UWorld | FINANCE

SimpleSheets™+

CFA® Exam Formulas | Level 1

2024 Edition

SimpleSheets+

Formulas at Your Fingertips

Quantitative Methods - Prereadings

Common Probability Distributions

- Discrete Uniform Distribution

  $F(x) = n \times p(x)$ for the nth observation.

- Binomial Distribution

  $P(X=x) = {}_nC_x (p)^x (1 - p)^{n-x}$

  where:
  p = probability of success
  1 – p = probability of failure
  ${}_nC_x$ = number of possible combinations of having x successes in n trials. Stated differently, it is the number of ways to choose x from n when the order does not matter.

- Mean of a Binomial Random Variable

  $\overline{B(n,p)} = np$

- Variance of a Binomial Random Variable

  $\sigma_x^2 = n \times p \times (1 - p)$

- The Continuous Uniform Distribution

  $P(X < a), P(X > b) = 0$

  $P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$

- Confidence Intervals

  For a random variable X that follows the normal distribution:
  The 90% confidence interval is $\bar{x} - 1.65s$ to $\bar{x} + 1.65s$
  The 95% confidence interval is $\bar{x} - 1.96s$ to $\bar{x} + 1.96s$
  The 99% confidence interval is $\bar{x} - 2.58s$ to $\bar{x} + 2.58s$

  The following probability statements can be made about normal distributions

  - Approximately 50% of all observations lie in the interval $\mu \pm (2/3)\sigma$
  - Approximately 68% of all observations lie in the interval $\mu \pm 1\sigma$
  - Approximately 95% of all observations lie in the interval $\mu \pm 2\sigma$
  - Approximately 99% of all observations lie in the interval $\mu \pm 3\sigma$

- z-Score

  z = (observed value − population mean)/standard deviation
  $= (x - \mu)/\sigma$

- Continuously Compounded Returns

  $EAR = e^{r_{cc}} - 1$ r_{cc} = continuously compounded annual rate

  $r_{cc} = \ln(1 + HPR)$

  (1 + HPR) simply equals (V_t/V_0). Therefore, the continuously compounded rate of return can also be calculated as:

  $r_{cc} = \ln\left(\frac{V_t}{V_0}\right)$

  $HPR_t = e^{r_{cc} \times t} - 1$

Sampling and Estimation

- Sampling Error

  Sampling error of the mean = Sample mean − Population mean
  $= \bar{x} - \mu$

- Standard Error of Sample Mean when Population Variance is known

  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

  where:
  $\sigma_{\bar{x}}$ = the standard error of the sample mean
  σ = the population standard deviation
  n = the sample size

- Standard Error of Sample Mean when Population Variance is not known

  $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

  where:
  $s_{\bar{x}}$ = standard error of sample mean
  s = sample standard deviation

- Confidence Intervals

  Point estimate ± (reliability factor × standard error)

  where:
  Point estimate = value of the sample statistic that is used to estimate the population parameter
  Reliability factor = a number based on the assumed distribution of the point estimate and the level of confidence for the interval (1 – α)
  Standard error = the standard error of the sample statistic (point estimate)

  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

  where:
  $\bar{x}$ = The sample mean (point estimate of population mean)
  $z_{\alpha/2}$ = The standard normal random variable for which the probability of an observation lying in either tail is σ / 2 (reliability factor)
  $\frac{\sigma}{\sqrt{n}}$ = The standard error of the sample mean

  $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

  where:
  $\bar{x}$ = sample mean (the point estimate of the population mean)
  $t_{\frac{\alpha}{2}}$ = the t-reliability factor
  $\frac{s}{\sqrt{n}}$ = standard error of the sample mean
  s = sample standard deviation

Quantitative Methods

Rates and Returns

- Arithmetic Mean

  $\mu = \sum_{i=1}^{N} x_i / N$ for a population

  $\bar{X} = \sum_{i=1}^{n} x_i / n$ for a sample

- Weighted Mean

  $\bar{X}_w = \sum_{i=1}^{n} w_i X_i$

- Geometric Mean

  $\bar{X}_G = \left[\prod_{i=1}^{n}(1+X_i)\right]^{\frac{1}{n}} - 1$

  which can be alternatively written for portfolio returns as:

  $R_G = \sqrt[n]{(1+R_1)(1+R_2)\ldots(1+R_n)} - 1$

- Holding Period Return

  $R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}} =$ Capital gain + Dividend yield

  $= \frac{P_T + D_T}{P_0} - 1$

  where:
  P_t = Price at the end of the period
  P_{t-1} = Price at the beginning of the period
  D_t = Dividend for the period

- Holding Period Returns for more than One Period

  $R = [(1+R_1) \times (1+R_2) \times \ldots \times (1+R_n)] - 1$

  where:
  $R_1, R_2, \ldots, R_n$ are sub-period returns

- Geometric Mean Return

  $R = \{[(1+R_1) \times (1+R_2) \times \ldots \times (1+R_n)]^{1/n}\} - 1$

- Time Weighted Rate of Return
  If we have annual returns data, the annualized time-weighted return can be calculated as the geometric mean of N annual returns, as follows:

  $r_{TW} = [(1+r_1) \times (1+r_2) \times \ldots \times (1+r_N)]^{1/N} - 1$

- Annualized Return

  $r_{annual} = (1 + r_{period})^n - 1$

  where:
  r = Return on investment
  n = Number of periods in a year

- Harmonic Return

  $\bar{X}_H = \frac{n}{\sum_{i=1}^{n}(1/X_i)}$ with $X_i > 0$

The Time Value of Money in Finance

- Effective Annual Rates

  $EAR = (1 + \text{Periodic interest rate})^N - 1$

- The Future Value of a Single Cash Flow

  $FV_N = PV(1+r)^N$

- The Present Value of a Single Cash Flow

  $PV = \frac{FV}{(1+r)^N}$

- The Present and Future Value of an Ordinary Annuity

  $PV_{Annuity}$: # periods N; % interest per period I/Y; amount $PMT \rightarrow PV$

  $FV_{Annuity}$: # periods N; % interest per period I/Y; amount $PMT \rightarrow FV$

- The Present and Future Value of an Annuity Due

  $PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1+r)$

  $FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1+r)$

- Present Value of a Perpetuity

  $PV_{Perpetuity} = \frac{PMT}{I/Y}$

- Continuous Compounding and Future Values

  $FV_N = PVe^{r_s \times N}$

- PV of a Coupon Bond (v1, p51, Eq 6)

  $PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \frac{PMT_3}{(1+r)^3} + \ldots \frac{(PMT_N + FV_N)}{(1+r)^N}$

- Periodic Payment for fully amortizing loan (v1, p54, Eq 8):

  $A = \frac{r \times \text{Principal}}{1 - (1+r)^{-N}}$

- Equity with a constant dividend (v1, p56, Eq 10)

  $PV_t = \frac{D_t}{r}$

- Equity with a dividend with constant growth (v1, p57, Eq 14)

  $PV_t = \frac{D_{t+1}}{(r-g)}$

- Bond Implied Return (v1, p61, Eq 18)

  $r = \left(\frac{FV_t}{PV}\right)^{\frac{1}{t}} - 1$

- Equity Implied Return (v1, p65, Eq 21)

  $r = \frac{D_{t+1}}{PV_t}$

- Equity Implied Growth (v1, p65, Eq 22)

  $g = r - \frac{D_{t+1}}{PV_t}$

Statistical Measures of Asset Returns

- Quantiles

  $L_y = (n+1)\frac{y}{100}$

- Mean Absolute Deviation

  $MAD = \frac{\sum_{i=1}^{n}|X_i - \bar{X}_i|}{n}$

- Sample Variance

  $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$

  $S = \sqrt{S^2}$

- Geometric Mean

  $X_G = \bar{X} - \frac{S^2}{2}$

- Target Downside Deviation

  $S_{Target} = \sqrt{\frac{\sum_{\text{for } X_i \le B}^{n}(X_i - B)^2}{n-1}}$

- Coefficient of Variation

  $CV = \frac{S}{\bar{X}}$

- Skewness

  $skewness \approx \left(\frac{1}{n}\right)\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{S^3}$

- Kurtosis

$$K_E = \left[\left(\frac{1}{n}\right)\frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{S^4}\right] - 3$$

- Covariance

$$COV_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

- Correlation

$$\rho XY = \frac{COV_{XY}}{S_X S_Y}, \text{ where } -1 \le \rho XY \le +1$$

Probability Trees and Conditional Expectations

- Odds for an Event

Where the odds *for* are given as "a to b", then:

$$P(E) = \frac{a}{(a+b)}$$

Where the odds *against* are given as "a to b", then:

$$P(E) = \frac{b}{(a+b)}$$

- Conditional Probabilities

$$P(A|B) = \frac{P(AB)}{P(B)} \text{ given that } P(B) \neq 0$$

- Multiplication Rule for Probabilities

$$P(AB) = P(A|B) \times P(B)$$

- Addition Rule for Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

- For Independent Events

$P(A|B) = P(A)$, or equivalently, $P(B|A) = P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$

$P(A \text{ and } B) = P(A) \times P(B)$

- The Total Probability Rule

$P(A) = P(AS) + P(AS^c)$

$P(A) = P(A|S) \times P(S) + P(A|S^c) \times P(S^c)$

- The Total Probability Rule for *n* Possible Scenarios

$$P(A) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + \cdots + P(A|S_n) \times P(S_n)$$

where the set of events $\{S_1, S_2, \ldots, S_n\}$ is mutually exclusive and exhaustive.

Portfolio Mathematics

- Expected Value

$$E(X) = P(X_1)X_1 + P(X_2)X_2 + \ldots P(X_n)X_n$$

$$E(X) = \sum_{i=1}^{n} P(X_i)X_i$$

where:
X_i = one of *n* possible outcomes.

- Variance and Standard Deviation

$$\sigma^2(X) = E\{[X - E(X)]^2\}$$

$$\sigma^2(X) = \sum_{i=1}^{n} P(X_i)\,[X_i - E(X)]^2$$

- The Total Probability Rule for Expected Value

1. $E(X) = E(X|S)P(S) + E(X|S^c)P(S^c)$
2. $E(X) = E(X|S_1) \times P(S_1) + E(X|S_2) \times P(S_2) + \ldots + E(X|S_n) \times P(S_n)$

where:
$E(X)$ = the unconditional expected value of X
$E(X|S_1)$ = the expected value of X given Scenario 1
$P(S_1)$ = the probability of Scenario 1 occurring
The set of events $\{S_1, S_2, \ldots, S_n\}$ is mutually exclusive and exhaustive.

- Covariance

$$Cov(XY) = E\{[X - E(X)][Y - E(Y)]\}$$

$$Cov(R_A, R_B) = E\{[R_A - E(R_A)][R_B - E(R_B)]\}$$

- Correlation Coefficient

$$Corr(R_A, R_B) = \rho(R_A, R_B) = \frac{Cov(R_A, R_B)}{(\sigma_A)(\sigma_B)}$$

- Expected Return on a Portfolio

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_N E(R_N)$$

where:

$$\text{Weight of asset } i = \frac{\text{Market value of investment } i}{\text{Market value of portfolio}}$$

- Portfolio Variance

$$Var(R_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(R_i, R_j)$$

- Variance of a 2 Asset Portfolio

$$Var(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B Cov(R_A, R_B)$$

$$Var(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B \rho(R_A, R_B)\sigma(R_A)\sigma(R_B)$$

- Variance of a 3 Asset Portfolio

$$Var(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + w_C^2\sigma^2(R_C) + 2w_A w_B Cov(R_A, R_B) + 2w_B w_C Cov(R_B, R_C) + 2w_C w_A Cov(R_C, R_A)$$

- Bayes' Formula

$$P(\text{Event} \mid \text{Information}) = \frac{P(\text{Information} \mid \text{Event}) \times P(\text{Event})}{P(\text{Information})}$$

- Roy's Safety-First Criterion

Minimize $P(R_P < R_T)$

where:
R_P = portfolio return
R_T = target return

- Safety-First

$$\text{Safety-First ratio (SF Ratio)} = \frac{E(R_P) - R_T}{\sigma_P}$$

Hypothesis Testing

- Test Statistic

$$\text{Test statistic} = \frac{\text{Sample statistic} - \text{Hypothesized value}}{\text{Standard error of sample statistic}}$$

- Power of a Test

Power of a test = 1 − P(Type II error)

- Decision Rules for Hypothesis Tests

Decision	**H_0 is True**	**H_0 is False**
Do not reject H_0	Correct decision	Incorrect decision **Type II error**
Reject H_0	Incorrect decision **Type I error** Significance level = P(Type I error)	Correct decision Power of the test = 1 – P(Type II error)

- Confidence Interval

$$\left[\begin{pmatrix}\text{sample}\\\text{statistic}\end{pmatrix} - \begin{pmatrix}\text{critical}\\\text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\\text{error}\end{pmatrix}\right] \le \begin{pmatrix}\text{population}\\\text{parameter}\end{pmatrix} \le \left[\begin{pmatrix}\text{sample}\\\text{statistic}\end{pmatrix} + \begin{pmatrix}\text{critical}\\\text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\\text{error}\end{pmatrix}\right]$$

$$\bar{x} \;-\; (z_{\alpha/2}) \; (s/\sqrt{n}) \;\le\; \mu_0 \;\le\; \bar{x} \;+\; (z_{\alpha/2}) \; (s/\sqrt{n})$$

- Summary

Type of test	Null hypothesis	Alternate hypothesis	Reject null if	Fail to reject null if	P-value represents
One tailed (upper tail) test	$H_0 : \mu \leq \mu_0$	$H_a : \mu > \mu_0$	Test statistic > critical value	Test statistic ≤ critical value	Probability that lies above the computed test statistic.
One tailed (lower tail) test	$H_0 : \mu \geq \mu_0$	$H_a : \mu < \mu_0$	Test statistic < critical value	Test statistic ≥ critical value	Probability that lies below the computed test statistic.
Two-tailed	$H_0 : \mu = \mu_0$	$H_a : \mu \neq \mu_0$	Test statistic < lower critical value Test statistic > upper critical value	Lower critical value ≤ test statistic ≤ upper critical value	Probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic.

- t-Statistic

$$\text{t-stat} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

where:
x = sample mean
μ_0 = hypothesized population mean
s = standard deviation of the sample
n = sample size

- z-Statistic

When the population is normally distributed and its variance is known:

$$\text{z-stat} = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$$

where:
x = sample mean
μ = hypothesized population mean
σ = standard deviation of the population
n = sample size

When the population's variance is unknown, but the sample size is large:

$$\text{z-stat} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

where:
x = sample mean
μ = hypothesized population mean
s = standard deviation of the sample
n = sample size

- Tests Concerning Differences Between Means with Two Independent Samples

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{1/2}}$$

where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

s_1^2 = variance of the first sample
s_2^2 = variance of the second sample
n_1 = number of observations in first sample
n_2 = number of observations in second sample
degrees of freedom = $n_1 + n_2 - 2$

- Paired Comparisons Test

$$t = \frac{\overline{d} - \mu_{dz}}{s_{\overline{d}}}$$

where:
$\overline{d}$ = sample mean difference
$s_{\overline{d}}$ = standard error of the mean difference = $\frac{s_d}{\sqrt{n}}$
s_d = sample standard deviation
n = the number of paired observations

- Chi Squared Test-Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

where:
n = sample size
s^2 = sample variance
σ_0^2 = hypothesized value for population variance

- Test-Statistic for the F-Test

$$F = \frac{s_1^2}{s_2^2}$$

where:
s_1^2 = Variance of sample drawn from Population 1
s_2^2 = Variance of sample drawn from Population 2

- Hypothesis Tests Concerning the Variance

Hypothesis Test Concerning	Appropriate Test Statistic
Variance of a single, normally distributed population	Chi-square stat
Equality of variance of two independent, normally distributed populations	F-stat

- Parametric Test for Correlation

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where:
r = sample correlation

- Non-Parametric Test for Correlation
Spearman rank correlation (r_s) measure

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$

where:
d_i = difference between ranks of each pair of observation

- Non-Parametric Test of Independence

The nonparametric test statistic for this test of independence is chi-square distributed:

$$X^2 = \sum_{i=1}^{m} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

m = the number of cells in the table (number of groups in the first class multiplied by the number of groups in the second class) = 3 * 3 = 0

O_{ij} = the number of observations in each cell of row i and column j (i.e., observed frequency)

E_{ij} = the expected number of observations in each cell of row i and column j, assuming independence (i.e., expected frequency)

Standardized residual = $\frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}}$