

**2025 CAIA<sup>®</sup>**  
Exam Prep

# SchweserNotes<sup>™</sup>

Methods and Models, Accessing Alternative  
Investments, and Due Diligence

**Level II** Book 2

**KAPLAN**  **SCHWESER**

# Book 2: Methods and Models, Accessing Alternative Investments, and Due Diligence

## **SchweserNotes™ 2025**

CAIA Level II



SCHWESERNOTES™ 2025 CAIA® LEVEL II BOOK 2: METHODS AND MODELS, ACCESSING ALTERNATIVE INVESTMENTS, AND DUE DILIGENCE

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# Readings and Learning Objectives

## TOPIC 4

CAIA Association. *CAIA Curriculum Level II Volume II*. Self-published, CAIA Association, 2024.

### Reading 4.1: Modeling Overview and Fixed Income Models

4.1.1: Demonstrate knowledge of underlying models of investment strategy.

Including:

- Compare normative strategies with positive strategies.
- Distinguish between theoretical and empirical models.
- Distinguish between applied versus abstract models.
- Compare cross-sectional versus time-series models.
- Discuss the importance of methodology in model building.

4.1.2: Demonstrate knowledge of equilibrium models and arbitrage-free models of the term structure.

Including:

- Contrast equilibrium fixed-income models with arbitrage-free models.
- Apply Vasicek's model.
- Contrast the Cox, Ingersoll, and Ross (CIR) model with Vasicek's model.
- Discuss the Ho and Lee model.

4.1.3: Demonstrate knowledge of the Black-Derman-Toy (BDT) model.

Including:

- Interpret a binomial BDT tree.
- Understand how to calibrate the level of rates based on average returns.
- Understand how to calibrate the spread of rates based on volatilities.
- Discuss BDT calibrations in general.

4.1.4: Demonstrate knowledge of credit risk and credit risk modeling.

Including:

- Distinguish types of credit events that may lead to an increase in credit risk.
- Explain exposure at default (EAD) and loss given default (LGD).
- Describe how adverse selection and moral hazard relate to credit risk.
- Discuss how probability of default (PD) and recovery rate (RR) affect credit risk.
- Calculate loss given default and expected loss from credit risk.
- Describe the basic concepts of credit risk modeling.
- Contrast the three approaches to credit risk modeling.

4.1.5: Demonstrate knowledge of the structural model approach through the lens of the Merton model.

Including:

- Describe Merton's structural model using the option-like nature (both call options and put options) of traditional corporate securities.
- Describe the inherent conflict of interest that exists between shareholders and bondholders.
- Evaluate advantages and disadvantages of the Merton model.
- Understand how binomial trees can be used to value structured products.

4.1.6: Demonstrate knowledge of the Merton model.

Including:

- Apply the Merton model to determine equity values and payoffs to bondholders for a given investment.
- Calculate the value of risky debt using the Black Scholes option pricing model in the Merton

model.

- Evaluate the use of Black-Scholes option pricing in the Merton model.
- Analyze the role of credit spreads in structural models and how the credit spread can be used to calculate the price of risky debt.
- Understand the four important properties of the Merton model.

4.1.7: Demonstrate knowledge of the Kealhofer, McQuown, and Vasicek (KMV) credit risk model.

Including:

- Describe the characteristics and application of the KMV model.
- Evaluate the credit score (the distance to default) for a given firm using the KMV model.
- Evaluate the expected default frequency for a given investment using the KMV model.

4.1.8: Demonstrate knowledge of reduced-form models.

Including:

- Describe the characteristics of reduced-form models.
- Discuss the role of default intensity in reduced-form models and calculate default intensity for a given firm.
- Demonstrate how default intensity can be incorporated into the valuation of risky debt.
- Analyze the relationship among credit spreads, default intensities, and recovery rates, and use two of these factors as variables to solve for the third for a given investment.
- Describe the two predominant reduced-form credit models.

4.1.9: Demonstrate knowledge of empirical credit models.

Including:

- Contrast empirical credit models with structural and reduced-form models.
- Describe the purpose and characteristics of the Altman Z-score model.
- Understand the five financial ratios that are used as inputs to determine Altman Z-scores.
- Evaluate Z-scores in Altman's credit scoring model.

## **Reading 4.2: Valuation and Hedging Using Binomial Trees**

4.2.1: Demonstrate knowledge of one-period binomial trees and risk-neutral modeling.

Including:

- Create a one-period binomial model of default risk with risk neutrality.
- Understand the modeling of a default risk premium.
- Apply p-measures and q-measures in risk-neutral modeling.
- Identify four key components of risk-neutral modeling.

4.2.2: Demonstrate knowledge of multi-period binomial trees, values, and mean rates.

Including:

- Construct a one-period trinomial tree model based on prices.
- Construct a two-period binomial tree model with compounded returns.
- Identify three fallacies generated by averaging compounded rates of return.
- Understand the advantages to using computer programming to model.

4.2.3: Demonstrate knowledge of forming a tree of stock prices with a binomial tree model.

Including:

- Understand the formation of a binomial tree of stock prices.

4.2.4: Demonstrate knowledge of valuation of convertible securities with a binomial tree model.

Including:

- Apply a binomial tree of prices to determine the value of options on equity.
- Create a tree of prices for a convertible bond's underlying stock.
- Interpret a tree of prices for the convertible bond's underlying stock.
- Understand how to value a convertible bond one period prior to its maturity.
- Determine, through backward induction, the current value of a convertible bond.

4.2.5: Demonstrate knowledge of valuing callable bonds with a tree model.

Including:

- Describe a two-period binomial interest rate tree.
- Understand how to model the spread between upward and downward shifting rates.
- Calculate the price of a straight bond using a two-period binomial tree.
- Calculate the price of a callable bond using a two-period binomial tree.

### **Reading 4.3: Multifactor Equity Pricing Models**

4.3.1: Demonstrate knowledge of multifactor asset pricing models.

Including:

- Explain multifactor asset pricing.
- Understand the role of marginal investor utility in the CAPM and how it relates to asset factors.
- Explain how multiple factors relate to “bad times.”
- Discuss factors based on expected utility or anomalies.
- Identify the three major categories of factors.
- Contrast theoretically versus empirically derived multifactor return models.
- Identify the fundamentals of empirical models.
- Discuss the tradability of factors and the intercepts.

4.3.2: Demonstrate knowledge of the original Fama-French model.

Including:

- Contrast the three Fama-French models.
- Calculate the original Fama-French model and the Fama-French-Carhart models.

4.3.3: Demonstrate knowledge of the three challenges of empirical multifactor models.

Including:

- Understand how factors can be falsely identified.
- Contrast factor correlation with factor causation.
- Explain why the CAPM may not be sufficient.

4.3.4: Demonstrate knowledge of factor investing.

Including:

- Discuss the emergence of return factor analysis.
- Identify how return factors are described.
- Explain how risk premiums vary across return factors.
- Explain how factor returns vary across market conditions.
- Explain the relationship between return factors and investability.
- Interpret risk allocation based on return factors.
- Understand performance with allocations based on return factors.

4.3.5: Demonstrate knowledge of the adaptive markets hypothesis (AMH) and time-varying volatility.

Including:

- Evaluate the practical implication of the Adaptive Markets Hypothesis.
- Explain how equity market volatility is predictable.
- Explain how volatility is negatively correlated with average returns.
- Discuss time-varying volatility in the context of multiple factors as well as higher moments.

4.3.6: Demonstrate knowledge of stochastic discount factors.

Including:

- Calculate traditional discount factors.
- Interpret stochastic discount factors.
- Calculate the present value of a cash flow in one period based on stochastic discount factors.
- Discuss the importance of stochastic discount factors.

### **Reading 4.4: Directional Strategies and Methods**

4.4.1: Demonstrate knowledge of efficiently inefficient markets.

Including:

- Define an efficient inefficient market and identify how and why it exists.
- Understand the two paradoxes of informational market efficiency.

4.4.2: Demonstrate knowledge of trend following technical directional strategies.

Including:

- Identify the metrics of technical analysis.
- Define the various trendsetting or momentum models.

4.4.3: Demonstrate knowledge of divergence technical directional strategies.

Including:

- Understand market divergence.
- Interpret the signal-to-noise ratio.
- Define market divergence and calculate the signal-to-noise ratio.
- Understand and calculate the market divergence index.
- Identify technical strategies based on machine learning.
- Interpret the risks of directional technical strategies.

4.4.4: Demonstrate knowledge of fundamental directional strategies.

Including:

- Define fundamental directional strategies.
- Understand the bottom-up approach of fundamental analysis.
- Identify four procedures within the fundamental investment process.
- Identify four mechanics of fundamental strategies.
- Understand the top-down approach of fundamental analysis.
- Describe schools of thought within top-down fundamental analysis.
- Discuss risks of directional fundamental strategies.

4.4.5: Demonstrate knowledge of directional strategies and behavioral finance.

Including:

- Describe the six sentiment indicators.
- Describe overconfidence and its role in finance.
- Explain behavioral biases from over-reliance on the past.
- Identify other potential sources of pricing anomalies.

4.4.6: Demonstrate knowledge of factors in directional trading.

Including:

- Contrast investment style classifications such as value and growth.
- Define directional trading based on momentum.
- Discuss emphasis on illiquidity premiums.

## **Reading 4.5: Multivariate Empirical Methods and Performance Persistence**

4.5.1: Demonstrate knowledge of statistical factors in principal component analysis.

Including:

- Define principal component analysis and its factors.
- Understand the basics of principal component analysis.
- Identify two primary outputs of principal component analysis.
- Interpret examples of applying and interpreting principal component analysis.
- Contrast principal component analysis and factor analysis.

4.5.2: Demonstrate knowledge of multifactor models and regression.

Including:

- Interpret the multifactor regression model such as the Fama-French model.
- Understand the two primary adverse effects of multicollinearity.
- Explain the selection of the number of factors and overfitting of a regression model.

4.5.3: Demonstrate knowledge of partial autocorrelations and regression.

Including:

- Understand return autocorrelation and partial autocorrelation.
- Estimate partial autocorrelation.
- Interpret partial autocorrelations of a return series based on appraisals.

4.5.4: Demonstrate knowledge of dynamic risk exposure models.

Including:

- Understand positions with nonlinear exposures.
- Understand the dummy variable approach to dynamic risk exposures.
- Define the separate regression approach to dynamic risk exposures.
- Describe the use of a quadratic model to explain market timing performance.

4.5.5: Demonstrate knowledge of approaches to modeling changing correlation.

Including:

- Define the conditional correlation modeling approach.
- Discuss examples of conditional correlations.
- Interpret variations on conditional empirical analyses.
- Describe and apply the rolling window modeling approach.

4.5.6: Demonstrate knowledge of multifactor approaches to understanding returns.

Including:

- Understand style analysis and fund groupings based on asset classes.
- Identify funds based on strategies.
- Describe funds based on market-wide factors.
- Understand funds based on specialized market factors.

4.5.7: Demonstrate knowledge of evidence on fund performance persistence.

Including:

- Understand performance persistence based on return correlations.
- Understand performance persistence based on risk-adjusted returns.
- Understand performance persistence based on portfolio returns.

## **Reading 4.6: Relative Value Methods**

4.6.1: Demonstrate knowledge of relative value methods.

Including:

- Understand the importance of market inefficiencies with respect to relative value strategies.
- Contrast pure arbitrage with risk arbitrage.
- Identify the limits to arbitrage.
- Interpret examples of nearly pure arbitrage.
- Discuss examples illustrating risk arbitrage opportunities.
- Identify the steps of pairs trading and types of pairs trading.

4.6.2: Demonstrate knowledge of statistical pairs trading of equities.

Including:

- Calculate statistical pairing with the co-integration approach.
- Understand the timing of trade entry opportunities.
- Define the nature and performance of pairs trading strategies.

4.6.3: Demonstrate knowledge of pairs trading in commodity markets based on spreads.

Including:

- Identify different commodity derivatives calendar spreads.
- State the three dimensions of commodity relative value strategies.
- Estimate the profitability of calendar spread trading.
- Understand processing spreads.
- Understand the two conditions that hold for producers that are hedgers.
- Evaluate substitution spreads.
- Describe quality spreads and location spreads.
- Interpret intramarket relative value strategies.

4.6.4: Demonstrate knowledge of pairs trading in rates from fixed income and currency markets.

Including:

- Understand and apply the concept of a carry trade such as covered interest rate parity.

4.6.5: Demonstrate knowledge of relative value market-neutral strategies and portfolio risks.

Including:

- Identify different risks of pairs trading strategies.
- Describe equity market-neutral strategies.
- Describe risks related to equity market neutrality.

## **Reading 4.7: Valuation Methods for Private Assets: The Case of Real Estate**

4.7.1: Demonstrate knowledge of depreciation tax shields.

Including:

- Understand the depreciation tax advantage.
- Calculate the present value of depreciation tax shields.
- Describe depreciation as generating an interest free loan.

4.7.2: Demonstrate knowledge of deferral of taxation of gains.

Including:

- Calculate after-tax return without tax deferral.
- Calculate after-tax returns with the tax deferral of gains.
- Understand the income tax benefits of leveraged real estate.

4.7.3: Demonstrate knowledge of how to compare after-tax returns for various taxation scenarios.

Including:

- Contrast stated tax rate and effective tax rate.
- Interpret real estate without taxation.
- Interpret after-tax returns when depreciation is not allowed.
- Identify the four principles of depreciation and returns.
- Calculate returns when accounting depreciation equals economic depreciation.
- Calculate returns when accounting depreciation is accelerated.
- Calculate returns when capital expenditures can be immediately and fully expensed.
- Understand the relationship between an investor's tax bracket and tax advantaged investments.

4.7.4: Demonstrate knowledge of various transaction-based indices and their biases.

Including:

- Apply the repeat-sales method.
- Identify advantages of the repeat-sales method.
- Identify disadvantages of the repeat-sales method.
- Analyze the hedonic pricing method.
- Identify steps in calculating a hedonic price index.
- Apply the hedonic pricing approach.
- Identify primary advantages of the hedonic pricing model.
- Identify primary disadvantages of the hedonic pricing model.
- Contrast the various indices and biases.

4.7.5: Demonstrate knowledge of appraisal-based indices.

Including:

- Contrast the various approaches to appraisals.
- Identify advantages of appraisal-based models.
- Identify disadvantages of appraisal-based models.
- Distinguish between the types of pricing errors.
- Understand the square root of N rule.

## **TOPIC 5**

2024.

## Reading 5.1: Hedge Fund Replication

5.1.1: Demonstrate knowledge of replication products and their potential benefits.

Including:

- Understand basics of hedge fund replication products.
- Evaluate the potential benefits to investors of using replication products.

5.1.2: Demonstrate knowledge of the case for using hedge fund replication.

Including:

- Estimate the risk and return of a given fund of hedge funds.
- Describe three theories for the increased beta and decreased alpha in hedge fund returns.
- Contrast the level of alpha that is generated by the aggregate of hedge fund managers with the alpha available to investors who select individual managers.
- Discuss how replication products can serve as a source of alpha or alternative beta.

5.1.3: Demonstrate knowledge of the benefits of replication products.

Including:

- Identify two reasons to use replication products.
- Evaluate issues regarding the benefits of fund replication.
- Understand potential unique benefits from hedge fund replication.

5.1.4: Demonstrate knowledge of factor-based approaches to replication.

Including:

- Identify primary issues in constructing a factor-based replication product.
- Identify the steps involved in factor-based replication.
- Understand current research on factor-based replication.
- Describe the payoff-distribution approach to factor replication.

5.1.5: Demonstrate knowledge of the algorithmic (bottom-up) approach.

Including:

- Understand the basics of the algorithmic (or bottom-up) approach.
- Describe the algorithmic approach to merger arbitrage factor replication.
- Describe the algorithmic approach to convertible arbitrage factor replication.
- Describe the algorithmic approach to momentum factor replication.

## Reading 5.2: Diversified Access to Hedge Funds

5.2.1: Demonstrate knowledge of evidence regarding hedge fund risk and returns.

Including:

- Interpret evidence regarding performance of hedge funds by strategies.
- Interpret evidence regarding the systematic and total risk of hedge funds.
- Interpret evidence regarding correlations and diversification of hedge funds.

5.2.2: Demonstrate knowledge of the approaches used by investors to gain hedge fund exposure.

Including:

- Discuss the advantages and disadvantages of the direct approach to obtaining hedge fund exposure in portfolios.
- Describe the five services provided as part of the delegated approach to obtaining hedge fund exposure in portfolios.
- Describe the index approach to obtaining hedge fund exposure in portfolios.

5.2.3: Demonstrate knowledge of the characteristics of funds of hedge funds.

Including:

- Understand the approach to manager selection of funds of hedge funds.
- Identify ways that funds of hedge funds can be grouped or categorized.
- Understand how typical hedge fund biases can be reduced when applied to funds of hedge funds.
- Contrast funds of hedge funds with multistrategy funds.

5.2.4: Demonstrate knowledge of approaches to fund of hedge funds portfolio construction.

Including:

- Distinguish between the six different approaches to constructing a fund of hedge funds portfolio.

5.2.5: Demonstrate knowledge of how funds of hedge funds add value for investors.

Including:

- Discuss three approaches used by funds of hedge funds managers to add value for their investors (i.e., through strategic allocation, through tactical allocation, and through fund selection).
- Analyze evidence regarding value added using these approaches by fund of hedge fund managers.

5.2.6: Demonstrate knowledge of hedge fund indices and alternative mutual funds.

Including:

- Identify factors contributing to the development of hedge fund indices and arguments presented against hedge fund index investing.
- Describe the desirable characteristics of investment indices and the challenges of creating representative, investable hedge funds indices.
- Discuss investable hedge fund indices.
- Describe the three potential benefits of offering alternative mutual funds.
- Describe the three benefits of alternative mutual funds to investors.
- Describe the three risks of alternative mutual funds.
- Describe the three advantages of exchange-traded alternative funds.

### **Reading 5.3: Access to Real Estate and Commodities**

5.3.1: Demonstrate knowledge of issues in private and listed investment access.

Including:

- Define financial market segmentation.
- Identify potential advantages of listed assets.
- Identify potential advantages of privately organized assets.
- Understand the relative amount of fees charged on investments.
- Describe the role that governance plays in the creation of wealth through private equity.

5.3.2: Demonstrate knowledge of unlisted real estate funds.

Including:

- Understand the role and purpose of open-end real estate funds.
- Understand the role and purpose of closed-end real estate funds.
- Describe real estate funds of funds.
- Justify the role of non-traded REITs.
- Understand the potential advantages of unlisted real estate funds.

5.3.3: Demonstrate knowledge of the return drivers of private equity real estate funds.

Including:

- Discuss how the return to private equity real estate funds varies by international exposure, GDP growth, vintage year, credit spreads, and public real estate returns.
- Discuss how diversification across vintage years impacts the risk of a portfolio of private real estate funds.

5.3.4: Demonstrate knowledge of listed real estate funds.

Including:

- Contrast REITs with REOCs.
- Interpret exchange-traded funds based on real estate indices.
- Identify potential advantages of listed real estate funds.
- Identify potential disadvantages of listed real estate funds.
- Understand the role and accessibility of global REITs.

5.3.5: Demonstrate knowledge of ownership methods of commodities.

Including:

- Describe the concept and process of direct physical ownership of commodities.
- Describe the concept and process of indirect ownership of commodities.
- Interpret commodity index swaps.
- Understand and interpret public commodity-based equities.
- Describe the ownership of commodities through bonds.
- Understand how commodity-based mutual funds and exchange-traded products allow for exposure.

5.3.6: Demonstrate knowledge of ETNs and private market commodity funds.

Including:

- Understand the process of financing commodity trade and production.
- Describe public and private commodity partnerships.
- Understand how commodity-linked investments operate.
- Understand how commodity-based hedge funds operate.

5.3.7: Demonstrate knowledge of leveraged and option-based structured commodity exposures.

Including:

- Identify exposures that leveraged and inverse commodity index-based products create.
- Identify exposures of leveraged notes.
- Identify the role of principal-guaranteed notes.

5.3.8: Demonstrate knowledge of key concepts in managing commodity exposure.

Including:

- Understand roll return in the context of commodity exposure.
- Describe potential cycles of commodity prices and returns.
- Describe the relationship between commodity prices and key economic variables.

## **Reading 5.4: The Risk and Performance of Private and Listed Assets**

5.4.1: Demonstrate knowledge of evidence regarding illiquidity premiums from listed assets.

Including:

- Understand a factor-pricing-based explanation for illiquidity premiums.
- Interpret empirical evidence of illiquidity premiums in US treasuries.
- Interpret empirical evidence of an illiquidity premium in US equities.

5.4.2: Demonstrate knowledge of private and listed real performance in real estate.

Including:

- State the case against unlisted real estate pools based upon historical performance.
- Explain the divergent performance between private properties and listed properties.
- State the case against unlisted real estate pools based upon risk-adjusted performance.

5.4.3: Demonstrate knowledge of challenges in the PME method to evaluate private asset performance.

Including:

- Understand and apply the interim internal rate of return.
- Explain why IRRs under the LN-PME method cannot be calculated in some cases.
- Identify why IRRs fail to adjust for scale and timing.
- Justify using the PME method in evaluating performance.
- Analyze how the PME method can be manipulated.

5.4.4: Demonstrate knowledge of multiple evaluation tools.

Including:

- Understand and apply simple cash flow multiples as an evaluative performance metric.
- Interpret private equity fund benchmark analysis.
- Understand how to apply a PME analysis to PE funds.
- Interpret results using multiple evaluation tools.

5.4.5: Demonstrate knowledge of IRR aggregation problems for portfolios.

Including:

- Calculate equal weighting IRRs or IIRRs as measures of performance.
- Calculate commitment weighting IRRs or IIRRs as measures of performance.
- Calculate pooled cash flows for weighting IRRs or IIRRs as measures of performance.
- Apply time-zero based pooling.
- Contrast the weighting approaches for IRR or IIRR.

5.4.6: Demonstrate knowledge of considerations when investing in private funds.

Including:

- Evaluate the key empirical findings regarding PE fund performance.
- Understand two propositions regarding allocating assets in private or listed markets.

## TOPIC 6

CAIA Association. *CAIA Curriculum Level II Volume III*. Self-published, CAIA Association, 2024.

### Reading 6.1: Selection of a Fund Manager

6.1.1: Demonstrate knowledge of the importance of fund selection across managers through time and the relationship between GPs and LPs.

Including:

- Compare the performance of high and low quartile PE fund managers through time.
- Understand the dynamic between PE GPs and LPs.
- Explain adverse selection in GP-LP relationships.
- Describe the life cycle aspect of the GP-LP relationship.
- Identify the entry and establish phase of PE Funds.
- Identify the build and harvest phase of PE Funds.
- Identify the decline or exit phase.

6.1.2: Demonstrate knowledge of fund return persistence.

Including:

- Describe the fund performance persistence hypothesis.
- Interpret evidence regarding fund performance persistence.
- Explain transition matrices and return persistence in PE funds.
- Understand the persistence of return persistence in PE funds.
- Identify challenges to the performance persistence hypothesis.
- Describe performance persistence implementation issues.

6.1.3: Demonstrate knowledge of the manager selection process and several issues inherent in fund management.

Including:

- Understand the fund manager selection process.
- Understand how moral hazard, adverse selection, and the holdup problem impact fund management.

6.1.4: Demonstrate knowledge of how to screen fund management.

Including:

- Identify questions regarding the nature of a fund's investment program.
- Identify questions regarding the investment objective of PE funds.
- Identify questions regarding the investment process of PE funds.
- Identify questions regarding the value added by the fund manager of PE funds.

6.1.5: Demonstrate knowledge of historical performance review.

Including:

- Identify critical decisions regarding performance review.
- Understand implications of relying on past performance.
- Discuss the importance of analyzing past assets under management.
- Interpret drawdown.
- State five classic statistical issues when using past data to predict the future.
- Apply statistical systems to understand portfolio risk management systems.

6.1.6: Demonstrate knowledge of manager selection and deal sourcing.

Including:

- Determine the wish list of fund characteristics.
- Recommend systems to assess a management team's competence.
- Understand how to source deals.

6.1.7: Demonstrate knowledge of fund culture.

Including:

- Understand the importance of a fund's culture.

## **Reading 6.2: Cases in Tail Risk**

6.2.1: Demonstrate knowledge of risks driven by market losses.

Including:

- Identify the reasons why Amaranth Advisors, LLC collapsed.
- Understand the processes that led to the collapse of Long-Term Capital Management.
- Identify the reasons why Carlyle Capital Corporation collapsed.
- Understand the concept of volatility of volatility derivatives.

6.2.2: Demonstrate knowledge of risks driven by leverage and behavioral biases.

Including:

- Evaluate the relationship between declining investment opportunities and leverage.
- Describe the link between behavioral biases and risk taking.

6.2.3: Demonstrate knowledge of the impact of trading technologies in financial crises.

Including:

- Discuss how the unwind hypothesis and crowded trades explain the Quant Meltdown of August 2007.
- Discuss how a circuit breaker can help prevent a flash crash.
- Discuss how technical issues at one large market participant can impact the financial markets.

6.2.4: Demonstrate knowledge of cases of failures that occurred due to fraud.

Including:

- Understand the reasons for the failures of Bayou Management.
- Understand the reasons for the failure of Bernie Madoff.
- Understand the reasons for the failure of Lancer Group.
- Understand the reasons for the failure of the venture capital startup Theranos.
- Evaluate the failures in diligence by venture capitalists invested with FTX.

6.2.5: Demonstrate knowledge of four major lessons from analysis of fund failures.

Including:

- Discuss the lessons that emerge from the analysis of various types of hedge fund failures.

## **Reading 6.3: Investment Process Due Diligence**

6.3.1: Demonstrate knowledge of investment due diligence.

Including:

- Describe different approaches to due diligence.
- Describe and compare quantitative due diligence and qualitative due diligence.
- Understand the importance of investment due diligence.
- State internal fund functions.
- Distinguish between investment process and operational due diligence.
- Understand costs and importance of due diligence.
- Identify the role of due diligence checklists and questionnaires.

6.3.2: Demonstrate knowledge of the investment strategy or investment mandate.

Including:

- Describe details of investment strategies.
- Discuss strategy drift within the investment mandate.
- Understand leverage within strategy drift.
- Understand how investment markets and securities are related.
- Describe the relationship between the due diligence process and competitive advantage.
- Identify key persons within investment strategies.

6.3.3: Demonstrate knowledge of investment implementation processes and accompanying risks.

Including:

- Discuss how to implement investment strategies.
- Interpret risks within investment processes.
- Understand how to detect investment process risks.

6.3.4: Demonstrate knowledge of asset custody and valuation.

Including:

- Understand the role of custodians in safeguarding assets.
- Describe the role of the current portfolio position in the due diligence process.
- Evaluate principles of fund asset valuation.
- Discuss conflicts of interest with respect to fund asset valuation.
- Identify challenges in listed asset valuation.
- Understand the relationship between asset level and fair asset values.
- Interpret internal valuation of assets.

6.3.5: Demonstrate knowledge of risk alert advantages and observations.

Including:

- Identify advantages of portfolio information aggregators.
- Understand risk alert observations on third party information regarding asset values.
- Understand risk alert observations on trends in due diligence.

6.3.6: Demonstrate knowledge of portfolio risk review.

Including:

- Understand the role of risk review.
- Identify the role of the chief risk officer.
- State general questions that must be asked in a risk review.
- Identify risks of special concern in the risk review.
- Understand the relationship between risk review and leverage.
- Understand how leverage magnifies losses and probabilities of various loss levels.
- Identify subscription and redemption risks.

6.3.7: Demonstrate knowledge of warning indicators and awareness signals in investments and risk management.

Including:

- Identify warning indicators and awareness signals with respect to investments.
- Identify warning indicators and awareness signals with respect to risk management.

## **Reading 6.4: Operational Due Diligence**

6.4.1: Demonstrate knowledge of risks and remedies in operations.

Including:

- Identify operational errors, agency conflicts, and operational fraud of a fund.
- Understand why operational due diligence is driven by operational risk.
- List the major components to controlling operational risk.
- Understand how investors can mitigate operational risk.
- Describe how perverse incentives can motivate the reporting of performance.
- Discuss oversight procedures of the trade life cycle.
- Explain the role of an SEC risk alert with respect to a fund's investment process.

6.4.2: Demonstrate knowledge of key operational activities and key elements of the ODD process.

Including:

- Understand due diligence with respect to the execution of trades.
- Understand due diligence with respect to posting of trades.
- Understand due diligence with respect to trade allocation.
- Understand due diligence with respect to trade reconciliation.
- Identify core elements of the ODD process.
- Understand explanations for the expanding scope of operational due diligence.
- Discuss the use of third-party sources for due diligence review.

6.4.3: Demonstrate knowledge of cash fund management and movement.

Including:

- List the primary purposes of fund cash.
- Analyze the use of cash to meet fund expenses.
- Analyze the use of cash to facilitate trading.
- Evaluate the reasons for analyzing cash flow to and from investors.
- Discuss the role of unencumbered cash.

6.4.4: Demonstrate knowledge of how to analyze external parties and check principals.

Including:

- Understand the role of fund prime brokers.
- Analyze the role of fund administrators.
- Understand the role of investigative due diligence.
- Describe various models for selecting personnel for investigation.
- List areas that are commonly included in background investigations.
- Understand how to organize and interpret information from investigations.
- Understand the process of asset verification.

6.4.5: Demonstrate knowledge of analysis of fund compliance.

Including:

- Understand the importance of personal trading compliance of fund employees.
- Identify common compliance risks regarding personal trading.
- Discuss compliance risks regarding nonpublic and inside information.
- Understand the role of electronic communication monitoring.
- Analyze the work of third-party compliance consultants.

6.4.6: Demonstrate knowledge of processes and role of on-site manager visits as well as meta risk and information technology.

Including:

- Understand how to select visit locations.
- Explain why desk reviews are not best practice.
- Identify the risk alert's three tasks on desk and site review.
- Understand the due diligence role played by information technology.
- Discuss five due diligence questions that surround information technology.
- Define meta risk.

6.4.7: Demonstrate knowledge of performing operational due diligence on emerging managers.

Including:

- Define emerging managers.
- Describe the difficulties an emerging manager may encounter in the operational due diligence process.

6.4.8: Demonstrate knowledge of funding, applying, and concluding ODD.

Including:

- Contrast approaches to resource allocation for operational due diligence.
- Understand how to document the operational due diligence process.
- State the relationship between due diligence and the operational decision.

## **Reading 6.5: Due Diligence of Terms and Business Activities**

6.5.1: Demonstrate knowledge of the document collection process in due diligence.

Including:

- Explain asset owners' objectives within allocations.

6.5.2: Demonstrate knowledge of unlisted manager-investor relationships.

Including:

- Apply the ILPA guiding principles to private fund structures.
- Support the use of the ILPA guiding principles in fund management.

6.5.3: Demonstrate knowledge of structural review of funds and fund managers.

Including:

- Understand the importance of legal fund structures.
- Describe how master-feeder trusts work.
- Understand how side pocket arrangements operate.
- Discuss the role of the documentation of registrations.
- Understand the role of fund manager organization and ownership.

6.5.4: Demonstrate knowledge of terms for liquid and illiquid private funds.

Including:

- Understand redemption terms.
- Describe potential benefits of lockups.
- Understand the relationships between the LPA, fund term, and distributions.
- Understand terminations and divorces within funds.
- Describe investment limits and legal liability limits.
- Understand investor relations.

6.5.5: Demonstrate knowledge of side letters to limited partnership agreements.

Including:

- Evaluate the various issues involving side letters.

6.5.6: Demonstrate knowledge of private-placement memorandums (PPM).

Including:

- State the key functions of the offering memorandum (OM) and PPM.
- Understand the function of side letters.
- Identify different purposes of legal counsel reviews and ODD document reviews.
- Analyze other common private placement memorandum terms.

6.5.7: Demonstrate knowledge of fund fees and expenses.

Including:

- Identify the timing of fee collections.
- Understand the role of fee offsets.
- Understand the contribution of GP's contribution with respect to fund risk taking.

6.5.8: Demonstrate knowledge of private fund audited financial statement reviews.

Including:

- Define the role of audited financial statements.
- Understand valuation policies.

6.5.9: Demonstrate knowledge of business activities, continuity planning, disaster recovery, and insurance.

Including:

- Understand the process of business continuity planning and disaster recovery.
- Describe the role of information technology in continuity planning and disaster recovery.
- Recognize the role of fund insurance in operational due diligence.

The following is a review of the Methods and Models principles designed to address the learning objectives set forth by CAIA Association®. Cross-reference to CAIA Association Reading 4.1.

## READING 4.1

# MODELING OVERVIEW AND FIXED INCOME MODELS

Topic 4

### EXAM FOCUS

While the CAIA Level I curriculum focused on single-factor models, the Level II curriculum emphasizes multifactor models. This reading introduces four different models for valuing fixed-income products. For the exam, understand the differences between, assumptions of, and uses for the Vasicek model, the Cox, Ingersoll, and Ross (CIR) model, the Ho-Lee model, and the Black-Derman-Toy (BDT) model. This reading also introduces several credit risk and credit scoring models. The structural and reduced-form models attempt to estimate default probabilities and credit spreads. The empirical model ignores the default process and instead uses financial statements to estimate a credit score. Be able to perform the basic credit default calculations as well as the more complex ones for the exam. These models include the Merton model and the Kealhofer, McQuown, and Vasicek (KMV) model.

### INVESTMENT STRATEGY MODELS

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**LO 4.1.1: Demonstrate knowledge of underlying models of investment strategy.**

**Including:**

- **Compare normative strategies with positive strategies.**
  - **Distinguish between theoretical and empirical models.**
  - **Distinguish between applied versus abstract models.**
  - **Compare cross-sectional versus time-series models.**
  - **Discuss the importance of methodology in model building.**
- 

Financial models are simplifications of reality that are designed to make sense of complex financial markets. There are two key variables involved. An **exogenous variable** comes from a source outside of the model, and it is assumed to be valid. This is essentially an independent variable. For an endowment fund, cash donations are an example of an exogenous variable. An **endogenous variable** comes from within the model. It is essentially a dependent variable. For an endowment fund, the percentage allocation to a new alternative asset class is an endogenous variable. This chapter will provide an overview of various models that use these two types of variables.

## Normative Strategies and Positive Strategies

A **normative model** attempts to explain how people and asset prices *should* behave. This mirrors the goals of fundamental analysis by exploring the drivers of rational decision-making under idealized scenarios. Alternatively, a **positive model** seeks to understand how people are *actually* behaving. This is more like technical analysis, which looks for mispricings through observed historical price patterns.

Both normative and positive models are used to predict future behavior by alternative asset managers. Trading strategies formed using a normative model anticipate that future prices will converge on normal values. This could be accomplished by using an arbitrage-free model like put-call parity. Trading strategies derived from a positive model attempt to infer future actions based on price patterns. Strategies using moving averages and price channels are examples of this approach.

## Theoretical and Empirical Models

**Theoretical models** draw conclusions about behavior from existing observations of underlying behavior. In this sense, they are similar to normative models. A theoretical model can use simplifying assumptions and observed relationships to deduce the price of a simple option; think put-call parity. This works well under simplified situations.

An **empirical model** makes predictions when behavior is complex, the number of data points is high, and the relationship between variables is changing over time. This model is a better choice for a complex security with overlapping layers of optionality. Because alternative assets commonly experience illiquidity, dynamic movements, changing risks, and other complexities, empirical models are often the better choice for alternative investors.

## Applied and Abstract Models

An **applied model** is designed to solve *actual* real-world challenges in the present day, and the commonly used Markowitz model is a perfect example. Investors use this model to identify efficient diversification opportunities for current application. Most asset pricing models fall into this category, and it is the type primarily used by alternative asset managers.

An **abstract model** (a.k.a. basic model) is structured to solve *hypothetical* real-world challenges set in the future. These models attempt to explain behavior under less realistic—but possible—scenarios, and they can lead to new innovations. Because they are hypothetical in nature, they are not widely used by alternative managers.

## Cross-Sectional and Time-Series Models

There are three different ways that analysts can consider data as it relates to time. First, **cross-sectional models** analyze relationships at a specific point in time. Second, a **time-series model** studies behavior over a period. Third, **panel data sets** (a.k.a. longitudinal data sets) combine the study of multiple variables with observation over a period.

Consider an example using real estate investment trusts (REITs). Perhaps an analyst wants to study a basket of REITs relative to an index that is simply an arithmetic average of each constituent REIT. The analyst could begin by considering the impact of changes in equity prices, Treasury yields, and mortgage rates on the index over time. This is a time-series model. Next, the analyst considers the differences in the long-term average returns of each REIT in the index as of today. This is a cross-sectional model. Then, the analyst regresses the returns of individual REITs against factors like geographic location, leverage, and property type. This would be panel data.

## Model Building Methodology

This section has presented models across four dimensions: normative versus positive, theoretical versus empirical, applied versus abstract, and cross-sectional versus time-series. To properly evaluate a manager's strategy, it is important to understand which model (or combination of models) is being applied. Manager 1 uses an equilibrium pricing model to identify trading opportunities based on deviations from the ideal value. Manager 2 devised a trading signal based on a month-end statistical pattern that has been observed to repeat. The first manager is using theoretical and normative models, while the second manager is using empirical and positive models. There is not a judgment on which is correct, but it is useful to understand the logic behind their differing approaches.

## EQUILIBRIUM AND ARBITRAGE-FREE MODELS

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**LO 4.1.2: Demonstrate knowledge of equilibrium models and arbitrage-free models of the term structure.**

**Including:**

- **Contrast equilibrium fixed-income models with arbitrage-free models.**
  - **Apply Vasicek's model.**
  - **Contrast the Cox, Ingersoll, and Ross (CIR) model with Vasicek's model.**
  - **Discuss the Ho and Lee model.**
- 

## Equilibrium Fixed-Income Models

The CAIA Level I curriculum presented theories that explain the shape of the yield curve (e.g., liquidity preference theory, market expectations theory, market segmentation theory, and preferred habitat theory). This chapter will introduce models to explain the evolution of bond values. **Equilibrium models of the term structure** (a.k.a. first-generation models) combine assumptions about fixed-income markets with economic logic to infer bond prices and the term structure of interest rates. Two such models were devised by Vasicek<sup>1</sup> and the team of Cox, Ingersoll, and Ross.<sup>2</sup>

## Vasicek Model

The **Vasicek model** is a single-factor model that assumes constant volatility and mean reversion:

$$\tilde{r}_{t+1} = r_t + k(\mu - r_t) + \sigma\tilde{\epsilon}_{t+1}$$

This model specifies that next period's short-term rate,  $\bar{r}_{t+1}$ , is equal to the current short-term rate,  $r_t$ , plus two adjustment factors. The first adjustment involves two constants,  $k$  and  $\mu$ , that are both positive values. The value represented by  $\mu$  is the long-term average of the short-term rate. This formula is mean reverting, which means that if the current value is below (above) the long-term average, then it is expected to increase (decrease) in value. The speed of the mean-reverting adjustment is modeled by  $k$ , where a higher variable means a faster movement toward the mean. The second adjustment factor introduces uncertainty and noise. The volatility of change in interest rates,  $\sigma$ , is further adjusted for noise,  $\bar{\varepsilon}_{t+1}$ , which is assumed to be normally distributed with a mean of zero and a standard deviation of 1.

The Vasicek model can be used to estimate expected interest rates,  $E(r_{t+1})$ :

$$E(r_{t+1}) = r_t + k(\mu - r_t) + \sigma E(\varepsilon_{t+1})$$

For example, if the long-term average rate is 4.25%, the current rate is 3.75%, the volatility of interest rates changes is 1%, and the speed of adjustment is assumed to be 0.65, then the estimated rate for next period is 4.075%:

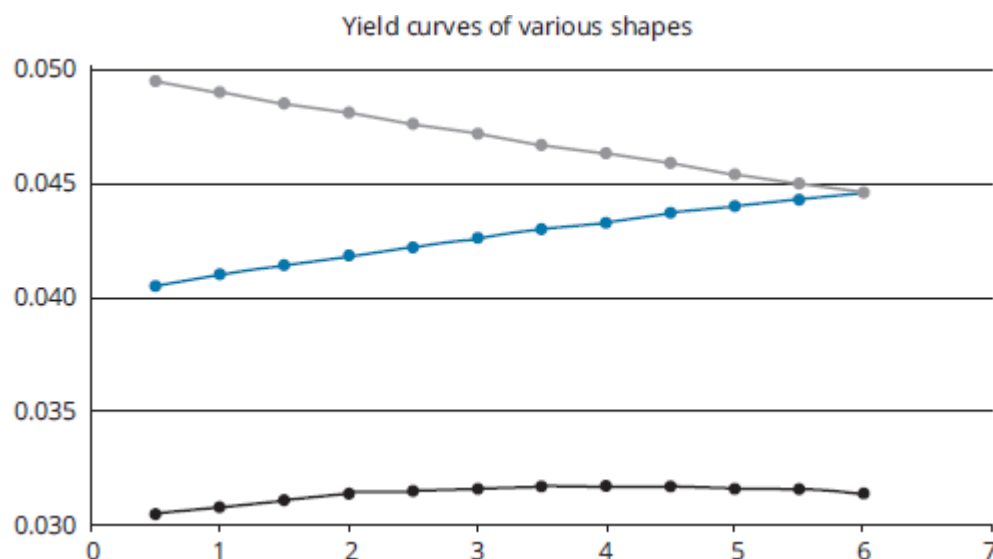
$$E(r_{t+1}) = 0.0375 + 0.65(0.0425 - 0.0375) + 0.01(0) = 4.075\%$$

This method assumes that the volatility of interest rate changes is constant, which is a primary criticism of the Vasicek model. This example also assumes zero noise.

In the Vasicek model, it is assumed that all bond prices are driven by a single factor: short-term interest rates. Essentially, this means that the only source of uncertainty modeled is random changes in short-term rates. Remember that the unbiased expectations theory argues that all fixed-income securities with a common time until maturity should trade at the same yield. Merging the Vasicek model logic with the unbiased expectations theory can be done to calculate a yield curve (i.e., the term structure of interest rates) from the yield to maturity on a zero-coupon bond.

Applying the Vasicek model can produce a yield curve that is downward sloping, upward sloping, or humped. This can be seen in Figure 1. The upward-sloping curve is derived from an example with a current short-term rate of 5%, a long-term mean of 7%, a speed adjustment of 0.1, and a standard deviation of 1%.

Figure 1: Illustration of the Vasicek Model

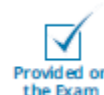


In this example, you can see that the general slope of the curve is driven by the relationship between the current rate and the long-term mean. The upward- (downward-) sloping line reflects an upward (downward) movement toward the mean. The humped line shows risk aversion.

### Cox, Ingersoll, and Ross Model

Another criticism of the Vasicek model is that it allows negative short-term rates. The **Cox, Ingersoll, and Ross model** (a.k.a. the CIR model) adjusts Vasicek’s original model to make the variance of the short-term rate proportional to the rate itself. This methodology disallows negative rates because as rates approach zero, their volatility also approaches zero. The CIR model is as follows:

$$r_{t+1} = r_t + k(\mu - r_t) + \sqrt{r_t} \sigma \tilde{\epsilon}_{t+1}$$



Notice that the three constants remain the same and that the last term has been altered from volatility to variance. This makes sense because the volatility of rates tends to be high if short-term rates are also high. The CIR model can also generate yield curves with different shapes.

### Arbitrage-Free Interest Rate Models

**Arbitrage-free models of the term structure** (a.k.a. second-generation models) use parameters that are based on the current yield curve. This method is used in support of the risk-neutral idea that arbitrage opportunities should not exist. The notable distinguishing factor for these models is that the current yield curve is used to infer parameters for the model. The result is that the theoretical output of these models is consistent with current observations. This means that a fixed-income derivative priced in this way will be consistent with the current yield curve’s structure, and arbitrage opportunities should not be present.

## Ho-Lee Model

In 1986, Ho and Lee<sup>3</sup> proposed the first arbitrage-free model for interest rates. The **Ho-Lee model** is a single-factor model that assumes a normal distribution for short-term rates and incorporates a drift parameter that directly connects the model with the current yield curve. Their model is as follows:

$$r_{t+1} = r_t + \theta_t + \sigma \tilde{\varepsilon}_{t+1}$$



The inclusion of current rates, and the combination of volatility and noise, is the same as the Vasicek model. The Ho-Lee model adds a term for time-dependent mean change,  $\theta_t$ , which is chosen to ensure that the model fits the initial yield curve. This time-dependent term is not constant.

The Ho-Lee model is the basis for a binomial model to calculate bond prices. Their process begins with the current zero-coupon bond, whose prices are used to calibrate the model to the parameters on the currently observed yield curve. After this initial point, the yield curve is assumed to be impacted by random changes in interest rates. Using this approach, bond prices can evolve in response to random changes in interest rates. The Ho-Lee model relies on the well-known, risk-neutral approach that current bond prices should be equal to the next period's observed price, discounted at the risk-free rate.

The primary advantage of the Ho-Lee model is that it is calibrated to fit the observed yield curve, meaning that arbitrageurs will enable the prices for fixed-income derivative products to be linked to observed bond prices in such a way that arbitrage opportunities should not exist. There are two primary disadvantages to their approach. First, their model allows negative rates. Second, it assumes a simplified binomial process for bond prices. Since their pioneering work was developed, other researchers have devised more sophisticated models that prevent negative rates and allow for a more dynamic process.

## BLACK-DERMAN-TOY MODEL

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**LO 4.1.3: Demonstrate knowledge of the Black-Derman-Toy (BDT) model.**

**Including:**

- Interpret a binomial BDT tree.
- Understand how to calibrate the level of rates based on average returns.
- Understand how to calibrate the spread of rates based on volatilities.
- Discuss BDT calibrations in general.

---

The **Black-Derman-Toy (BDT) model**<sup>4</sup> is commonly used to value fixed-income derivatives. It remains consistent with the current yield curve while also factoring for the implied volatility on interest rate caplets. As such, it is useful for calculating spot rates, forward rates, or discounting factors.

## Binomial BDT Trees

Consider how the BDT model works using a two-period (two-year) binomial tree. In this example, the current spot rate,  $r_0$ , is 4.5%. Assume that there is a 50% probability of a move up to  $r_u$  and a 50% probability of a move down to  $r_d$ . A binomial tree can be used to illustrate this relationship:

$$r_0 = 4.5\% \begin{cases} r_u = 5.03\% \\ r_d = 3.75\% \end{cases}$$

It is important to note that in this example, the yield on a two-year zero-coupon bond is 4.45% (this is the connection with the current yield curve), and the lognormal volatility of the short rate in one year is 14.72%. This model assumes that all rates are continuously compounded. The BDT model requires that up rate and down rate are both calibrated based on two key principles: the average two-period return of the two paths must equal the return of the two-year zero-coupon bond, and the spread between the two paths must be consistent with the implied volatility of a time-relevant interest rate caplet. You will see these calculations play out in the next two sections.

## Calibrating the Level of Rates

In this model, calibration ensures that arbitrage opportunities do not exist. In this section, you will see how the up rate and down rate are calibrated with the two-year zero-coupon bond to prevent potential arbitrage. Recall that the up rate is 5.03%, the down rate is 3.75%, and the yield on a two-year zero-coupon bond is 4.45%. The two-year bond has a total two-year return of 9.09%:

$$(1.0445)^2 = 9.09\%$$

The average return of the up path and the down path,  $\bar{R}_{u,d}$ , is as follows:

$$\bar{R}_{u,d} = 0.5[(1 + r_0)(1 + r_u) + (1 + r_0)(1 + r_d)] - 1$$

$$\bar{R}_{u,d} = 0.5[(1.045)(1.0503) + (1.045)(1.0375)] - 1 = 9.09\%$$

This process confirms that the first requirement of the BDT model is met. The average two-period return of the two paths equals the return of the two-year zero-coupon bond.

## Calibrating the Spread of Rates

The second requirement is that the spread between the up rate and the down rate is consistent with the observed volatility on a time-relevant interest rate caplet. The spread requirement is fulfilled through the following formula, which is also consistent with the continuously compounded constraint:

$$r_u = r_d e^{2\sigma}$$

In the case of this ongoing example, the implied volatility is 14.72%, and the relationship holds:

$$r_u = 0.0375 e^{2 \times 0.1472} = 5.03\%$$

## Summary of BDT Calibrations

The BDT model can calculate spot rates or forward rates to value fixed-income derivatives. It remains consistent with the current yield curve and the observed implied volatility on time-relevant interest rate caplets. The spot rates in the current yield curve drive the *level* of the rates projected throughout the binomial tree. The implied volatility on the interest rate caplets drives the *spread* between the up rate and the down rate.

In a two-period tree, the rates can be solved with moderate complexity. In practice, a BDT model can have 30 or more time steps, which requires an algorithm to properly construct. The intuitive constraints of the BDT model enable arbitrage-free pricing of fixed-income derivatives.

## ECONOMICS OF CREDIT RISK

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**LO 4.1.4: Demonstrate knowledge of credit risk and credit risk modeling.**

**Including:**

- Distinguish types of credit events that may lead to an increase in credit risk.
  - Explain exposure at default (EAD) and loss given default (LGD).
  - Describe how adverse selection and moral hazard relate to credit risk.
  - Discuss how probability of default (PD) and recovery rate (RR) affect credit risk.
  - Calculate loss given default and expected loss from credit risk.
  - Describe the basic concepts of credit risk modeling.
  - Contrast the three approaches to credit risk modeling.
- 

**Credit** is simply money or funds granted by a creditor to a borrower. **Credit risk** is the risk of loss from a counterparty due to a credit event. For example, credit risk occurs if the debtor is unable or unwilling to meet its contractual obligations. **Credit events** include the following:

- **Bankruptcy** refers to the dissolution or insolvency of an entity when it is unable to meet its obligations.
- A **credit downgrade** occurs when a rating is downgraded by a credit rating agency.
- **Failure to make payments** in a timely manner arises when a borrower fails to make scheduled principal or interest payments, even if it is not in bankruptcy or in distress.
- **Corporate events** include mergers or spinoffs, which could weaken an entity's financial condition and capacity to service its obligations.
- **Government actions** include capital controls or restrictions by governments.

Several measures are critical in measuring credit risk. **Exposure at default (EAD)** measures a creditor's potential loss if a credit event occurs. **Loss given default (LGD)** measures the loss (and therefore recovery) in a default scenario. LGD is typically less than EAD.

## Adverse Selection and Credit Risk

Borrowers often have more information than lenders, which is referred to as **adverse selection**. Adverse selection raises a lender's risk. In turn, the lender may choose to offset this risk through higher interest rates. However, as lenders raise interest rates, the borrower's cost of financing increases, therefore increasing credit risk. To offset this risk, lenders, such as large institutions, require additional credit protection, including collateral, limits on loan amounts, and verification of a borrower's credit profile and history.

## Moral Hazard and Credit Risk

Adverse selection arises before completion of a financial transaction. In contrast, moral hazard arises after completion of a financial *transaction*. **Moral hazard** arises when the borrower takes on more risk knowing that the counterparty (lender) bears the risk of the transaction. For example, commercial banks take deposits they know are guaranteed by the government to make risky loans to clients. Lenders can protect against these risks through monitoring, various restrictions, and limiting loan sizes.

## Probability of Default

**Probability of default (PD)** refers to the probability that the borrower will not meet its contractual obligations. Both moral hazard and adverse selection affect PD. Lenders use their expertise, credit spread data, and historical data to gauge PD. Lenders often use credit ratings and credit models to estimate PD. A historical observation that 0.05% of Ba2 firms default would imply that a bond rated Ba2 would have a 0.05% chance of default within the next year.

## Expected Credit Loss

The **recovery rate (RR)** ties the EAD and LGD together and is measured as the percentage of EAD that can be recovered. Note, however, that an RR of 70% does not necessarily guarantee that the lender will recover 70% of its loan at the time of default. The recovery process can be lengthy, and therefore, the more relevant measure is the present value of the recovered amount:

$$\text{recovery rate} = \text{present value of recovered sum} / \text{EAD}$$

For example, the RR is 35% on a loan of \$175 million, to be recovered over four years using a discount rate of 6.5%. The implied RR is found to be 27.21%:

$$\text{amount to be recovered} = 0.35 \times \$175 \text{ million} = \$61.25 \text{ million}$$

$$\text{present value of planned recovery} = \frac{\$61.25}{(1 + 0.065)^4} = \$47.611 \text{ million}$$

$$\text{recovery rate} = \frac{\$47.611}{\$175} = 27.21\%$$

LGD, EAD, PD, and RR are all related and can be used together to calculate the expected credit loss:

$$\text{LGD} = \text{EAD} \times (1 - \text{RR})$$

$$\text{E[Loss]} = \text{LGD} \times \text{PD} = \text{EAD} \times (1 - \text{RR}) \times \text{PD}$$

For example, the EAD is \$175 million, the RR is estimated to be 27.21%, and the estimated probability of loss is 0.75%. This yields an LGD of \$127.38 million and an expected loss of \$955,000:

$$\text{LGD} = \$175 \times (1 - 0.2721) = \$127.38$$

$$\text{E[Loss]} = \$127.38 \times 0.0075 = \$0.955$$

## Credit Risk Modeling

While investors in government bonds are primarily exposed to interest rate risk, investors in bonds of levered entities face a wide range of risks. They are exposed to credit risk, credit event risk, and the risk of deterioration in the entity's credit quality. Default is an extreme example of a credit event. However, default does not necessarily represent the final state because entities may go through a process of workout or restructuring.

There are three types of credit risk modeling approaches:

1. *Structural credit risk models.* These models draw an explicit relationship between capital structure and default from the perspective of the equity owner. Within the capital structure, the value of assets equals the value of equity plus debt, where equity is modeled as a call option on the entity's assets with a strike price equal to the face value of the bonds. Bondholders, by contrast, have a risk-free bond and a short position in a put option on the entity's assets.
2. *Reduced-form models.* These models view default as a random external factor. The models, therefore, view default as a random event that can be quantified using economic and statistical models.
3. *Empirical models.* The empirical approach to credit risk modeling does not directly evaluate the entity or its surroundings given the difficulty in forecasting risk factors. Instead, empirical models produce a credit score that is used to rank entities by creditworthiness.

## STRUCTURAL CREDIT RISK MODELS

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**LO 4.1.5: Demonstrate knowledge of the structural model approach through the lens of the Merton model.**

**Including:**

- Describe Merton's structural model using the option-like nature (both call options and put options) of traditional corporate securities.
  - Describe the inherent conflict of interest that exists between shareholders and bondholders.
  - Evaluate advantages and disadvantages of the Merton model.
  - Understand how binomial trees can be used to value structured products.
- 

## Merton's Structural Model

**Structural credit risk models** explicitly factor credit risk by using option theory. They model risk through the lens of the factors that drive the underlying asset's volatility coupled with the structuring of the relevant cash flows. Robert Merton pioneered the option-like perspective of a firm's capital structure. His thinking can be used to consider the option-like characteristics of credit risk analysis.

Consider a simple leveraged firm with two sources of capital: a single-issue zero-coupon bond and a single class of equity. Using the **call option view of capital structure**, the equity in a levered firm behaves like a call option with a strike price equal to the face value of the firm's debt and an expiration date equal to the maturity of the debt. If the firm does well, then the call option will pay off. The firm will fully pay its debtholders, and the firm will be solely owned by the equityholders who will then enjoy upside potential. However, if the firm performs poorly, then the firm may go bankrupt (in the most extreme cases). In this circumstance, the call option on the firm's assets will expire worthless and whatever remains of the firm is owned only by the debtholders. This situation is depicted in the following equation:

$$\text{equity of levered firm} = \text{call option on the firm's assets}$$

The call option view can also lead to an understanding of the firm's debt. You may recall the *balance sheet identity*, which states that the firm's assets must equal the combination of its debts and equity. This relationship is why the report is called a *balance sheet*. If the firm's assets are equal to the combination of debt and equity, then the firm's debt can be seen as the combination of owning the firm's assets and writing a call option on those assets. This is essentially a covered call strategy and is depicted in the following relationship:

$$\text{debt of levered firm} = \text{firm assets} - \text{call option on the firm's assets}$$

Analogous to this logic is the opportunity to view debt in terms of a put option. Due to put-call parity, a call option is equivalent to combining a riskless bond (that fully funds all of the firm's assets) and a long position in a put option. Therefore, the **put option view of capital structure** views the equity holders as owning the firm's assets with riskless financing and owning a put option to transfer those assets to the debtholders. Likewise, the debt of the firm can be seen as combining the riskless bond with writing a put option as depicted in the following relationship:

$$\text{debt of levered firm} = \text{riskless bond} - \text{put option on the firm's assets}$$

From the vantage point of a put option, the equityholders have limited liability in the event of bankruptcy. If this worst-case scenario did materialize, then the debtholders would pay the strike price of the put (the face value of the riskless bond) to the equity holders. The debtholders would then be left with a pool of assets with depleted values.

Using an option-like view to credit risk analysis is very useful. This view permits a credit analyst to use an arbitrage-free option pricing model to value complex structured products. The Black-Scholes option pricing model can therefore be used to evaluate complex credit instruments.

The advantages of Merton's structural model include the following:

- This structural model relies upon observable data (i.e., stock price volatility or the implied volatility on an option) from equity markets. Because equity markets are more liquid than fixed-income markets, they provide marginally more reliable data.
- The structural model is able to handle multiple debt securities from the same issuer, including convertible debt and bonds with varying seniority.

The disadvantages of Merton's structural model include the following: