UpperMark™ Study Handbook

CAIA® Level II

Volume 3

Topic 7: Volatility and Complex Strategies

Topic 8: Universal Investment Considerations

Topic 9: Emerging Topics

UpperMark™

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It is recommended that candidates use any test preparation product together with the original reading materials suggested in the CAIA Study Guide.[1]

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Preface

Volume 3 of the UpperMark™ *Study Handbooks* provides a comprehensive and concise account of each learning objective (L.O.) in Topics 7-9 of the CAIA Level II Study Guide. The *Study Handbook* is compiled using the reference materials recommended by the CAIA Association and, as in Volume 1, is organized as follows.

- Each Chapter in the Study Guide is presented as a separate chapter, keywords are indicated in ***bold italics***, and learning objective sub-bullets are indicated by underlined, capitalized subheadings (e.g., ROLE OF INVESTMENT OBJECTIVES AND CONSTRAINTS).
- The list of keywords and learning objectives from the Study Guide is provided at the end of each chapter.
- Space is provided at the end of each chapter for you to record your Personal Study Notes.
- A set of practice exam questions (multiple-choice and constructed-response) is provided at the end of each chapter. A considerably larger set of practice questions (multiple-choice and constructed-response questions, and constructed-response question sets ["Essays"]) is in our TestBank software.

Supplementary information is included in footnotes.

The CAIA Equation Exception List is not included in this Volume, since no equations on that list are from Topics 7-9.

We wish you the best with your exam preparation.

Padideh Jalali, Ph.D., CAIA
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Table of Contents

Topic 7

Volatility and Complex Strategies

Topic 7 is composed of four Readings on volatility and complex strategies.

1. Reading 7.1 discusses volatility as a factor exposure, volatility models, and implied volatility structures.
2. Reading 7.2 describes volatility, correlation, and dispersion products and strategies.
3. Reading 7.3 discusses the concept of complexity in investment products and describes several structured products.
4. Reading 7.4 reviews international real estate investing and its key risks and challenges.

Reading 7.1

Volatility as a Factor Exposure

This Reading discusses measures of volatility and describes different volatility exposures, option Greeks (e.g., vega), modelling volatility processes, and implied volatility structures.

Learning Objectives

7.1.1 Demonstrate knowledge of measures of volatility.

i. Understand differences between implied volatility and realized volatility
ii. Identify limitations of realized volatility as a measure of dispersion
iii. State the properties of realized volatility

Keywords

1. Implied return volatility
2. Realized return volatility

7.1.2 Demonstrate knowledge of volatility and the vegas, gammas, and thetas of options.

i. Describe option vegas
ii. Interpret the scaling of the vega of an option
iii. Interpret and apply vega as an option for finite shifts
iv. Understand how vega shifts as underlying variables change
v. Interpret option gammas
vi. Understand the interrelationships between option vegas, gammas, and thetas

7.1.3 Demonstrate knowledge of exposures to volatility as a factor.

i. Contrast long volatility with short volatility
ii. Understand distinctions between positive vega and long volatility exposures
iii. Explain how volatility can be used to hedge risk
iv. Understand volatility as an unobservable but unique risk factor
v. Understand how long volatility carries a negative risk premium
vi. Explain how short volatility earns a positive risk premiums

Keywords

1. Long volatility
2. Negative volatility risk premium
3. Short volatility
4. Volatility derivatives

7.1.4 Demonstrate knowledge of modeling volatility processes

i. Understand volatility processes with jump risk
ii. Construct volatility processes and regime changes
iii. Discuss reasons why volatility strategies recover
iv. Identify reasons why volatility mean reversion cannot be arbitraged

Keywords

1. Mixture model or a regime switching model
2. Regime change
3. Volatility clustering
4. Volatility diffusion risk
5. Volatility jump risk
6. Volatility risk

7.1.5 Demonstrate knowledge of implied volatility structures

i. Describe methods of computing implied volatility
ii. Identify structures regarding implied volatility and moneyness
iii. Identify an implied volatility surface
iv. Explain key reasons for implied volatility structures and surfaces
v. Discuss reasons for high implied volatility and out-of-the-money puts

Keywords

1. Implied volatility structure
2. Options volatility surface
3. Smile or a smirk
4. Volatility skew

L.O. 7.1.1 **DEMONSTRATE KNOWLEDGE OF MEASURES OF VOLATILITY.**

This L.O. describes implied and realized volatilities.

DIFFERENCES BETWEEN IMPLIED VOLATILITY AND REALIZED VOLATILITY

Implied return volatility is the volatility inferred from an option price under assumptions including risk-neutrality, the validity of an option pricing model, and the accuracy of the model's inputs (other than volatility). Implied volatility (IV) is typically estimated using market prices. The option pricing model used makes assumptions about the underlying asset's return process. For instance, in the Black-Scholes option pricing model, the option's underlying asset is assumed to follow a geometric Brownian motion (GBM) in which the asset's instantaneous returns have the following traits.

1. Constant variance over time (i.e., are homoscedastic)
2. Normally distributed
3. Uncorrelated over time

Realized return volatility is the actual variation in a given time period using a specific return measurement interval (e.g., weekly return granularity). It is typically measured as the standard deviation of returns. In practice, observed returns are discrete; they are not continuous and do not have the properties of a GBM process.

Since options are exposed to changes in volatility, IVs should differ from expected realized volatilities to reflect volatility-related risk premiums. Therefore, IV does not represent an accurate, unbiased consensus of realized volatility expected by market participants due to risk premiums.

LIMITATIONS OF REALIZED VOLATILITY AS A MEASURE OF DISPERSION

Realized volatility is not a complete description of dispersion; it is an estimate that describes the dispersion of a frequency return distribution of a sample. Realized volatility has three limitations.

1. It does not describe the shape of the return distribution.
2. Assets with identical realized volatilities may have markedly different underlying returns; i.e., they may be trending, mean reverting, or minimally autocorrelated.
3. It does not describe whether most of the dispersion occurred near a particular price of the underlying asset or during a particular time period in the sample.

For instance, in a set of assets with the same realized return volatility, one asset may have mostly moderate returns, another may have one extremely large return and many small returns, another may trend, and another may mean-revert. Despite having the same realized volatility, these assets will not have the same results for most volatility strategies. In addition, the granularity of the return interval can substantially alter its return characteristics.

PROPERTIES OF REALIZED VOLATILITY

Sinclair (2013) presents six observations regarding realized volatility, many of which are assumptions of volatility arbitrage strategies and risk management techniques.

1. Realized volatility slowly mean-reverts (thus is not constant) and clusters.
   - Thus, volatility is often modeled using generalized autoregressive conditional heteroscedasticity (GARCH) and regime switching models.
2. Realized volatility is typically low for a period of time until a market shock occurs, and then it increases and remains at the higher level for a period of time.
3. Short-term realized volatility can be high, but, in the long run, reverts to a long-term mean.
4. High volatility increases investors' risk aversion, which indicates that high volatility is generally negatively correlated with risky asset returns.
5. Equity market realized volatility is negatively correlated with stock prices (i.e., rises in bear markets; declines in bull markets).
6. Realized equity volatility increases faster in bear markets than in bull markets.

L.O. 7.1.2 **DEMONSTRATE KNOWLEDGE OF VOLATILITY AND THE VEGAS, GAMMAS, AND THETAS OF OPTIONS.**

This L.O. discusses vega, gamma, and theta; which reflect sensitivities of assets to changes in underlying factors.

OPTION VEGA

Vega indicates the sensitivity of an option value to a change in the volatility of the option's underlying asset. Some volatility strategies involve positions with predetermined levels of sensitivity to changes in implied volatility (i.e., with target exposures to vega), while minimizing exposure to changes in the underlying asset value (i.e., minimizing delta). Some traders may aim to earn a volatility risk premium using short vega positions.

The vega of a call or put option on an asset is the partial derivative of the option value with respect to its volatility (i.e., rate of change in the option value with respect to an infinitesimal shift in volatility). From the Black-Scholes option pricing model, the vega, v, of a call or put on an asset, S, may be expressed as:

$$v = \frac{\partial p}{\partial \sigma} = SN'(d)\sqrt{T}, \tag{1}$$

where p is the option value, σ is the underlying asset's volatility, $N'(d)$ is the (non-cumulative) probability density function of the normal distribution at d, and T is the option's time to expiration or tenor.

SCALING THE VEGA OF AN OPTION

In practice, vega is commonly scaled to reflect the risk of a one basis point change in volatility. As such, it is expressed as vega per basis point: $\frac{v}{100} = \frac{SN'(d)\sqrt{T}}{100}$.

- Vega in Equation (1) measures an option value's instantaneous rate of response to a change in one percentage point in volatility; e.g., a change from 20% to 21% (i.e., 0.20 to 0.21).
- Vega per basis point measures an option value's instantaneous rate of response to a change in one basis point in volatility; e.g. a change from 20% to 20.01% (i.e., 0.20 to 0.2001).

Practitioners and financial data providers typically report vega scaled per basis point.

Example 1

A put option on a non-dividend-paying stock valued at $50 has 0.25 years to expiration. If $N'(d)$ is 0.2, what is the vega and vega per basis point of this option?

Vega: $v = \frac{\partial p}{\partial \sigma} = SN'(d)\sqrt{T} = \$50 \times 0.2 \times \sqrt{0.25} = \$5$.

Vega per basis point = Vega/100 = $5/100 = $0.05.

The vega per basis point may be interpreted as the option value rising towards an increase of $0.05 as the option's implied volatility rises towards an increase of 0.01% (i.e., 0.0001).

The interpretation is not expressed as the option value increasing by $0.05 because the relationship between option value and volatility is non-linear and vega is only a precise measure when volatility makes infinitesimal shifts. For large shifts in volatility, higher-order derivatives are needed to determine accurate changes in option values.

VEGA AS AN APPROXIMATION FOR FINITE SHIFTS

The linear (first-order) approximate change in option value may be expressed as:

$$\Delta p \approx v\Delta\sigma.$$

Example 2

A call option on a non-dividend-paying stock has a vega per basis point of $0.005. What is the first-order approximate change in the call value for a decline in volatility from 0.27 to 0.24?

$\Delta p \approx v\Delta\sigma = \$0.005\Delta\sigma$

$\Delta\sigma$ (in basis points) = 0.27 – 0.24 = –0.03 = –3% = –300 basis points[1]

$\Rightarrow \Delta p \approx \$0.005\Delta\sigma = \$0.005 \times -300 = -\$1.50$

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[1] 1 basis point = (1/100) of 1% [or 0.01%]; therefore, 1% = 100 bps.

HOW VEGA SHIFTS AS UNDERLYING VARIABLES CHANGE

Four observations may be made about vega given by Equation (1): $v = SN'(d)\sqrt{T}$.

1. Vega is always positive for a long call or put option.
   - This is because all three terms on the right side of Equation (1) are positive.
2. Vega is the same for a call and a put with the same underlying asset, strike price, time to expiration, and implied volatility are equal.
   - In put-call parity (i.e., Call – Put = Stock – Bond), IV does not appear on the right side of the equation. This indicates that the effects of IV on the left side (i.e., "Call – Put") must cancel out and, to do so, they must be equal. Thus, "Stock – Bond" on the right side may be considered a financed long stock position with a vega of zero and "Call – Put" has a net vega of zero.
3. Vega of an option approaches zero as the option's time to expiration approaches zero.
   - This is clear in Equation (1): as T approaches zero, v approaches zero.
4. Vega of an option approaches zero as the underlying asset value approaches zero or infinity.
   - Therefore, vega approaches zero for deep out-of-the-money (OTM) and in-the-money (ITM) options because $N'(d)$ in Equation (1) approaches zero when S is very small or very large (i.e., towards either tail of the normal distribution).

Changes in option value with respect to IVs and/or tenor are illustrated in Figure 1, where the differences in the call values are primarily driven by the quantity $\sigma\sqrt{T}$. The three curves represent different IVs and/or different tenors. An up or down shift in volatility moves the option value curve higher or lower. As time passes, all else equal, option values move down to lower curves.

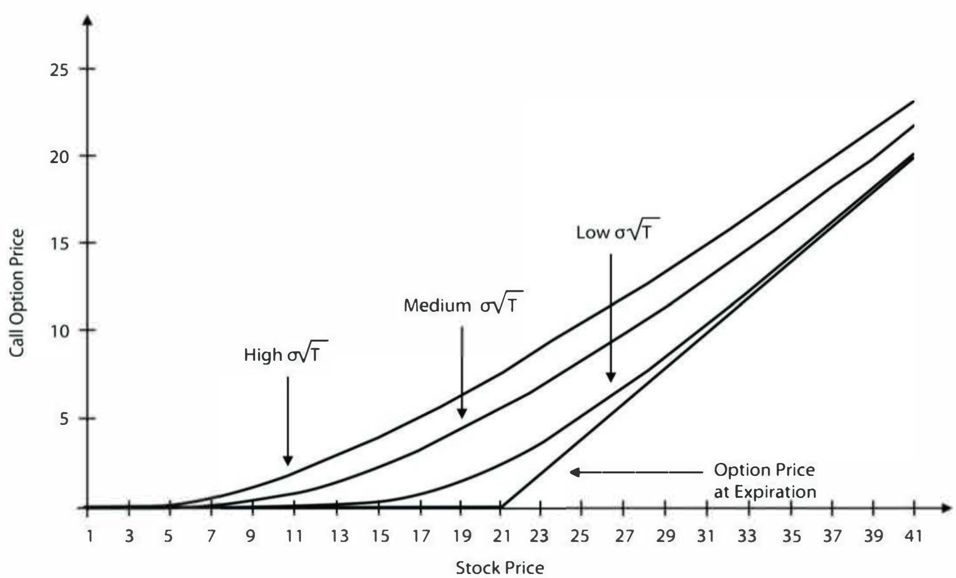

Figure 1: Call Option Values at Three Levels of Implied Volatility and/or Tenor

OPTION GAMMA

An option's gamma is the second-order partial derivative of its value with respect to the underlying asset value. Gamma may also be expressed as the first-order partial derivative of an option's delta with respect to the underlying asset value (since delta is the first-order partial derivative of an option's value with respect to its underlying asset value).

From the Black-Scholes model, the gamma of an option, γ, may be expressed as:

$$\gamma = \frac{N'(d)}{S\sigma\sqrt{T}} = \frac{v}{\sigma S^2 T}.$$

- Gamma is the same for a call and a put with the same underlying asset, strike price, and tenor.
- The terms in the gamma formula are all non-negative; therefore, gamma and vega are positive for simple long call or put options.
- Long option positions have long volatility and positive gamma, and short option positions are short volatility and have negative gamma.
- Graphically, an option's gamma depicts the degree of curvature in the relationship between option price and the underlying asset price. In Figure 1, the lowest curve has relatively sharp curvature when the option is near-the-money, which corresponds to a high value of gamma. The highest curve has less sharp curvature, corresponding to a low (but positive) gamma.
  - Gamma is highest for near-the-money options and smallest (or zero) for ITM or OTM options.

INTERRELATIONSHIPS BETWEEN OPTION GAMMAS, VEGAS, AND THETAS

Gamma provides long gamma call option holders with the desirable combination of increasing rates of gain as the option's underlying asset moves up and decreasing rates of loss as the underlying asset moves down. Holders of puts experience the desirable combination of increasing rates of gain as the underlying asset moves down and decreasing rates of loss as the underlying asset moves up.

- A portfolio with large positive gamma benefits greatly from large directional moves in the underlying asset in one direction but suffers relatively smaller losses from directional moves in the other direction.
- If options are competitively priced, there is no "free lunch". As such, positive gamma portfolios (that tend to be long options) have negative theta, which means that the option value declines as time passes, all else equal (i.e., changes in the underlying asset price and volatility are minimal).

  Vega reflects the sensitivity of options to volatility that is driven by gamma.
- Vega is small for options with near-zero gamma.
- An option's positive gamma makes anticipated higher volatility result in higher option values.

Therefore, gamma (which comes from asymmetric payoffs) is the primary reason that options sell for a premium above their intrinsic value (i.e., value at expiration). This positive time value to a call option is due to the call's virtually unlimited gain potential and relatively limited losses. It will decay over time (measured by option's theta) if anticipated volatility does not occur.

L.O. 7.1.3 **DEMONSTRATE KNOWLEDGE OF EXPOSURES TO VOLATILITY AS A FACTOR.**

This L.O. discusses the sign of an option's vega and whether the option is long or short volatility.

LONG VOLATILITY VS. SHORT VOLATILITY

The notion of an investment (e.g., option) being long or short volatility refers to the investment's empirically-observed correlation with volatility.

- When an investment's returns are negatively correlated with the volatility level of market returns, the position is ***short volatility*** (or short vol); i.e., short the volatility of the market.
- When an investment's returns are positively correlated with the volatility level of market returns, the position is ***long volatility*** (or long vol).

However, long option positions are not always long volatility.

- Consider that long at-the-money (ATM) equity options are generally long volatility with respect to the underlying assets' volatility, and short ATM options are short volatility. However, short-dated deeply in-the-money (DITM) call options on an equity index behave like the underlying index and, since an equity index has negative correlation with equity market volatility, a long position in the index is short volatility. Therefore, long DITM call options on an equity index are short volatility (similar to the index). The reason for this is that the effect of declining equity market prices on the value of a DITM equity call option (via its positive delta) generally dominates the value-increasing effect of the increased volatility on the value of the DITM option (via its positive vega).
- Long equity options are always long equity-market volatility (and short options are always short volatility) in delta-neutral portfolios, where the effects of directional moves in the equity market have been hedged.

Numerous securities or derivatives have been developed to be either long or short volatility. Thus, portfolios can be long or short volatility based on their direct option exposures and through exposures to products engineered to have volatility exposure.

POSITIVE VEGA VS. LONG VOLATILITY EXPOSURES

The concepts of positive vega and long volatility are related in that they both describe the value of an asset increasing when volatility increases. However, there are distinctions between the concepts. These are discussed below.

As the partial derivative of an option with respect to the implied volatility of the option's underlying asset, vega represents the option's response to changes in volatility, holding all other variables constant (e.g., value of the option's underlying asset).

- The vega of an asset is positive when the partial derivative of the asset price with respect to implied volatility is positive.
- The vega of a long simple option is positive and of a short simple option is negative.
- The vega of a simple long or short traditional asset position (e.g., in an equity index) is zero.