



PERFORMANCE MEASUREMENT

CFA[®] Program Curriculum
2027 • LEVEL III CORE • VOLUME 3

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How to Use the CFA Program Curriculum

The CFA® Program exams measure your mastery of the core knowledge, skills, and abilities required to succeed as an investment professional. These core competencies are the basis for the Candidate Body of Knowledge (CBOK™). The CBOK consists of four components:

A broad outline that lists the major CFA Program topic areas (www.cfainstitute.org/programs/cfa/curriculum/cbok/cbok)

Topic area weights that indicate the relative exam weightings of the top-level topic areas (www.cfainstitute.org/en/programs/cfa/curriculum)

Learning outcome statements (LOS) that tell you the specific knowledge, skills, and abilities you should gain from each curriculum topic area. You will find these statements at the start of each learning module and lesson. We encourage you to review the information about the LOS on our website (www.cfainstitute.org/programs/cfa/curriculum/study-sessions), including the descriptions of LOS “command words” on the candidate resources page at www.cfainstitute.org/-/media/documents/support/programs/cfa-and-cipm-los-command-words.ashx.

The CFA Program curriculum that candidates receive access to upon exam registration.

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The curriculum, including the practice questions, is the basis for all exam questions. The curriculum is selected/developed specifically to provide candidates with the knowledge, skills, and abilities reflected in the CBOK.

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Your exam registration fee includes access to the CFA Institute Learning Ecosystem (LES). This digital learning platform provides access to all the curriculum content and practice questions. The LES is organized as a series of learning modules consisting of short online lessons and associated practice questions. This tool is your source for all study materials, including practice questions and mock exams. The LES is the primary method by which CFA Institute delivers your curriculum experience. Here, you will find additional practice questions to test your knowledge, including some interactive questions.

DESIGNING YOUR PERSONAL STUDY PROGRAM

An orderly, systematic approach to exam preparation is critical. You should dedicate a consistent block of time every week to reading and studying. Review the LOS both before and after you study curriculum content to ensure you can demonstrate

the knowledge, skills, and abilities described by the LOS and the assigned learning module. Use the LOS as a self-check to track your progress and highlight areas of weakness for later review.

Successful candidates report an average of more than 300 hours preparing for each exam. Your preparation time will vary based on your prior education and experience, and you will likely spend more time on some topics than on others.

ERRATA

The curriculum development process is rigorous and involves multiple rounds of reviews by content experts. Despite our efforts to produce a curriculum that is free of errors, we must make corrections in some instances. Curriculum errata are periodically updated and posted by exam level and test date on the Curriculum Errata webpage (www.cfainstitute.org/en/programs/submit-errata). If you believe you have found an error in the curriculum, you can submit your concerns through our curriculum errata reporting process found at the bottom of the Curriculum Errata webpage.

OTHER FEEDBACK

Please send any comments or suggestions to info@cfainstitute.org, and we will review your feedback thoughtfully.

Performance Measurement

LEARNING MODULE

1

Portfolio Performance Evaluation

by Marc A. Wright, CFA, and Charles Mitchell Conover, CFA, CIPM.

Marc A. Wright, CFA, is at Russell Investments (USA). Charles Mitchell Conover, CFA, CIPM, is at the University of Richmond (USA).

LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain the following components of portfolio evaluation and their interrelationships: performance measurement, performance attribution, and performance appraisal
<input type="checkbox"/>	describe attributes of an effective attribution process
<input type="checkbox"/>	contrast return attribution and risk attribution; contrast macro and micro return attribution
<input type="checkbox"/>	describe returns-based, holdings-based, and transactions-based performance attribution, including advantages and disadvantages of each
<input type="checkbox"/>	interpret the sources of portfolio returns using a specified attribution approach
<input type="checkbox"/>	interpret the output from fixed-income attribution analyses
<input type="checkbox"/>	discuss considerations in selecting a risk attribution approach
<input type="checkbox"/>	identify and interpret investment results attributable to the asset owner versus those attributable to the investment manager
<input type="checkbox"/>	discuss uses of liability-based benchmarks
<input type="checkbox"/>	describe types of asset-based benchmarks
<input type="checkbox"/>	discuss tests of benchmark quality
<input type="checkbox"/>	describe the impact of benchmark misspecification on attribution and appraisal analysis
<input type="checkbox"/>	describe problems that arise in benchmarking alternative investments

LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	calculate and interpret the Sortino ratio, the appraisal ratio, upside/downside capture ratios, maximum drawdown, and drawdown duration
<input type="checkbox"/>	describe limitations of appraisal measures and related metrics
<input type="checkbox"/>	evaluate the skill of an investment manager

1

INTRODUCTION

Performance evaluation is one of the most critical areas of investment analysis. Performance results can be used to assess the quality of the investment approach and suggest changes that might improve it. They are also used to communicate the results of the investment process to other stakeholders and may even be used to compensate the investment managers. Therefore, it is of vital importance that practitioners who use these analyses understand how the results are generated. By gaining an understanding of the details of how these analyses work, practitioners will develop a greater understanding of the insights that might be gathered from the analysis and will also be cognizant of the limitations of those approaches, careful not to infer more than what is explicit or logically implicit in the results.

We will first consider the broad categories of performance measurement, attribution, and appraisal, differentiating between the three and explaining their interrelationships. Next, we will provide practitioners with tools to evaluate the effectiveness of those analyses as we summarize various approaches to performance evaluation. We will cover returns-based, holdings-based, and transactions-based attribution, addressing the merits and shortcomings of each approach and providing guidance on how to properly interpret attribution results. Again, by reviewing how each approach generates its results, we reveal strengths and weaknesses of the individual attribution approaches.

Next, we will turn to the subject of benchmarks and performance appraisal ratios. We will review the long-standing tests of benchmark quality and differentiate market indexes from benchmarks. We will also review different ratios used in performance appraisal, considering the benefits and limitations of each approach.

Lastly, we will provide advice on using these tools to collectively evaluate the skill of investment managers. This advice relies heavily on understanding the analysis tools, the limitations of the approaches, the importance of data to the quality of the analysis, and the pitfalls to avoid when making recommendations.

PERFORMANCE EVALUATION AND ATTRIBUTION

2

- explain the following components of portfolio evaluation and their interrelationships: performance measurement, performance attribution, and performance appraisal
- describe attributes of an effective attribution process
- contrast return attribution and risk attribution; contrast macro and micro return attribution
- describe returns-based, holdings-based, and transactions-based performance attribution, including advantages and disadvantages of each

Performance evaluation includes three primary components, each corresponding to a specific question we need to answer to evaluate a portfolio's performance:

- Performance measurement—what was the portfolio's performance?
- Performance attribution—how was the performance achieved?
- Performance appraisal—was the performance achieved through manager skill or luck?

We will consider each of these components on their own and the interrelationships between them.

Performance measurement provides an overall indication of the portfolio's performance, typically relative to a benchmark. In its simplest form, performance measurement is the calculation of investment returns for both the portfolio and its benchmark. This return calculation is a critical first step in the performance evaluation process, building the foundation on which performance evaluation is based. The investment return tells us what the portfolio achieved over a specific period, irrespective of peer or benchmark performance. For purposes of this reading, we will call this the *absolute return*. But it also provides the basis to understand the difference between the portfolio return and its benchmark return, the **excess return**.

In addition to return, performance measurement must consider the risk incurred to achieve that return. We measure risk using a variety of *ex post* (looking back in time) and *ex ante* (looking forward in time) techniques. For *ex post*, we might consider the volatility or standard deviation of the past returns, along with many other performance appraisal ratios considered later in this reading. The calculation of a portfolio's value at risk (VaR) at a point in time is an example of an *ex ante* measure. These measures of risk allow us to quantify the risk in a portfolio and better assess the performance.

Performance attribution then builds on the foundation of the investment returns and risk, helping us explain *how* that performance was achieved or that risk was incurred. Performance attribution can be used to explain either absolute returns or relative returns. It can be used to understand what portion of returns was driven by active manager decisions and what portion was a result of exposures not specifically targeted by the portfolio manager. Performance attribution can also be used to decompose the excess return into its component sources, where it is used to help explain why a manager over- or underperformed the target benchmark. Similarly, risk attribution can be used to decompose the risk incurred in the portfolio.

The third component of performance evaluation, performance appraisal, makes use of risk, return, and attribution analyses to draw conclusions regarding the *quality* of a portfolio's performance. Performance appraisal attempts to distinguish manager skill from luck. Did the portfolio manager's decisions help achieve a better outcome, or

was the outcome due to market changes outside of the manager's control? If superior results can be attributed to skill, there is a higher likelihood that the manager will generate superior performance in the future. The analysis may affirm the management process or may contain insights for improving the process. This is a key feedback loop in the investment management process.

EXAMPLE 1

Performance Evaluation

1. Performance attribution:

- A. measures the excess performance of a portfolio.
- B. explains the proportion of returns due to manager skill.
- C. explains how the excess performance or risk was achieved.

Solution:

C is correct. Performance attribution identifies the drivers of investment returns. A is not correct because measuring the excess performance of a portfolio is the subject of performance measurement. B is not correct because it is performance appraisal that distinguishes skill from luck.

2. Performance appraisal:

- A. identifies the sources of under- or outperformance.
- B. decomposes a portfolio's risk and return into their constituent parts.
- C. uses the results of risk, return, and attribution analyses to assess the quality of a portfolio's performance.

Solution:

C is correct. Performance appraisal combines all the techniques of performance measurement and attribution to assess the quality of performance. Both A and B describe performance attribution.

Performance Attribution

As previously described, performance attribution is a critical component of the portfolio evaluation process. Used by senior management, client relationship specialists, risk controllers, operations staff, portfolio managers, and sales and marketing professionals, attribution analysis provides important insights to the investment decision-making process. Clients and prospects also use attribution analysis as part of their evaluation of that process. Effective performance attribution analysis requires a thorough understanding of the investment decision-making process and should reflect the active decisions of the portfolio manager.

An effective performance attribution process must

- account for *all* of the portfolio's return or risk exposure,
- reflect the investment decision-making process,
- quantify the active decisions of the portfolio manager, and
- provide a complete understanding of the excess return/risk of the portfolio.

If the return or risk quantified by the attribution analysis does not account for all the return or risk presented to the client, then at best the attribution is incomplete and at worst the quality of the attribution analysis is brought into doubt. If the attribution does not reflect the investment decision-making process, then the analysis will be of little value to either the portfolio manager or the client. For example, if the portfolio manager is a genuine bottom-up stock picker who ignores sector benchmark weights, then measuring the impact of sector allocation against these weights is not measuring decisions made as part of the investment process; sector effects are merely a byproduct of the manager's investment decisions.

Performance attribution includes return attribution and risk attribution (although in practice, "performance attribution" is often used to mean "return attribution"). **Return attribution** analyzes the impact of active investment decisions on *returns*; **risk attribution** analyzes the *risk* consequences of those decisions. Depending on the purpose of the analysis, risk may be viewed in absolute or benchmark-relative terms. For example, when risk relative to a benchmark is the focus, a risk attribution analysis might identify and evaluate a portfolio's deviations from a benchmark's exposures to risk factors.

Performance attribution provides a good starting point for a conversation with clients, explaining both positive and negative aspects of recent performance. Return attribution analysis is particularly important when performance is weak; portfolio managers must demonstrate an understanding of their performance, provide a rationale for their decisions, and generate confidence in their ability to add value in the future. When it accurately reflects the investment decision-making process, return attribution provides quality control for the investment process and provides senior management with a tool to manage a complex business with multiple investment strategies.

The attribution process described earlier—understanding the drivers of a manager's returns and whether those drivers are consistent with the stated investment process—is a common application of attribution analysis. But attribution can also be conducted to evaluate the asset owner's tactical asset allocation and manager selection decisions (called **macro attribution**) or to evaluate the impact of the portfolio manager's decisions on the performance of the asset owner's total fund (called **micro attribution**). A defined-benefit pension plan makes the decision to allocate a given percentage of the fund to each asset class and decides which manager(s) to hire for each asset class. Macro attribution measures the effect of the sponsor's choice to deviate from the strategic asset allocation, including the effect of "gaps" between the strategic asset allocation and its implementation (e.g., where the sum of the managers' benchmarks is equal to something other than the benchmark index).

Micro attribution measures the impact of portfolio managers' allocation and selection decisions on total fund performance.

Performance attribution may be either returns based, holdings based, or transactions based. The decision to use one set of inputs rather than another depends on the availability of data as well as the investment process being measured.

Returns-based attribution uses only the total portfolio returns over a period to identify the components of the investment process that have generated the returns. Returns-based attribution is most appropriate when the underlying portfolio holding information is not available with sufficient frequency at the required level of detail. For example, one might use returns-based attribution to evaluate hedge funds, because it can be difficult to obtain the underlying holdings of hedge funds. Returns-based attribution is the easiest method to implement, but because it does not use the underlying holdings, it is the least accurate of the three approaches and the most vulnerable to data manipulation.

Unlike returns-based attribution, **holdings-based attribution** references the beginning-of-period holdings of the portfolio. Calculated with monthly, weekly, or daily data, the accuracy of holdings-based attribution improves when using data with

shorter time intervals. For longer evaluation periods, we link together the attribution results for the shorter measurement periods. Because holdings-based attribution fails to capture the impact of any transactions made during the measurement period, it may not reconcile to the actual portfolio return. For example, in a daily holdings-based attribution, securities are included at the end of the day they are purchased and excluded at the end of the day they are sold. If the transaction price is significantly different from the closing price, the attribution analysis can differ significantly from the actual performance.

The residual caused by ignoring transactions might be described as a timing or trading effect. Holdings-based analysis is most appropriate for investment strategies with little turnover (e.g., passive strategies). Holdings-based analysis may be improved by valuing the portfolio with the same prices used to calculate the underlying benchmark index, removing one potential difference between the portfolio and benchmark returns that is not a management effect.

The third approach, **transactions-based attribution**, uses both the holdings of the portfolio and the transactions (purchases and sales) that occurred during the evaluation period. For transaction-based attribution, both the weights and returns reflect *all transactions* during the period, including transaction costs. Transaction-based attribution is the most accurate type of attribution analysis but also the most difficult and time-consuming to implement. To obtain meaningful results, the underlying data must be complete, accurate, and reconciled from period to period. Because all the data are available, the entire excess return can be quantified and explained. The return used in the attribution analysis will reconcile with the return presented to the client, and attribution analysis can be used as a diagnostic tool to identify errors.

The choice of attribution approach depends on the availability and quality of the underlying data, the reporting requirements for the client, and the complexity of the investment decision-making process.

EXAMPLE 2

Performance Attribution

1. Effective attribution analysis must:

- A. use intraday transaction data.
- B. reconcile to the total portfolio return or risk exposure.
- C. measure the contribution of security and sector selection decisions.

Solution:

B is correct. An effective attribution process accounts for all of the portfolio's return or risk exposure. A is not correct; an attribution analysis is improved with intraday transaction data, but an effective attribution analysis can be produced with a returns- or holdings-based approach. C is not correct because an attribution process that measures the sector selection effects of a bottom-up stock-picker does not measure the effectiveness of the investment decision-making process.

2. Which of the following most accurately describes macro attribution?

- A. Attribution analysis at the portfolio level
- B. Attribution analysis of the fund sponsor decisions
- C. Attribution analysis of asset allocation decisions

Solution:

B is correct. Macro attribution measures the effect of the sponsor's choice to deviate from the strategic asset allocation and the sponsor's manager selection decisions. A is not correct because attribution analysis at the portfolio level may be either macro attribution or micro attribution. C is not correct because macro attribution measures both asset allocation and manager selection decisions of the asset owner.

3. Risk attribution differs from return attribution in that it:

- A. is not conducted relative to a benchmark.
- B. quantifies the risk consequences of the investment decisions.
- C. quantifies the investment decisions of the investment manager.

Solution:

B is correct. Risk attribution, unlike return attribution, attempts to quantify the risk consequences of the investment decisions. A is not correct because risk attribution may be conducted on either an absolute or a relative basis. C is not correct because risk attribution does not capture the return impact of a manager's investment decisions.

4. An analyst is *most likely* to use returns-based attribution when:

- A. the portfolio has a low turnover.
- B. the holdings for the portfolio are not available.
- C. she wants the analysis to be as accurate as possible.

Solution:

B is correct. Returns-based attribution is typically used when the holdings data are not available. Neither A nor C is correct because returns-based attribution is the least accurate of the three approaches.

RETURN ATTRIBUTION

3

- interpret the sources of portfolio returns using a specified attribution approach

Return attribution allows us to look across a specific time horizon and identify which investment decisions have either added value to or detracted value from the portfolio, relative to its benchmark. As feedback to the portfolio management process, return attribution quantifies the active decisions of portfolio managers and informs management and clients. In this way, return attribution can be thought of as “backward looking” or *ex post*, meaning that it is used to evaluate the investment decisions for some historical time horizon.

Return attribution is a set of techniques used to identify the sources of excess return of a portfolio against its benchmark, quantifying the consequences of active investment decisions.

Specific return attribution approaches have been designed to evaluate particular types of assets. In this section, we will consider two common approaches for equity attribution: Brinson–Fachler and factor-based attribution. We will also review the output and findings from a typical fixed-income attribution approach.

Practitioners may also encounter the concept of geometric attribution and arithmetic attribution, two approaches to measuring attribution effects over longer periods. **Arithmetic attribution** approaches are designed to explain the **excess return**, the arithmetic difference between the portfolio return, R , and its benchmark return, B .

When using an arithmetic attribution approach, the attribution effects will sum to the excess return. Arithmetic approaches are straightforward for a single period, for which there is no difference between the sum of the attribution effects and the excess return. However, when combining multiple periods, the sub-period attribution effects will *not* sum to the excess return. Because the excess return is calculated by *geometrically* linking the sub-period returns, adjustments must be made to “smooth” the *arithmetic* sub-period attribution effects over time. Multiple smoothing approaches exist in the industry, including algorithms suggested by David Cariño (1999) and Jose Menchero (2000).

Geometric attribution approaches extend the arithmetic approaches by attributing the geometric excess return (G), as defined below:

$$G = \frac{1+R}{1+B} - 1 = \frac{R-B}{1+B}$$

Note that the geometric excess return is simply the arithmetic excess return divided by the wealth ratio of the benchmark (1 plus the return on the benchmark during the period).

In a geometric attribution approach, the attribution effects will compound (multiply) together to total the geometric excess return. Because the attribution effects compound together to exactly equal the geometric excess return, the compounding works across multiple periods. Therefore, no smoothing is required to adjust the geometric attribution effects across multiple periods.

Practitioners typically choose arithmetic attribution approaches when they want to use the attribution analysis with non-practitioner clients or in marketing reports. With results that add up to the total excess return for all periods, arithmetic approaches are more intuitively understood. Geometric approaches tend to be limited to practitioners who understand the approach and who appreciate that they do not have to adjust the attribution effects over time.

A Simple Return Attribution Example

Suppose a portfolio’s return for the past year was 5.24% and the portfolio’s benchmark return for that same period was 3.24%. In this case, the portfolio achieved a positive arithmetic excess return of 2.00% ($5.24\% - 3.24\% = 2.00\%$) over the past year.

To understand how the 2.00% was achieved, we apply return attribution. In this example, return attribution will quantify two typical sources of excess return: *security selection* and *asset allocation*. Security selection answers the question, Was the return achieved by selecting securities that performed well relative to the benchmark or by avoiding benchmark securities that performed relatively poorly? Asset allocation answers the question, Was the return achieved by choosing to overweight an asset category (e.g., economic sector or currency) that outperformed the total benchmark or to underweight an asset category that underperformed the total benchmark? (The term “allocation” is used somewhat differently here. It is not measuring the plan sponsor’s asset allocation decision but, rather, the *manager’s* decision to allocate among countries, sectors, or, in cases where the manager has a broad mandate, asset classes.)

Models of equity return attribution often attempt to separate the investment process into those two key decisions—selection and allocation—assigning each a magnitude and direction (plus or minus) for both decisions. For instance, for the portfolio referenced previously, we might calculate the return attribution results shown in Exhibit 1:

Exhibit 1: Total Portfolio Return Attribution Analysis (Time Period: Past 12 Months)

Portfolio Return	Benchmark Return	Excess Return	Allocation Effect	Selection Effect
5.24%	3.24%	2.00%	-0.50%	2.50%

As we noted, the investment decisions generated a positive excess return of 200 basis points (bps) relative to the benchmark. We use the “return attribution analysis” to see how this 200 bps was generated. First, note that the *negative* allocation effect indicates that the allocation decisions over the past 12 months, whatever they were, had a negative impact on the total portfolio performance. They *subtracted* 50 bps from the excess return. In contrast, the *positive* selection effect indicates that the security selection decisions—decisions to overweight or underweight securities relative to their benchmark weights—*added* 250 bps to the excess return. Our return attribution analysis implies that the portfolio manager’s security selection decision was far superior to his or her asset allocation decision for the past 12 months.

Equity Return Attribution—The Brinson–Hood–Beebower Model

The foundations of return attribution were established in two articles, one written by Brinson and Fachler (1985) and the other by Brinson, Hood, and Beebower (1986). The Brinson–Fachler model is more widely used in performance attribution today, but we introduce the Brinson–Hood–Beebower (BHB) model first to lay an important foundation.

BHB is built on the assumption that the total portfolio and benchmark returns are calculated by summing the weights and returns of the sectors within the portfolio (Equation 1) and the benchmark (Equation 2):

$$\text{Portfolio return } R = \sum_{i=1}^{i=n} w_i R_i \quad (1)$$

$$\text{Benchmark return } B = \sum_{i=1}^{i=n} W_i B_i \quad (2)$$

where

w_i = weight of the i th sector in the portfolio

R_i = return of the portfolio assets in the i th sector

W_i = weight of the i th sector in the benchmark

B_i = return of the benchmark in the i th sector

n = number of sectors or securities

The sum of the weights in both the portfolio and the benchmark must equal 100%. The presence of leverage would require a position with a negative weight (borrowings or short positions) to balance to 100%.

Attribution analysis quantifies each of the portfolio manager's active decisions that explain the difference between the portfolio return, R , and the benchmark return, B . Note that for this example, we are concerned with only single-period, single-currency return attribution models.

Exhibit 2 provides data for a three-sector domestic equity portfolio, used to illustrate the BHB model.

Exhibit 2: BHB Model Illustration—Portfolio and Benchmark Data

Sector	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Energy	50%	50%	18%	10%
Health care	30%	20%	-3%	-2%
Financials	20%	30%	10%	12%
Total	100%	100%	10.1%	8.2%

Total portfolio return $R = (50\% \times 18\%) + (30\% \times -3\%) + (20\% \times 10\%) = 10.1\%$

Total benchmark return $B = (50\% \times 10\%) + (20\% \times -2\%) + (30\% \times 12\%) = 8.2\%$

Thus, the excess return is 1.9% ($10.1\% - 8.2\% = 1.9\%$), or 190 bps.

We will use the weights and returns data shown in Exhibit 2 to calculate the basic attribution effects using the BHB model, including the allocation effect, the security selection effect, and the interaction effect. The allocation effect refers to the value the portfolio manager adds (or subtracts) by having portfolio sector weights that are different from the benchmark sector weights. A sector weight in the portfolio greater than the benchmark sector weight would be described as *overweight*, and a sector weight less than the benchmark sector weight would be described as *underweight*.

To calculate allocation, we first calculate the contribution to allocation (A_i) for each sector. The contribution to allocation in the i th sector is equal to the portfolio's sector weight minus the benchmark's sector weight, times the benchmark sector return:

$$A_i = (w_i - W_i)B_i \quad (3)$$

Using the data from Exhibit 2, we calculate individual sector allocation effects as follows:

- Energy: $(50\% - 50\%) \times 10\% = 0.0\%$
- Health care: $(30\% - 20\%) \times -2.0\% = -0.2\%$
- Financials: $(20\% - 30\%) \times 12\% = -1.2\%$

To find the total portfolio allocation effect, A , we sum the individual sector contributions to allocation:

$$A = \sum_{i=1}^{i=n} A_i \quad (4)$$

Total allocation effect = $0.0\% - 0.2\% - 1.2\% = -1.4\%$

We can then use the results to state the following conclusions:

- The portfolio weight in the energy sector is equal to the benchmark weight; therefore, there is no contribution to allocation in energy.

- In health care, the portfolio manager held a higher weight than the benchmark (30% versus 20%), but the sector underperformed the aggregate benchmark (−2.0% versus 8.2%). Therefore, the decision to overweight health care lowered the overall excess return; the contribution to allocation is −0.2%.
- In financials, the portfolio manager chose to underweight versus the benchmark (20% versus 30%). But because financials outperformed the aggregate benchmark (12% versus 8.2%), the decision to underweight financials also lowered the overall excess return; the contribution to allocation is −1.2%.
- Overall, the combined allocation effect for this portfolio was −1.4%, demonstrating that the weighting decisions negatively contributed to the performance of the portfolio.

The other attribution effect in the BHB model is security selection—the value the portfolio manager adds by holding individual securities or instruments within the sector in different-from-benchmark weights.

To calculate selection, we first calculate the contribution to selection (S_i) for each sector. The contribution to selection in the i th sector is equal to the benchmark sector weight times the portfolio's sector return minus the benchmark's sector return.

$$S_i = W_i(R_i - B_i) \quad (5)$$

Using the data from Exhibit 2, we calculate individual sector selection effects as follows:

- Energy: $50\% \times (18\% - 10\%) = 4.0\%$
- Health care: $20\% \times (-3\% - (-2.0\%)) = -0.2\%$
- Financials: $30\% \times (10\% - 12\%) = -0.6\%$

To find the total portfolio selection effect, S , we sum the individual sector contributions to selection:

$$S = \sum_{i=1}^{i=n} S_i \quad (6)$$

$$\text{Total selection effect} = 4.0\% + (-0.2\%) + (-0.6\%) = 3.2\%$$

We can use the results to state the following conclusions:

- The portfolio's energy sector outperformed the benchmark's energy sector by 800 bps (18% − 10%); 800 bps times the benchmark weight of 50% for this sector results in a 4.0% contribution to selection.
- The portfolio's health care sector underperformed the benchmark's health care sector by 100 bps [(-3%) − (-2%)]; 100 bps times the benchmark weight of 20% for this sector results in a contribution of −0.2%.
- The portfolio's financials sector underperformed the benchmark's financials sector by 200 bps (10% − 12%); 200 bps times the benchmark weight of 30% to this sector results in a contribution of −0.6%.
- Overall, the combined selection effect for this portfolio was 3.2%.

In the BHB model, selection and allocation do not completely explain the arithmetic difference. For example, in the attribution analysis based on Exhibit 2, allocation (−1.4%) and selection (3.2%) together represent just 1.8% of the arithmetic difference between the portfolio return of 10.1% and the benchmark return of 8.2%; 0.1% is missing. To explain this remaining difference in the excess return, the BHB model uses a third attribution effect, called “interaction.” The **interaction effect** is the effect resulting from the interaction of the allocation and selection decisions combined.

To calculate interaction, we first calculate the contribution to interaction for each sector. The contribution to interaction in the i th sector is equal to the portfolio sector weight minus the benchmark sector weight, times the portfolio sector return minus the benchmark sector return:

$$I_i = (w_i - W_i)(R_i - B_i) \quad (7)$$

Using the data from Exhibit 2, we calculate individual sector selection effects as follows:

- Energy: $(50\% - 50\%) \times (18\% - 10\%) = 0.0\%$
- Health care: $(30\% - 20\%) \times (-3\% - -2.0\%) = -0.1\%$
- Financials: $(20\% - 30\%) \times (10\% - 12\%) = 0.2\%$

To find the total portfolio interaction effect, we sum the individual sector contributions to interaction:

$$I = \sum_{i=1}^{i=n} I_i \quad (8)$$

$$\text{Total interaction effect} = 0.0\% + -0.1\% + 0.2\% = 0.1\%$$

We can use the results to state the following conclusions:

- For the energy sector, the portfolio weight equals the benchmark weight and thus there is no contribution to interaction.
- Because the manager had an overweight to a sector in which selection was negative, the contribution from interaction in health care was also negative, -0.1% .
- In the financials sector, the manager was underweight by 10% and selection was negative. The effect of being underweight in a sector in which the manager underperforms leads to a contribution from interaction of $+0.2\%$.
- Total contribution from interaction is $+0.1\%$, representing the combined effect of the interaction of the selection and allocation effects.

EXAMPLE 3

Interpreting the Results of a BHB Attribution

BHB Attribution Analysis Results Table

Region	Portfolio Return	Benchmark Return	Portfolio Weight	Benchmark Weight	Allocation	Selection	Inter-action	Total
Americas	2.80%	1.20%	30%	30%	0.00%	0.48%	0.00%	0.48%
APAC	-1.50%	-0.50%	20%	30%	0.05%	-0.30%	0.10%	-0.15%
EMEA	0.70%	1.50%	50%	40%	0.15%	-0.32%	-0.08%	-0.25%
Total	0.89%	0.81%	100%	100%	0.20%	-0.14%	0.02%	0.08%

Use the table above to answer the following questions.

1. Why is the contribution to selection for Europe, the Middle East, and Africa (EMEA) negative?
 - A. The total benchmark return is less than the total portfolio return.
 - B. The manager selected securities in EMEA that underperformed the benchmark.

- C. The manager underweighted an outperforming sector.

Solution:

B is correct. The manager selected securities that underperformed the benchmark, with a portfolio return for EMEA of 0.7% versus a benchmark return for EMEA of 1.5%.

2. Why is the contribution to allocation for Asia Pacific (APAC) equal to +5 bps?

- A. The benchmark weight and the portfolio weight are equal.
 B. The manager has an overweight position in an overperforming region.
 C. The manager has an underweight position in an underperforming region.

Solution:

C is correct. The manager is underweight in APAC, 20% versus a benchmark weight of 30%. The APAC portion of the portfolio underperformed, with a -0.50% benchmark return versus the total benchmark return of 0.81% .

3. Which of the following conclusions from the above attribution analysis is *most* correct?

- A. The manager's security selection decisions were better in the Americas than in APAC.
 B. The manager's security selection decisions were better in EMEA than in APAC.
 C. The manager's allocation decisions were better in APAC than in EMEA.

Solution:

A is correct. As reflected in the contribution to selection, the manager's security selection decisions were better in the Americas (0.48%) than in APAC (-0.30%).

4. Which of the following conclusions from the above attribution analysis is *most* correct?

- A. Overall, the manager made better allocation decisions than selection decisions.
 B. Overall, the manager made better selection decisions than allocation decisions.
 C. Contribution from interaction was most noticeable in the Americas.

Solution:

A is correct. Overall, the manager made better allocation decisions (0.20%) than selection decisions (-0.14%).

Brinson–Fachler Model

The Brinson–Fachler (BF) model differs from the BHB model only in how individual sector allocation effects are calculated.

In the BHB model, all overweight positions in sectors with positive returns will generate positive allocation effects irrespective of the overall benchmark return, whereas all overweight positions in negative markets will generate negative allocation effects. Thus, overweighting a sector i that earns a positive return, $B_i > 0$, results in a positive allocation effect, $A_i = (w_i - W_i)B_i > 0$, even when the sector return is less than the overall benchmark return (i.e., $B_i < B$). When the sector return is negative, $0 > B_i$, overweighting produces a negative allocation effect, $A_i = (w_i - W_i)B_i < 0$.

Clearly, if the portfolio manager is overweight in a negative market that has outperformed the overall benchmark, the effect should be positive.

The BF model solves this problem by modifying the asset allocation factor to compare returns with the overall benchmark as follows:

$$B_S - B = \sum_{i=1}^{i=n} (w_i - W_i) B_i = \sum_{i=1}^{i=n} (w_i - W_i) (B_i - B) \quad (9)$$

Because $\sum_{i=1}^{i=n} w_i = \sum_{i=1}^{i=n} W_i = 1$, the constant B can be introduced. The contribution to asset allocation in the i th sector is now:

$$A_i = (w_i - W_i) (B_i - B) \quad (10)$$

Note that in Equation 10, the allocation effect at the portfolio level, $B_S - B$, is unchanged from the BHB model.

The contribution to arithmetic excess return from sector allocation for the portfolio data shown in Exhibit 2 is $B_S - B = 6.8\% - 8.2\% = -1.4\%$. Revised BF sector allocation effects are calculated for the portfolio data in Exhibit 2 as follows, using $A_i = (w_i - W_i)(B_i - B)$:

Energy	$(50\% - 50\%) \times (10\% - 8.2\%) = 0.0\%$
Health care	$(30\% - 20\%) \times (-2.0\% - 8.2\%) = -1.02\%$
Financials	$(20\% - 30\%) \times (12\% - 8.2\%) = -0.38\%$
Total	$0.0\% - 1.02\% - 0.38\% = -1.4\%$

The impact in health care is much greater. In addition to being overweight in a negative market, which costs -0.2% , the portfolio manager is correctly penalized the opportunity cost of not being invested in the overall market return of 8.2% , generating a further cost of $10\% \times -8.2\% = -0.82\%$ and resulting in a total impact of -1.02% . To describe it another way, the portfolio is 10% overweight in a market that is underperforming the overall market by -10.2% (i.e., $-2.0\% - 8.2\%$) and generating a loss of -1.02% .

The impact in financials is much smaller. Although being underweight in a positive market cost -1.2% , we must add back the opportunity cost of being invested in the overall market return of 8.2% , generating a contribution of $-10\% \times -8.2\% = 0.82\%$ and resulting in a total impact of -0.38% . To describe it another way, the portfolio is 10% underweight in an industry that is outperforming the overall market by 3.8% (i.e., $12.0\% - 8.2\%$), generating a loss of -0.38% . As expected, at the portfolio level, the allocation effect of -1.4% remains the same as that calculated with the BHB model.

The revised attribution effects are summarized in Exhibit 3.

Exhibit 3: BF Return Attribution Results

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection	Interaction
Energy	50%	50%	18%	10%	0.0%	4.0%	0.0%
Health care	30%	20%	-3%	-2%	-1.02%	-0.2%	-0.1%

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation	Selection	Interaction
Financials	20%	30%	10%	12%	-0.38%	-0.6%	0.2%
Total	100%	100%	10.1%	8.2%	-1.4%	3.2%	0.1%

EXAMPLE 4**Allocation Using the BF Model****Exhibit 4: Sample Portfolio Data**

	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
Technology	20%	30%	-11.0%	-10.0%
Telecommunications	30%	40%	-5.0%	-8.0%
Utilities	50%	30%	-8.0%	-5.0%
Total	100%	100%	-7.7%	-7.7%

1. Using the BF method, the allocation effect of utilities based on the portfolio data in Exhibit 4 is:

- A. -1.50%.
- B. 0.54%.
- C. 1.35%.

Solution:

B is correct: $(w_i - W_i)(B_i - B) = (50\% - 30\%)(-5.0\% + 7.7\%) = 0.54\%$. The portfolio is 20% overweight in a sector outperforming the overall benchmark by 2.7%, therefore contributing 0.54% to the overall allocation effect.

A is incorrect: $W_i B_i = 30\% \times -5.0\% = -1.5\%$ is the contribution to the benchmark return from utilities.

C is incorrect: $w_i(B_i - B) = 50\% \times (-5.0\% + 7.7\%) = +1.35\%$. Only the portfolio weight of 50% has been used, not the overweight position of 20%.

FACTOR-BASED AND FIXED-INCOME RETURN ATTRIBUTION

4

- interpret the sources of portfolio returns using a specified attribution approach
- interpret the output from fixed-income attribution analyses

As we have seen, return attribution allows us to analyze a portfolio's excess return by comparing the accounting information (weights and returns) in the portfolio with the information in the benchmark. The Brinson–Fachler model focuses on security selection, asset allocation, and the interaction of selection and allocation. But what if we want to assess other decisions within the investment process?

Another type of return attribution uses fundamental factor models to decompose the contributions to excess return from *factors*. Fundamental factor analysis allows us to quantify the impact of specific active investment decisions within the portfolio, showing how they add or remove value relative to the benchmark. We want to remove the effects of the market to identify the excess return generated by the active investment decisions. To do that, we return to our definition of excess return: Excess return = $R - B$.

Many different factor models can be used to decompose excess returns. The choice of factor model is driven by which aspects of the investment process you want to measure. One of the factor models commonly used in equity attribution analyses is the Carhart four-factor model, or simply the **Carhart model**, given in Equation 11 (Carhart 1997). The Carhart model explains the excess return on the portfolio in terms of the portfolio's sensitivity to a market index (RMRF), a market-capitalization factor (SMB), a book-value-to-price factor (HML), and a momentum factor (WML).

$$R_p - R_f = a_p + b_{p1}RMRF + b_{p2}SMB + b_{p3}HML + b_{p4}WML + E_p \quad (11)$$

where

R_p and R_f = the return on the portfolio and the risk-free rate of return, respectively

a_p = “alpha” or return in excess of that expected given the portfolio's level of systematic risk (assuming the four factors capture all systematic risk)

b_p = the sensitivity of the portfolio to the given factor

RMRF = the return on a value-weighted equity index in excess of the one-month T-bill rate

SMB = small minus big, a size (market-capitalization) factor (SMB is the average return on three small-cap portfolios minus the average return on three large-cap portfolios)

HML = high minus low, a value factor (HML is the average return on two high-book-to-market portfolios minus the average return on two low-book-to-market portfolios)

WML = winners minus losers, a momentum factor (WML is the return on a portfolio of the past year's winners minus the return on a portfolio of the past year's losers)

E_p = an error term that represents the portion of the return to the portfolio, p , not explained by the model

By analyzing the results of a factor return attribution analysis, we can identify the investment approach and infer the relative strengths and/or weaknesses of the investment decisions. For example, using the Carhart factor model, we calculate the following results for a hypothetical manager.

Exhibit 5: Sample Carhart Factor Model Attribution

Factor	Factor Sensitivity				Contribution to Active Return	
	Portfolio	Benchmark	Difference	Factor Return	Absolute	Proportion of Total
	(1)	(2)	(3)	(4)	(3) × (4)	Active
RMRF	0.95	1.00	-0.05	5.52%	-0.28%	-13.30%
SMB	-1.05	-1.00	-0.05	-3.35%	0.17%	8.10%
HML	0.40	0.00	0.40	5.10%	2.04%	98.40%
WML	0.05	0.03	0.02	9.63%	0.19%	9.30%
			A. Factor tilts return =		2.12%	102.40%
			B. Security selection =		-0.05%	-2.40%
			C. Active return (A + B) =		2.07%	100.00%

This attribution analysis yields information about this portfolio's investment approach, how the manager generated excess return, and his or her ability to consistently add value relative to the benchmark.

Let's first look at the analysis of the benchmark (column 2). The sensitivity to RMRF of 1 indicates that the assigned benchmark has average market risk, consistent with it being a broad-based index. The benchmark's negative sensitivity to SMB indicates a large-cap orientation. Assuming, of course, that the benchmark is a good fit for the manager's stated strategy, we can describe the approach as large cap without a value/growth bias (HML is zero) or a momentum bias (WML is close to zero).

Let's now look at where the portfolio manager's approach differed from that of the benchmark. Based on the factor sensitivities shown in column 1 (positive sensitivity to HML of 0.40) and the differences relative to the benchmark shown in column 3, we can see that the manager likely had a value tilt but was otherwise relatively neutral to the benchmark. We would expect the portfolio to hold more value-oriented stocks than the benchmark, and we would want to evaluate the contribution of this tilt.

We can examine the effects of this decision by looking at the balance of the table. Positive active exposure to the HML factor—the bet on value stocks—contributed 204 bps to the realized active return, about 98% of the 207 bps of total realized active return. The manager's minor active exposures to small stocks and momentum also contributed positively to return, whereas the active exposure to RMRF was a drag on performance. However, because the magnitudes of the exposures to RMRF, SMB, and WML were relatively small, the effects of those bets were minor compared with the value tilt (HML).

What about the manager's ability to contribute return through stock selection? Again, assuming that the benchmark is a good fit for the manager's investment process, the overall active return from security selection is the portion of return not explained by factor sensitivities. In this period, the contribution from selection was slightly negative (-0.05%).

In the aggregate, the manager's positive active return was largely the result of the large active bet on HML (+0.40) and a high return to that factor during the period (+5.10%). Is this type of tilt consistent with the manager's stated investment process? If yes, the manager can be credited with an active decision that contributed positively to return. If no, then the excess return in the period is unlikely to result from manager skill but, rather, is a byproduct of luck. What does the manager's investment process say about the role of security selection? If the manager does not profess skill in

security selection but instead focuses on sector or factor allocation, then the minimal contribution of security selection should not be perceived as a negative reflection on manager skill.

EXAMPLE 5

Factor-Based Attribution

Use the data from Exhibit 5 to answer the following questions.

1. Which of the following statements is *not* correct?
 - A. The manager's slight small-cap tilt contributed positively to return.
 - B. The manager's slight momentum tilt contributed positively to return.
 - C. The manager's below-benchmark beta contributed negatively to return.

Solution:

A is the correct answer. The negative coefficient on SMB indicates that the manager had a slight large-cap bias relative to the benchmark. The slight tilt on WML (+0.02) combined with a positive return to the factor resulted in a positive contribution to return. The below-benchmark beta of RMRF (−0.05) combined with a positive return to the factor resulted in a negative contribution to return.

2. What investment approach, not taken by the portfolio manager, could have delivered more value to the portfolio during the investment period?
 - A. A momentum-based approach
 - B. A growth-oriented approach
 - C. A small-cap-based approach

Solution:

A is correct. Had the manager overweighted momentum stocks during the period, the momentum factor (WML) return of 9.63% would have contributed significant positive performance to the portfolio.

Fixed-Income Return Attribution

Fixed-income portfolios are driven by decisions made with respect to credit risk and positioning along the yield curve. Building on work by Groupe de Reflexion en Attribution de Performance, or GRAP, outlined in Giguère (2005) and Murira and Sierra (2006), we will discuss three typical approaches to fixed-income attribution:

- Exposure decomposition—duration based
- Yield curve decomposition—duration based
- Yield curve decomposition—full repricing based

Candidates are not responsible for *calculating* fixed-income attribution but should be able to interpret the results of a fixed-income attribution analysis.

Exposure Decomposition—Duration Based

Exposure decomposition is a top-down attribution approach that seeks to explain the active management of a portfolio relative to its benchmark, typically working through a hierarchy of decisions from the top to the bottom. These decisions might include portfolio duration bets, yield curve positioning or sector bets, each relative to the benchmark. The term “exposure decomposition” relates to the decomposition of portfolio risk exposures by means of grouping a portfolio’s component bonds by specified characteristics (e.g., duration, bond sector). The term “duration based” relates to the typical use of duration to represent interest rate exposure decisions.

Models that take an exposure decomposition approach are similar to Brinson-type equity attribution models, where we might group the portfolio by its market value weights in different economic sectors. In this case, however, we group the portfolio by its market value weights in duration buckets (i.e., exposure to different ranges of duration). This approach simplifies the data requirements and allows straightforward presentation of results relative to other fixed-income approaches. For these reasons, the exposure decomposition approach is used primarily for marketing and client reports, where an important benefit is that users can easily understand and articulate the results of active portfolio management.

Yield Curve Decomposition—Duration Based

The duration-based yield curve decomposition approach to fixed-income attribution can be either executed as a top-down approach or built bottom-up from the security level. This approach estimates the return of securities, sector buckets, or years-to-maturity buckets using the known relationship between duration and changes in yield to maturity (YTM), as follows:

$$\% \text{ Total return} = \% \text{ Income return} + \% \text{ Price return},$$

where $\% \text{ Price return} \approx -\text{Duration} \times \text{Change in YTM}$.

Duration measures the sensitivity of bond price to a change in the bond’s yield to maturity. So, the percentage price return of a bond will be approximately equal to the negative of its duration for each 100 bp change in yields. The change in yield to maturity of the portfolio or instrument can be broken down into yield curve factors and spread factors to provide additional insights. These factors represent the changes in the risk-free government curve (e.g., changes in level, slope, and curvature) and in the premium required to hold riskier sectors and bonds. When they are combined and applied to the duration, we can determine a percentage price change for each factor.

For example, a manager may have a view as to how the yield curve factors will change over time. We can use the attribution analysis to determine the value of the yield curve views as they unfold over time.

This approach is applied to both the portfolio and the benchmark to identify contributions to total return from changes in the yield to maturity. Comparing the differences between the benchmark’s return drivers and the portfolio’s return drivers gives us the *effect of active portfolio management decisions*.

In this regard, this group of models is quite different from the exposure decomposition. One consequence of this difference is that we require more data points to calculate the separate absolute attribution analyses for the portfolio and the benchmark. Thus, the yield decomposition approach exchanges better transparency for more operational complexity. These models are typically used when preparing reports for analysts and portfolio managers, rather than in marketing or client reports.

Yield Curve Decomposition—Full Repricing

Instead of estimating price changes from changes in duration and yields to maturity, bonds can be repriced from zero-coupon curves (spot rates). Recall that a bond’s price is the sum of its cash flows discounted at the appropriate spot rate for each cash

flow's maturity. The discount rate to compute the present value depends on the yields offered on the market for comparable securities and represents the required yield an investor expects for holding that investment. Typically, we discount each cash flow at a rate from the spot curve that corresponds to the time the cash flow will be received.

As with the duration-based approaches, instruments can be repriced following incremental changes in spot rates, whether resulting from changes in overall interest rates, spreads, or bond-specific factors. This bottom-up security-level repricing can then be translated into a contribution to a security's return and aggregated for portfolios, benchmarks, and active management.

This full repricing attribution approach provides more precise pricing and allows for a broader range of instrument types and yield changes. It also supports a greater variety of quantitative modeling beyond fixed-income attribution (e.g., *ex ante* risk). This approach is better aligned with how portfolio managers typically view the instruments. However, it requires the full capability to reprice all financial instruments in the portfolio and the benchmark, including the rates and the characteristics of the instrument. Its complex nature can make it more difficult and costly to administer operationally and can make the results more difficult to understand, particularly for non-fixed-income professionals.

All three approaches can be applied to single-currency and multi-currency portfolios. We can most clearly demonstrate the principles of fixed-income attribution by using a single-currency domestic portfolio, without digressing into the relative merits of the various multi-currency approaches. Therefore, this example is a single-currency example.

Fixed-Income Attribution—Worked Example

Let's begin with an example of exposure decomposition analysis.

Exhibit 6 shows a breakdown of the portfolio and the benchmark by weights, duration, and each bucket's contribution to duration, aggregated by sector and duration buckets. For this example, the short-, mid-, and long-duration buckets are defined as follows:¹

Bucket	Duration
Short	Less than or equal to 5
Mid	Greater than 5 and less than or equal to 10
Long	Greater than 10

¹ Note that the practitioner should take care when selecting the upper and lower bands of each duration bucket. By grouping bonds of different durations in the same bucket, one is measuring the combined impact of those bonds relative to the combined impact of similar bonds in the benchmark. In this example (Exhibit 6 and the related discussion), for instance, a bond with a duration of 5.5 is treated the same as a bond with a duration of 9.5 in terms of its relative impact on the portfolio versus its benchmark.