Last Revised: 07/25/2025

2026 Level 2 - Derivatives

Pricing and Valuation of Forward Commitments

a. describe the carry arbitrage model without underlying cashflows and with underlying cashflows

b. describe how equity forwards and futures are priced, and calculate and interpret their no-arbitrage value

c. describe how interest rate forwards and futures are priced, and calculate and interpret their no-arbitrage value

d. describe how fixed-income forwards and futures are priced, and calculate and interpret their no-arbitrage value

e. describe how interest rate swaps are priced, and calculate and interpret their no-arbitrage value

f. describe how currency swaps are priced, and calculate and interpret their no-arbitrage value

g. describe how equity swaps are priced, and calculate and interpret their no-arbitrage value

LOSs will match between the video and the PDFs, but may be in a different order than the CFAI readings

Forwards/Futures Prices

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⇒ Arbitrage-free pricing & valuation

assumptions/
- replicating instruments are identifiable and investable
- market frictions are nil
- short selling is allowed with full use of proceeds
- borrowing & lending are available at a known risk-free rate

Notation: S – underlying
F – forward V – value of forward
f – futures v – value of futures

$V_0 = 0$
- at contract initiation, the value of a futures/forward contract = 0 (i.e. no money changes hands)
- at expiration, $F_T = f_T = S_T$ (called convergence)

$V_T = F_T - F_0 = S_T - F_0$ (long) $V_T = F_0 - F_T = F_0 - S_T$ (short)

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- futures/ $v_t = f_t - f_{t-1}$ (before)
$v_t = 0$ (after)
- futures are marked-to-market daily

1/ no underlying cash flows $F_0 = S_0e^{rT}$ – cont. comp.
or/ $F_0 = S_0(1 + r)^T$ – periodic comp.

e.g./ $S_0 = 100$
$r_f = 5\%$
$T = 1$ yr.

Step #1. Borrow $100 for one year
Step #2. Buy S_0
Step #3. Sell F_0
Step #4. Payback loan of S_0e^{rT} at time T

$F_0 = S_0e^{rT} = 100e^{.05(1)} = 105.127$
$F_0 = S_0(1 + r)^T = 100(1.05)^1 = 105$

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e.g./ $S_0 = 100$
$r_f = 5\%$
$T = 1$ yr.

$F_0 = S_0 e^{rT} = 100e^{.05(1)} = 105.127$
or/ $F_0 = S_0(1 + r)^T = 100(1.05)^1 = 105$

general rule : buy low, sell high

Case 1: $F_0 = 110$
- sell F_0 (i.e. short)
- borrow \$100, buy S_0
- at T, deliver S_0 for \$110
- pay back \$105
- at time 0, borrow $5/1.05 = 4.762$ (get paid today)

carry arbitrage

Case 2: $F_0 = 90$
- Sell S_0 for \$100, invest @ 5%
- Buy F_0 (i.e. long)
- at T, take delivery of S for \$90, cover short position
- profit = \$15
- at time 0, borrow $15/1.05 = 14.286$ (get paid today)

reverse carry arbitrage

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An Australian stock paying no dividends is trading in Australian dollars for A\$63.31, and the annual Australian interest rate is 2.75% with annual compounding. Based on the current stock price and the no-arbitrage approach, what is the equilibrium three-month forward price?

$S_0 = 63.31$
$r_f = 2.75\%$
$T = .25$

$F_0 = S_0(1 + r)^T$
$= 63.31(1.0275)^{.25} = 63.74$

If the interest rate immediately falls 50 bps to 2.25%, the three-month forward price will: ↓

$63.31(1.0225)^{.25} = 63.66$

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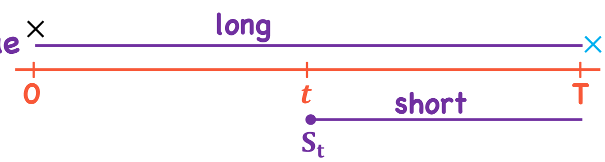
1/ no underlying cash flows Key point: the quoted forward price does <u>not</u> directly reflect expectations of future underlying prices

⇒ Value



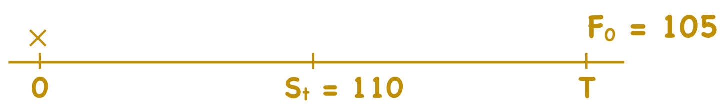
$F_0 = S_0(1 + r)^T$

$F_t = S_t(1 + r)^{T-t}$

$$V_t = \frac{F_t - F_0}{(1+r)^{T-t}} \quad \text{or/} \quad V_t = S_t - F_0 / (1+r)^{T-t}$$

e.g./ $F_0 = 105$
$T = 1$ yr.
$S_t = 110$
$t = 9$ mos.
$r_f = 5\%$

$V_0 = 0$



$$110 - 105 / (1.05)^{.25} = 6.2729$$

$$F_t = S_t(1+r)^{T-t} = 110(1.05)^{.25} = 111.3499$$

$$V_t = \frac{(111.3499 - 105)}{(1.05)^{.25}} = 6.2729$$

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2/ Underlying with cash flows

γ – gamma ⇒ benefits
θ – theta ⇒ costs

$$F_0 = [S_0 + PV(\theta) - PV(\gamma)](1+r)^T$$

PV(θ): increase F_0; PV(γ): decrease F_0

e.g./ $S_0 = 100$
$r_f = 5\%$
$T = 1$
CF = 2.9277 @ t = .5

$$F_0 = \left(100 + 0 - \frac{2.9277}{(1.05)^{.5}}\right)(1.05)^1 = 102$$

- 3 mos. later $S_t = 105$

$$F_t = \left(105 + 0 - \frac{2.9277}{(1.05)^{.25}}\right)(1.05)^{.75} = 105.9134$$

$$V_t = (105.9134 - 102) / (1.05)^{.75} = 3.7728$$

$$\left(105 - \frac{2.9277}{(1.05)^{.25}}\right) - 102 / (1.05)^{.75} = 3.7728$$

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2/ Underlying with known yield

(known yield: assumed to be continuous)

$$F_0 = S_0 e^{(r_c+\theta-\gamma)T}$$

r_c, θ, γ — all continuous rates

Recall/ $(1+r)^T = e^{r_c T} \Rightarrow \ln[(1+r)^T] = r_c T$

$$\frac{T\ln(1+r)}{T} = r_c$$

$$r_c = \ln(1+r)$$

e.g./ r_f = 5% annual

$r_c = \ln(1.05) = 4.897\%$ & $e^{.04897} = 1.05$

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A. Equities/

The continuously compounded dividend yield on the EURO STOXX 50 is 3%, and the current stock index level is 3,500. The continuously compounded annual interest rate is 0.15%. Based on the carry arbitrage model, the three-month futures price will be *closest* to:

$\gamma = 3\%$ $r_c = .15\%$ $F_0 = S_0 e^{(r_c-\gamma)T}$

$S_0 = 3500$ $T = .25$ $= 3500e^{(.0015-.03).25} = 3475.15$

Suppose Nestlé common stock is trading for CHF70 and pays a CHF2.20 dividend in one month. Further, assume the Swiss one-month risk-free rate is 1.0%, quoted on an annual compounding basis. Assume that the stock goes ex-dividend the same day the single stock forward contract expires. Thus, the single stock forward contract expires in one month.

$$F_0 = (S_0 - PV(\gamma))(1+r)^T$$

$$F_0 = S_0(1+r)^T - \gamma = 70(1.01)^{1/12} - 2.20 = 67,858$$

$$(S_0 - PV(\gamma))(1.01)^{1/12} \rightarrow 67,858$$

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Suppose we bought a one-year forward contract at 102 and there are now three months to expiration. The underlying is currently trading for 110, and interest rates are 5% on an annual compounding basis. If there are no other carry cash flows, the forward value of the existing contract will be closest to:

0 — 9 (t) — T
$F_0 = 102$
$S_t = 110$

$$F_t = 110(1.05)^{.25} = 111.3499$$
$$-102$$
$$9.3499$$

$$110 - 102 / (1.05)^{.25} = 9.2365$$

$$9.3499 / (1.05)^{.25} = 9.2365$$

If a dividend payment is announced between the forward's valuation and expiration dates, assuming the news announcement does not change the current underlying price, the forward value will most likely:

↓ $[S_0 + PV(\theta) = PV(\gamma)]$ **decrease**

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B. Interest Rates

- forward rate agreement (OTC)
  - underlying is an interest rate on a deposit

    long = floating rate receiver (pays fixed)
    short = fixed rate receiver (pays floating)

  - no exchange of notional amount
  - FRA price is the fixed interest rate that eliminates arbitrage

    3x9 FRA – the rate on a deposit that begins in 3 months and lasts for 6 months

L(90)
L(270)
0 — 3 — 9
f(3,6) = FRA fixed rate

$(30/360)$

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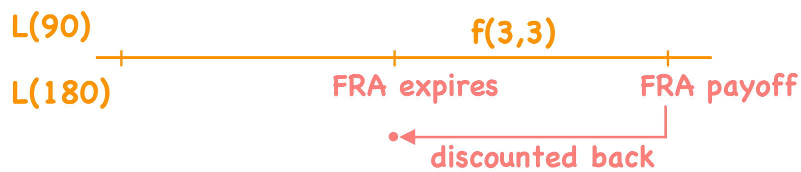
B. Interest Rates



advanced set, advanced settled (FRAs)

- advanced set, settled in arrears
- swaps
- interest rate options

- receive floating

$$\frac{NA(\text{floating rate} - \text{fixed rate})\left(\text{days}/360\right)}{\left[1 + \text{floating rate}\left(\text{days}/360\right)\right]}$$

- receive fixed

$$\frac{NA(\text{fixed rate} - \text{floating rate})\left(\text{days}/360\right)}{\left[1 + \text{floating rate}\left(\text{days}/360\right)\right]}$$

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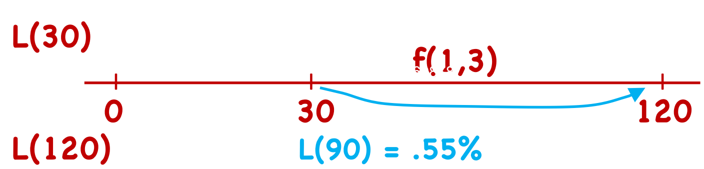
B. Interest Rates

In 30 days, a UK company expects to make a bank deposit of £10,000,000 for a period of 90 days at 90-day Libor set 30 days from today. The company is concerned about a possible decrease in interest rates. Its financial adviser suggests that it negotiates today, at Time 0, a 1 × 4 FRA, an instrument that expires in 30 days and is based on 90-day Libor. The company enters into a £10,000,000 notional amount 1 × 4 receive-fixed FRA that is advanced set, advanced settled. The appropriate discount rate for the FRA settlement cash flows is 0.40%. After 30 days, 90-day Libor in British pounds is 0.55%.

rec. fx. = fl.



The interest actually paid at maturity on the UK company's bank deposit will be closest to:

$$10M\left(.0055\left(90/360\right)\right) = 13,750$$

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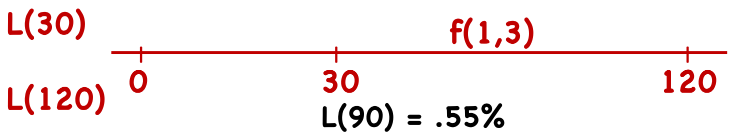
B. Interest Rates



If the FRA was initially priced at 0.60%, the payment received to settle it will be closest to:

rec. fx. – pay fl.

$$\frac{NA(\text{fx. rate} - \text{fl. rate})(90/360)}{1 + .004\,(90/360)} = \frac{10M(.006 - .0055)(90/360)}{1 + .004\,(90/360)}$$

$$= 1248.75$$

If the FRA was initially priced at 0.50%, the payment received to settle it will be closest to:

$$\frac{10M(.005 - .0055)(90/360)}{1 + .004\,(90/360)} = \frac{-1250}{1.001} = -1248.75$$

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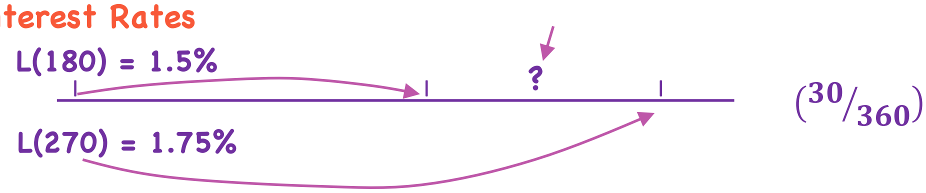
B. Interest Rates



6 x 9 FRA fixed rate = ?

$$[1 + .0175(270/360)] = [1 + .015(180/360)][1 + x(90/360)]$$

$$\frac{1 + .0175(270/360)}{1 + .015(180/360)} = 1 + x(90/360)$$

$$1.005583127 = 1 + x(90/360)$$

$$x = 2.233\%$$

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B. Interest Rates

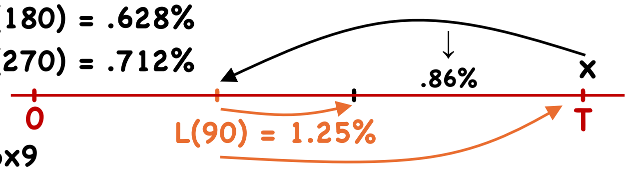
Suppose we entered a receive-floating 6 x 9 FRA at a rate of 0.86%, with notional amount of C$10,000,000 at Time 0. The six-month spot Canadian dollar (C$) Libor was 0.628%, and the nine-month C$ Libor was 0.712%. Also, assume the 6 x 9 FRA rate is quoted in the market at 0.86%. After 90 days have passed, the three-month C$ Libor is 1.25% and the six-month C$ Libor is 1.35%.

Assuming the appropriate discount rate is C$ Libor, the value of the original receive-floating 6 x 9 FRA will be closes to:

NA(fl. - fx.)

L(180) = .628%

L(270) = .712%



L(180) = 1.35%

$$\left[\frac{1+.0135\left(^{180}/_{360}\right)}{1+.0125\left(^{90}/_{360}\right)}-1\right]\times 4$$

$$=.01445$$

(rec. fl., pay fx.) (pay fl., rec fx.)

-.86% +1.45%

$$\frac{10M(.0145-.0086)\,^{90}/_{360}}{1+.0135\left(^{180}/_{360}\right)}=14,651.10$$

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C. Fixed Income Forwards

1. Full price = Clean Price + Accrued Interest
   (quoted price)
2. Fixed Income futures contracts are based on a generic bond i.e. Treasury Bond Futures – one contract involves the delivery of $100,000 face value, 6% semi-annual coupon

∴ underlying ≠ deliverable – short position decides what to deliver

e.g./ T-Bond futures ⇒ any gov't. bond > 15 yrs. to maturity as of Day 1 of the delivery period

∴ need a conversion factor – quoted price a bond would have on the first day of the delivery period on the assumption that the interest rate for all maturities = 6%

(Quoted futures price × CF) + AI } defines the price to be paid to the short side on delivery

(Quoted futures price: T-bond quote)