2026 Level 1 - Portfolio Management

Portfolio Risk and Return: Part I

a. describe characteristics of the major asset classes that investors consider in forming portfolios

b. calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data

c. explain risk aversion and its implications for portfolio selection

d. calculate and interpret portfolio standard deviation

e. describe the effect on a portfolio's risk of investing in assets that are less than perfectly correlated

f. describe and interpret the minimum-variance and efficient frontiers of risky assets and the global minimum-variance portfolio

g. explain the selection of an optimal portfolio, given an investor's utility (or risk aversion) and the capital allocation line

Return Measures

LOS a, b, c
- calculate
- interpret
- compare
- describe

- all financial assets have 2 common characteristics
  1. an expected return
  2. uncertainty regarding that return - risk

∴ all financial assets can be described by risk & return

Return - typically derived from 2 sources
- income
- cap. gains/losses

① Holding Period Return

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$$

$P_t - P_{t-1}$ - cap. gains/losses

$\frac{D_t}{P_{t-1}}$ - div. yield

LOS a, b, c
- calculate
- interpret
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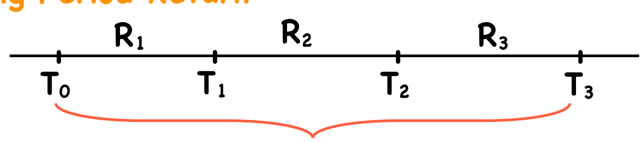
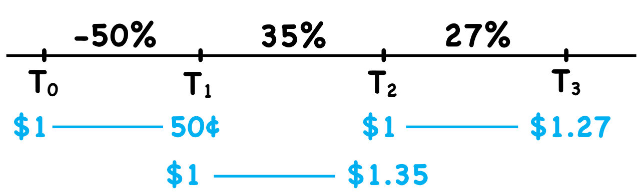
① Holding Period Return



$$HPR = [(1 + R_1)(1 + R_2)(1 + R_3)] - 1$$

② Arithmetic Mean

assumes no compounding



$$\bar{R} = \frac{-.5 + .35 + .27}{3} = 4\%$$

$$\bar{R}_i = \frac{1}{T}\sum_{i=1}^{T} R_i$$

tells us only the avg. return over a given random 1-period time frame

LOS a, b, c
- calculate
- interpret
- compare
- describe

③ Geometric Mean

considers compounding

	-50%	35%	27%
T_0	T_1	T_2	T_3

\$1 ⟶ 50¢ ⟶ 67.5¢ ⟶ 85.725%

- represents growth over a given time period

$$R_{G_i} = \sqrt[T]{\prod_{i=1}^{T}(1+R_{t_i})} - 1 \quad \text{or/} \quad \left[\prod_{t=1}^{T}(1+R_{t_i})\right]^{1/T} - 1$$

$$[(.50)(1.35)(1.27)]^{1/3} - 1 = 0.949953 - 1 = -0.050046$$

LOS a, b, c
- calculate
- interpret
- compare
- describe

④ Money-weighted Return (IRR)

CF_0 = -10,000
CF_1 = -1,000
CF_2 = -1000
CF_3 = 11,557

	-50%	35%	27%
T_0	T_1	T_2	T_3
10,000	1,000	1,000	11,557

10,000 ⟶ 5000; 5000 + 1,000 = 6000 ⟶ 8100; 8100 + 1,000 = 9100 ⟶ 11,557

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

-1.35%

CF_0 = -100
CF_1 = -950
CF_2 = 350
CF_3 = 1270

	-50%	35%	27%
T_0	T_1	T_2	T_3
100	950	350	1270

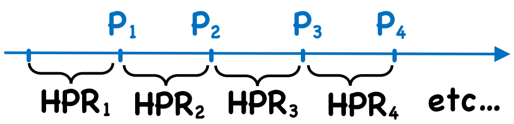
100 ⟶ 50; 50 + 950 = 1000 ⟶ 1350; 1350 − 350 = 1000 ⟶ 1270

IRR = 26.108%

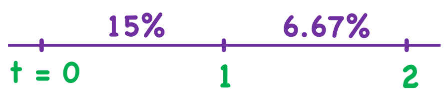
- accurately reflects what a specific investor earned but/ lacks comparability

Portfolio Return

time-weighted rr → measures the compounded rate of growth of $1 over the measurement period.



$$(1 + \text{twrr})^n = (1 + \text{HPR}_1)(1 + \text{HPR}_2) \ldots (1 + \text{HPR}_n)$$



$$(1 + \text{twrr})^2 = (1 + \text{HPR}_1)(1 + \text{HPR}_2)$$

$$(1 + \text{twrr})^2 = (1.15)(1.0667)$$

$$1 + \text{twrr} = \sqrt{(1.15)(1.0667)}$$

$$\text{twrr} = \sqrt{(1.15)(1.0667)} - 1$$

$$= 10.76\%$$

Return Measures

LOS a, b, c
- calculate
- interpret
- compare
- describe

e.g./

Year	AUM	R
1	30M	15%
2	45M	-5%
3	20M	10%
4	25M	15%
5	35M	3%

① HPR

$$[(1.15)(.95)(1.10)(1.15)(1.03)] - 1 = .4235 \text{ or } 42.35\%$$

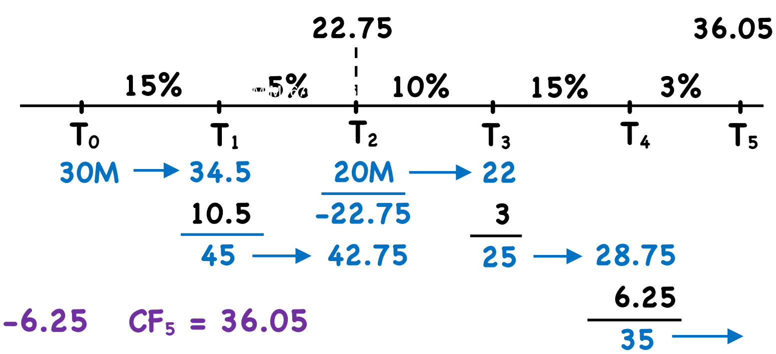
② Arithmetic $\bar{R}$

$$\frac{15 - 5 + 10 + 15 + 3}{5} = 7.6\%$$

③ Geometric $\bar{R}$

$$[\text{HPR} + 1]^{1/5} - 1 = (1.4235)^{1/5} - 1 = 7.317\%$$

④ mwrr



IRR = 5.86%

$CF_0 = -30$

$CF_1 = -10.5$

$CF_2 = 22.75$

$CF_3 = -3$ $CF_4 = -6.25$ $CF_5 = 36.05$

Annualized Returns/

LOS a, b, c
- calculate
- interpret
- compare
- describe

$$r_{ann} = (1 + r_{days})^{365/days}$$

52

e.g./ .2% ⇒ 1 week $\quad (1.002)^{365/7} - 1 = 1.1098 - 1 = 10.98\%$

.4% ⇒ 15 days $\quad (1.004)^{365/15} - 1 = 1.10201 - 1 = 10.201\%$

14.2% ⇒ 1½ yrs. $\quad (1.142)^{2/3} - 1 = 1.09225 - 1 = 9.255\%$

e.g. 2/

6.2% ⇒ 100 days $\quad (1.062)^{365/100} - 1 = 24.55\%$

2% ⇒ 4 weeks $\quad (1.02)^{365/28} - 1 = 29.45\% \quad (52/4) \quad 29.36\%$

5% ⇒ 3 mos. $\quad (1.05)^{12/3} - 1 = 21.55\%$

Portfolio Return/ weighted average of the individual returns

LOS a, b, c
- calculate
- interpret
- compare
- describe

$$R_P = \sum_{i=1}^{N} W_i R_i \quad \text{where} \quad \sum_{i=1}^{N} W_i = 1$$

(W_i → inv.)

Other Return Measures/

1) Gross and net returns

gross returns ⇒ Total return - trading fees
(Total return - trading fees: basis for comparing manager performance)

net return ⇒ gross return - mgmt./admin. fees
(gross return - mgmt./admin. fees: what the investor earns)

(smaller funds disadvantaged here)

LOS a, b, c
- calculate
- interpret
- compare
- describe

Other Return Measures/

② Pre-tax & After-tax Nominal Return

- components of the gain matter

cap. gains
- s.t.
- l.t. → pref. tax treatment

income
- interest
- dividends ⇒ pref. tax treatment

③ Real Returns

$$(1 + r) = (1 + r_f) \times (1 + \pi) \times (1 + RP)$$

nominal, real risk-free, inflation premium, risk premium

$$\frac{(1 + r)}{(1 + \pi)} = \underbrace{(1 + r_f) \times (1 + RP)}_{\text{real 'risky' rate}}$$

LOS a, b, c
- calculate
- interpret
- compare
- describe

Other Return Measures/

④ Leveraged Returns

- either by use of derivatives or margin
- gains are magnified, as are losses
- must also account for margin loan interest

e.g./

YR.	AUM	net R
1	30M	15%
2	45M	-5%
3	20M	10%
4	25M	15%
5	35M	3%

HPR = 42.35% $\bar{R}_A$ = 7.6% $\bar{R}_G$ = 7.32%

t = 20%
π = 2%

Calculate: real after-tax R_5

$$\frac{[1 + (.03 \times .8)]}{1.02} - 1 = .0039$$

.39%

Measures of Risk

LOS d
- calculate
- interpret

Variance/ - a measure of the dispersion of returns

$$\sigma^2 = \frac{\sum_{i=1}^{n}(R_i - \mu)^2}{n}$$

since we typically don't know pop. parameters, we use sample stats.

$$s^2 = \frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1}$$

Standard Deviation/

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^{n}(R_i - \mu)^2}{n}}$$

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1}}$$

LOS d
- calculate
- interpret

Variance of a Portfolio of Assets/

- need the variance of each asset

\+ the covariance of each asset with each other

⇒ variances & covariances are additive

$$\underbrace{\sigma^2(X_i) = \sum_{i=1}^{n} P(X_i)\,[X_i - E(X)]^2}_{\text{var. of each asset}}$$

$$\underbrace{\mathrm{COV}(R_iR_j) = E\left[\left(R_i - E(R_i)\right)\left(R_j - E(R_j)\right)\right]}_{\text{covar. of each asset with each other}}$$

$$\underbrace{\sigma^2(R_P) = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i\, W_j\, \mathrm{Cov}(R_iR_j)}_{\text{port. var. = sum of all the vars. + all the covars.}}$$

LOS d
- calculate
- interpret

Variance of a Portfolio of Assets/

$$\sigma^2(R_P) = \sum_{i=1}^{n} W_i^2 \, Var(R_i) + \sum_{i=1}^{n}\sum_{j=1}^{n} W_i \, W_j \, Cov(R_i R_j)$$

variances + covariances

$\Rightarrow Cov(R_i R_j) = P_{ij}\sigma_i\sigma_j$

e.g./ 2 asset portfolio

$$\sigma^2(R_P) = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2w_1w_2Cov(R_1R_2)$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots \quad + 2w_1w_2 \bullet P_{12}\sigma_1\sigma_2$$

and $\sigma(R_P) = \sqrt{\sigma^2(R_P)} = \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2w_1w_2Cov(R_1R_2)}$

LOS d
- calculate
- interpret

$Asset_i$	W_i	$E(R_i)$	σ_2
S&P500	80%	9.93%	16.21%
MSCI	20%	18.20%	33.11%

$Cov(R_iR_j) = .5\%$

R_P = ? $W_1R_1 + W_2R_2$
= .8(.0993) + .2(.1820)
= 0.1158 or 11.58%

$\sigma^2(R_P)$ = ? $W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2w_1w_2Cov(R_1R_2)$

$(.8)^2(.1621)^2 + (.2)^2(.3311)^2 + 2(.8)(.2)(.005)$

.01682 + .00439 + .00160

= .02281

$\sigma^2(R_P) = \sqrt{.02281} = 15.10\%$

$Cov(R_1R_2) = P_{12}\sigma_1\sigma_2$

$.005 = P_{12}(.1621)(.3311)$

$P_{12} = \frac{.005}{.05367} = .093$

0

(-1 → +1)

Expected Return vs. Historical Return

LOS d
- calculate
- interpret

$1 + E(R) = (1 + r_f) \times [1 + E(\pi)] \times [1 + E(RP)]$

- based on actual results
- as a practical matter, we often assume that historical mean return is an adequate representation of expected return

Other Investment Characteristics/

Skewness
kurtosis
} #25 - Quant.

Risk Aversion

LOS e
- explain

Sure thing
$25

Gamble
.5 → $50
.5 → $0

E(R) = $25

preferences

1) risk aversion
- take the sure thing
- max. return for a given level of risk, and min. for a given level of return

2) risk-seeking
- take the gamble
- get satisfaction from the uncertainty

3) risk neutral
- indifferent
- seek higher returns regardless of risk