%$ .

Calculate the implied three-year forward rate for a loan starting two years from now [i.e.,  $f(2,3)$ ].

**Answer:**

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

$$[1 + f(2,3)]^3 = [1 + 0.06]^5 / [1 + 0.04]^2$$

$$f(2,3) = 7.35\%$$

Note that the forward rate  $f(2,3) > S_5$  because the yield curve is upward sloping.

If the yield curve is upward sloping, [i.e.,  $S_{(j+k)} > S_j$ ], then the forward rate corresponding to the period from  $j$  to  $k$  [i.e.,  $f(j,k)$ ] will be greater than the spot rate for maturity  $j+k$  [i.e.,  $S_{(j+k)}$ ]. The opposite is true if the curve is downward sloping.

---

### LOS 25.b: Describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping.

---

A **par rate** is the yield to maturity of a bond trading at par. Par rates for bonds with different maturities make up the **par rate curve** or simply the **par curve**. By definition, the par rate will be equal to the coupon rate on the bond. Generally, par curve refers to the par rates for government or benchmark bonds.

By using a process called **bootstrapping**, spot rates or zero-coupon rates can be derived from the par curve. Bootstrapping involves using the output of one step as an input to the next step. We first recognize that (for annual-pay bonds) the one-year spot rate ( $S_1$ ) is the same as the one-year par rate. We can then compute  $S_2$  using  $S_1$  as one of the inputs. Continuing the process, we can compute the three-year spot rate  $S_3$  using  $S_1$  and  $S_2$  computed earlier. Let's clarify this with an example.

#### EXAMPLE: Bootstrapping spot rates

Given the following (annual-pay) par curve, compute the corresponding spot rate curve:

Maturity	Par Rate
1	1.00%
2	1.25%
3	1.50%

**Answer:**

$S_1 = 1.00\%$  (given directly).

If we discount each cash flow of the bond using its yield, we get the market price of the bond. Here, the market price is the par value. Consider the 2-year bond.

$$100 = \frac{1.25}{(1.0125)} + \frac{101.25}{(1.0125)^2}$$

Alternatively, we can also value the 2-year bond using spot rates:

$$100 = \frac{1.25}{(1 + S_1)} + \frac{101.25}{(1 + S_2)^2} = \frac{1.25}{(1.01)} + \frac{101.25}{(1 + S_2)^2}$$

$$100 = 1.2376 + \frac{101.25}{(1 + S_2)^2}$$

$$98.7624 = \frac{101.25}{(1 + S_2)^2}$$

Multiplying both sides by  $[(1 + S_2)^2 / 98.7624]$ , we get  $(1 + S_2)^2 = 1.0252$ .

Taking square roots, we get  $(1 + S_2) = 1.01252$ .  $S_2 = 0.01252$  or 1.252%

Similarly,  $100 = \frac{1.50}{(1 + S_1)} + \frac{1.50}{(1 + S_2)^2} + \frac{101.50}{(1 + S_3)^3}$

Using the values of  $S_1$  and  $S_2$  computed earlier,

$$100 = \frac{1.50}{(1.01)} + \frac{1.50}{(1.01252)^2} + \frac{101.50}{(1 + S_3)^3}$$

$$100 = 2.9483 + \frac{101.50}{(1 + S_3)^3}$$

$$97.0517 = \frac{101.50}{(1 + S_3)^3}$$

$$(1 + S_3)^3 = 1.0458$$

$$(1 + S_3) = 1.0151 \text{ and hence } S_3 = 1.51\%$$



### MODULE QUIZ 25.1

- When the yield curve is downward sloping, the forward curves are *most likely* to lie:
  - above the spot curve.
  - below the spot curve.
  - either above or below the spot curve.
- The model that equates buying a long-maturity zero-coupon bond to entering into a forward contract to buy a zero-coupon bond that matures at the same time is known as the:
  - forward rate model.
  - forward pricing model.
  - forward arbitrage model.



Video covering this content is available online.

## MODULE 25.2: SPOT AND FORWARD RATES, PART 2

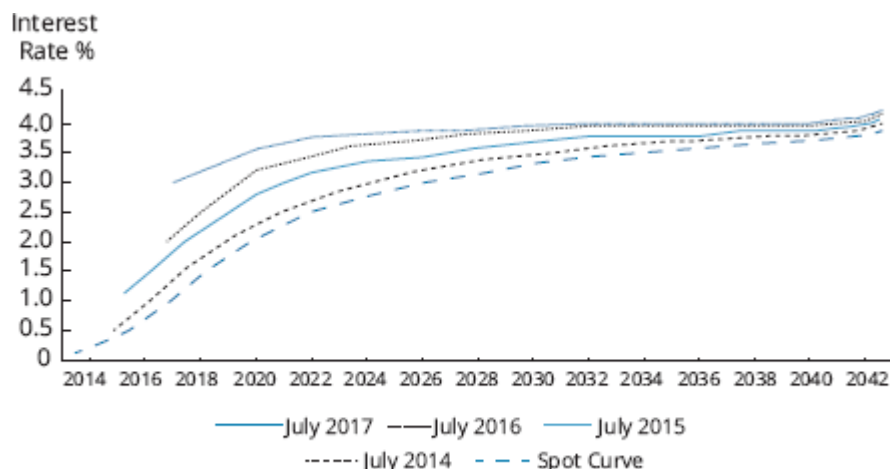
**LOS 25.c: Describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management.**

### RELATIONSHIPS BETWEEN SPOT AND FORWARD RATES

For an upward-sloping spot curve, the forward rate rises as  $j$  increases. (For a downward-sloping yield curve, the forward rate declines as  $j$  increases.) For an upward-sloping spot curve, the forward curve will be above the spot curve as shown in Figure 25.1. Conversely, when the spot curve is downward sloping, the forward curve will be below it.

Figure 25.1 shows spot and forward curves as of July 2013. Because the spot yield curve is upward sloping, the forward curves lie above the spot curve.

**Figure 25.1: Spot Curve and Forward Curves**



Reproduced from Level II CFA Curriculum learning module "The Term Structure and Interest Rate Dynamics," Exhibit 2, with permission from CFA Institute.

From the forward rate model:

$$(1 + S_T)^T = (1 + S_1)[1 + f(1, T - 1)]^{(T-1)}$$

which can be expanded to:

$$(1 + S_T)^T = (1 + S_1) [1 + f(1,1)] [1 + f(2,1)] [1 + f(3,1)] \dots [1 + f(T - 1,1)]$$

In other words, the spot rate for a long-maturity security will equal the geometric mean of the one period spot rate and a series of one-year forward rates.

### Forward Price Evolution

If the future spot rates actually evolve as forecasted by the forward curve, the forward price will remain unchanged. Therefore, a change in the forward price indicates that the future spot rate(s) did not conform to the forward curve. When

spot rates turn out to be lower (higher) than implied by the forward curve, the forward price will increase (decrease). A trader expecting lower future spot rates (than implied by the current forward rates) would purchase the forward contract to profit from its appreciation.

For a bond investor, the return on a bond over a one-year horizon is always equal to the one-year risk-free rate *if the spot rates evolve as predicted by today's forward curve*. If the spot curve one year from today is not the same as that predicted by today's forward curve, the return over the one-year period will differ, with the return depending on the bond's maturity.

An active portfolio manager will try to outperform the overall bond market by predicting how the future spot rates will differ from those predicted by the current forward curve.

### EXAMPLE: Spot rate evolution

Jane Dash, CFA, has collected benchmark spot rates as shown here:

Maturity	Spot Rate
1	3.00%
2	4.00%
3	5.00%

The expected spot rates at the end of one year are as follows:

Year	Expected Spot
1	5.01%
2	6.01%

Calculate the one-year holding period return of a:

1. 1-year zero-coupon bond.
2. 2-year zero-coupon bond.
3. 3-year zero-coupon bond.

### Answer:

First, note that the expected spot rates provided just happen to be the forward rates implied by the current spot rate curve.

Recall that:

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

Hence:

$$[1 + f(1,1)]^1 = \frac{(1 + S_2)^2}{(1 + S_1)} = \frac{(1.04)^2}{(1.03)} \rightarrow f(1,1) = 0.0501 \text{ and}$$

$$[1 + f(1,2)]^2 = \frac{(1 + S_3)^3}{(1 + S_1)} = \frac{(1.05)^3}{(1.03)} \rightarrow f(1,2) = 0.0601$$

1. The price of a one-year zero-coupon bond given the one-year spot rate of 3% is  $1 / (1.03)$  or 0.9709.

After one year, the bond is at maturity and pays \$1 regardless of the spot rates.

Hence the holding period return =  $\left(\frac{1.00}{0.9709}\right) - 1 = 3\%$

2. The price of a two-year zero-coupon bond given the two-year spot rate of 4%:

$$P_2 = \frac{1}{(1 + S_2)^2} = \frac{1}{(1.04)^2} = 0.9246$$

After one year, the bond will have one year remaining to maturity, and based on a one-year expected spot rate of 5.01%, the bond's price will be  $1 / (1.0501) = \$0.9523$

Hence, the holding period return =  $\left(\frac{0.9523}{0.9246}\right) - 1 = 3\%$

3. The price of three-year zero-coupon bond given the three-year spot rate of 5%:

$$P_3 = \frac{1}{(1 + S_3)^3} = \frac{1}{(1.05)^3} = 0.8638$$

After one year, the bond will have two years remaining to maturity.

Based on a two-year expected spot rate of 6.01%, the bond's price will be  $1 / (1.0601)^2 = \$0.8898$

Hence, the holding period return =  $\left(\frac{0.8898}{0.8638}\right) - 1 = 3\%$

Hence, regardless of the maturity of the bond, the holding period return will be the one-year spot rate if the spot rates evolve consistent with the forward curve (as it existed when the trade was initiated).

If an investor believes that future spot rates will be lower than corresponding forward rates, then she will purchase bonds (at a presumably attractive price) because the market appears to be discounting future cash flows at "too high" of a discount rate.

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#### LOS 25.d: Describe the strategy of rolling down the yield curve.

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### “ROLLING DOWN THE YIELD CURVE”

The most straightforward strategy for a bond investor is *maturity matching*— purchasing bonds that have a maturity equal to the investor's investment horizon.

However, with an upward-sloping interest rate term structure, investors seeking superior returns may pursue a strategy called **riding the yield curve** (also known as **rolling down the yield curve**). Under this strategy, an investor will purchase bonds with maturities longer than his investment horizon. In an upward-sloping yield curve, shorter maturity bonds have lower yields than longer maturity bonds. As the bond approaches maturity (i.e., rolls down the yield curve), it is valued using successively lower yields and, therefore, at successively higher prices.

If the yield curve remains unchanged over the investment horizon, riding the yield curve strategy will produce higher returns than a simple maturity matching strategy, increasing the total return of a bond portfolio. The greater the difference

between the forward rate and the spot rate, and the longer the maturity of the bond, the higher the total return.

Consider Figure 25.2, which shows a hypothetical upward-sloping yield curve and the price of a 3% annual-pay coupon bond (as a percentage of par).

**Figure 25.2: Price of a 3%, Annual Pay Bond**

Maturity	Yield	Price
5	3	100
10	3.5	95.84
15	4	88.88
20	4.5	80.49
25	5	71.81
30	5.5	63.67

A bond investor with an investment horizon of five years could purchase a bond maturing in five years and earn the 3% coupon but no capital gains (the bond can be currently purchased at par and will be redeemed at par at maturity). However, assuming no change in the yield curve over the investment horizon, the investor could instead purchase a 30-year bond for \$63.67, hold it for five years, and sell it for \$71.81, earning an additional return beyond the 3% coupon over the same period.

In the aftermath of the financial crisis of 2007–08, central banks kept short-term rates low, giving yield curves a steep upward slope. Many active managers took advantage by borrowing at short-term rates and buying long maturity bonds. The risk of such a leveraged strategy is the possibility of an increase in spot rates.



### MODULE QUIZ 25.2

1. If the future spot rates are expected to be lower than the current forward rates for the same maturities, bonds are *most likely* to be:
  - A. overvalued.
  - B. undervalued.
  - C. correctly valued.
2. The strategy of rolling down the yield curve is *most likely* to produce superior returns for a fixed income portfolio manager investing in bonds with maturity higher than the manager's investment horizon when the spot rate curve:
  - A. is downward sloping.
  - B. in the future matches that projected by today's forward curves.
  - C. is upward sloping.

## MODULE 25.3: THE SWAP RATE CURVE



Video covering this content is available online.

**LOS 25.e: Explain the swap rate curve and why and how market participants use it in valuation.**

### THE SWAP RATE CURVE

In a plain vanilla interest rate swap, one party makes payments based on a fixed rate while the counterparty makes payments based on a floating rate. The fixed rate in an interest rate swap is called the **swap fixed rate** or **swap rate**.

If we consider how swap rates vary for various maturities, we get the **swap rate curve**, which has become an important interest-rate benchmark for credit markets.

Market participants prefer the swap rate curve as a benchmark interest rate curve rather than a government bond yield curve for the following reasons:

- Swap rates reflect the credit risk of commercial banks rather than the credit risk of governments.
- The swap market is not regulated by any government, which makes swap rates in different countries more comparable. (Government bond yield curves additionally reflect sovereign risk unique to each country.)
- The swap curve typically has yield quotes at many maturities, while the U.S. government bond yield curve has on-the-run issues trading at only a small number of maturities.

Wholesale banks that manage interest rate risk with swap contracts are more likely to use swap curves to value their assets and liabilities. Retail banks, on the other hand, are more likely to use a government bond yield curve.

Given a notional principal of \$1 and a swap fixed rate  $SFR_T$ , the value of the fixed rate payments on a swap can be computed using the relevant (e.g., *MRR*) spot rate curve. For a given swap tenor  $T$ , we can solve for  $SFR$  in the following equation.

$$\sum_{t=1}^T \frac{SFR_T}{(1+S_t)^t} + \frac{1}{(1+S_T)^T} = 1$$

In the equation,  $SFR$  can be thought of as the coupon rate of a \$1 par value bond given the underlying spot rate curve.



#### PROFESSOR'S NOTE

Prior to the 2023 curriculum, CFA Institute referenced LIBOR (the “London Interbank Offered Rate”) as the underlying interest rate for derivative instruments. However, concerns about LIBOR being manipulated have led to LIBOR being replaced in recent years with other reference rates, such as the Federal Reserve Bank of New York’s “SOFR” (Secured Overnight Financing Rate), and the Bank of England’s “SONIA” (Sterling Overnight Index Average).

To reflect this change, the CFA curriculum now uses the generic term MRR (“market reference rate”) to refer to these various LIBOR replacements.

#### EXAMPLE: Swap rate curve

Given the following MRR spot rate curve, compute the swap fixed rate for a tenor of 1, 2, and 3 years (i.e., compute the swap rate curve).

Maturity	Spot Rate
1	3.00%
2	4.00%
3	5.00%

#### Answer:

1.  $SFR_1$  can be computed using the equation: