

Rates and Returns

2. INTEREST RATES AND TIME VALUE OF MONEY

The Time Value of Money

Cash now is preferred over future cash.

Interest Rate (r)

is used to compare financial instruments with varied payment timings.

Interest Rate Interpretations Ways:

1. **Required Return:** The minimum expected return on an investment.
2. **Discount Rate:** Used for calculating present value from future values.
3. **Opportunity Cost:** The foregone return from spending rather than saving or investing.

Determinants of Interest Rates

$r = \text{Real Risk-free rate} + \text{Inflation Premium} + \text{Default risk Premium} + \text{Liquidity premium} + \text{Maturity Premium}$

1. **Real Risk-Free Rate:** Interest rate in a no-inflation, risk-free scenario.
2. **Inflation Premium:** Adjustment for expected inflation.
3. **Default Risk Premium:** Compensates for the issuer's potential default.
4. **Liquidity Premium:** Covers risks related to quick asset liquidation.
5. **Maturity Premium:** Accounts for increased risks in long-term investments.

3. RATES OF RETURN

Two types of financial asset returns are:
 1) periodic income (cash dividends or interest payments)
 2) capital gain or loss (changes in the price of a financial asset).

Holding Period Return (HPR)

Measures the return over a specific period.

Calculated as:

$$R = \frac{(P_1 - P_0) + I_1}{P_0}$$

For periods longer than a year, HPR is compounded annually.

Arithmetic Mean Return

Sum of all returns divided by the total observations.

Advantages: Easy to compute, commonly used, and facilitates standard deviation calculations to assess variability.

Geometric Mean Return

Compound growth rate over multiple periods.

Suitable for a "buy-and-hold" strategy.

Calculated as:

$$\bar{R}_{Gi} = \sqrt[t]{(1 + R_{i1}) \times \dots \times (1 + R_{iT})} - 1$$

Advantage: Provides a more accurate measure of long-term investment returns.

Harmonic Mean Return

Sum of all returns divided by the total observations.

Used in cost averaging strategies, where fixed amounts are invested periodically.

Advantage: Harmonic mean is lower than geometric & arithmetic means when data is variable.

Additional Means for Handling Data Variability

Trimmed Mean: Excludes a small % of the extreme values from both ends of a dataset before calculating the mean.

Winsorized Mean: Assigns specified values to the extreme ends of a dataset before averaging, mitigating the impact of outliers.

4. MONEY-WEIGHTED AND TIME-WEIGHTED RETURN

Metric	Money-Weighted Return (MWR)	Time-Weighted Return (TWR)
Definition	Compound growth rate of all funds over the entire evaluation period	Compound rate of growth for one unit of initially invested money over a specified evaluation period
Formula	$\sum_{t=0}^T \frac{CF_t}{(1 + IRR)^t} = 0$	Time weighted return = $r_{twr} = [(1 + r_{t,1}) \times (1 + r_{t,2}) \times \dots \times (1 + r_{t,n})]^{1/N} - 1$
Representation	Internal Rate of Return (IRR)	Actual rate of return earned by the portfolio manager
Consideration of Cash Flows	Takes into account the timing and size of cash flows	Does not consider the timing and size of cash flows
Comparative Analysis	Can be used to compare different investments with varying cash flow patterns	Cannot be used to compare different investments
Common Usage	-	More commonly used to evaluate portfolio manager performance
Sensitivity to Cash Flows	Sensitive to timing and size of cash flows	Not sensitive to timing and size of cash flows
Comparative Capability	Can compare performance of different investments	Cannot compare performance of different investments
Limitations	Cannot compare returns between different individuals or investment opportunities	Requires determining account value for each cash flow, potentially incurring costs

5. ANNUALIZED RETURN

Annualized Return

- Return that would have been earned if a given investment was compounded over a one-year period.
 $r_{\text{annual}} = (1 + r_{\text{period}})^c - 1$
- Converts short-term returns to an annual basis,
- Standardizes return comparisons across time and investment types.
- Compounded differently based on the period (monthly, quarterly, weekly, daily)
- Limitation:** Assumes constant reinvestment at similar rates.

Continuously Compounded Rates of Return

- Returns that would have been earned if an investment was compounded continuously, rather than at discrete intervals.
- For Single Period:**
 $r_{t, t+1} = \ln(1 + \text{holding period return})$ or
 $r_{t, t+1} = \ln(\text{price relative}) = \ln(S_{t+1} / S_t)$
- For Multiple Period:**
 $R_{0,T} = \ln(S_T / S_0)$
- Continuously compounded returns are always lower than associated holding period returns
- Offers a refined view of returns, especially valuable in financial modeling and analysis.

6. OTHER MAJOR RETURN MEASURES AND THEIR APPLICATIONS

Gross and Net Return

Gross Return: Total return minus direct trading expenses. It is used to evaluate investment skills.

Net Return: Gross return minus all managerial and administrative expenses. Net return reflects the actual return earned by investors.

Pre-Tax and After-Tax Nominal Return

There is different taxation for capital gains and income returns. Long-term capital gains typically receive favorable tax treatment

After-Tax Nominal Return: is calculated by subtracting taxes on dividends, interest, and realized gains from the total return.

Tax minimization strategies include selecting tax-favored securities and reducing trading turnover

Real Returns

Real returns:

- accounts for inflation and risk premium.
- provide a more accurate gauge of purchasing power

$$\text{Real Return} = (1 + r) = (1 + r_{rF}) \times (1 + \pi) \times (1 + RP)$$

After-Tax Real Return are useful for comparing returns across different periods and countries.

Leveraged Return

Amplifies underlying security returns through futures contracts or borrowing funds.

Magnifies returns but heightens risk. - Impact varies based on leverage method.