Question 1 of 9

The following table reports long-term annual return components for US Treasury bonds and US corporate bonds:

Asset class	Income return	Price appreciation
US Treasuries	0.85%	4.25%
US corporates	1.00%	5.75%

If a portfolio allocates 70% to US Treasury bonds and 30% to US corporate bonds, then based on this data, the portfolio's annualized outperformance relative to US Treasury bonds is *closest* to:

A. 0.45%

B. **0.50%** ✓

C. 1.16%

Explanation:

General observations on long-term asset returns

Corporate bonds vs. government bonds	• Corporate bonds generate somewhat higher returns than government bonds
Equities vs. fixed income	• Over long horizons, equities have delivered higher average returns than fixed-income securities • The higher returns from equities are associated with greater volatility in performance
Small-cap vs. large-cap stocks	• Historically, smaller-capitalization companies have at times produced higher returns than large-capitalization firms • This small-cap return advantage may decline or disappear over time
Risk vs. return	• Investments with higher expected returns typically involve higher levels of risk or variability in returns

Total return includes **income** plus **price appreciation**. For portfolios, these return components combine across assets according to their portfolio weights.

In this scenario, the portfolio return is calculated as the weighted average of the individual asset returns:

Steps	Calculations
Calculate total returns for each asset class	US Treasuries = 0.0085 + 0.0425 = 0.0510 US Corporate bonds = 0.0100 + 0.0575 = 0.0675
Calculate the portfolio return	= (0.70 × 0.0510) + (0.30 × 0.0675) = 0.05595
Calculate portfolio outperformance relative to US Treasuries	= 0.0559 − 0.0510 = 0.00495 ≈ 0.005 ≈ 0.50%

(Choice A) 0.45% results from using only the price appreciation components and ignoring income return.

(Choice C) 1.16% results from reversing the portfolio weights.

Things to remember:
Total return includes income return plus price appreciation, and portfolio return equals the weighted average of the constituent asset returns.

Question 2 of 9

The following information relates to a US Treasury bill:

Price	985
Face value	1,000
Days to maturity	120
Days in a year	365

The price of the T-bill implies an annualized interest rate *closest* to:

A. 4.57%

B. 4.62%

C. 4.70% ✓

Explanation:

Annualized rate of return

$$r_{annual} = (1 + r_{period})^c - 1$$

r_{period} = Return for the period

c = Annualization factor based on the length of the holding period

Risk-free government securities, such as US Treasury bills (ie, T-bills), are commonly used as benchmarks for market interest rates, as their prices reflect prevailing interest rate conditions. Since bond prices and interest rates move in opposite directions, interest rates can be derived from bond prices.

To determine the **annualized** risk-free **rate**, the first step is to calculate the **total return** of the T-bill. A T-bill is issued at a discount and does not pay periodic interest, so the total **return** comes solely from the **difference between** the **purchase price** and the **amount** received at **maturity** (ie, P_1).

Steps	Calculations
Calculate return	$r = \frac{P_1 - P_0 + \text{Inc}}{P_0} = \frac{1{,}000 - 985 + 0}{985} \approx 0.0152$
Annualize return	$r = (1 + r_{period})^{\frac{365}{120}} - 1 \approx 1.047 - 1 \approx 0.047 \approx 4.7\%$

The T-bill's annualized return is therefore approximately 4.70%.

(Choice A) 4.57% results from using simple annualization with a 360 day year.

(Choice B) 4.62% results from using simple annualization by multiplying the holding period return by 365/120 rather than compounding.

Things to remember:
For a discount security such as a Treasury bill, the total return equals the price gain between purchase and maturity, and the annualized return is obtained by compounding the total return over the fraction of the year remaining to maturity.

Question 3 of 9

The following table shows a company's sales growth over a five-year period:

Year	Sales Growth Rate
2008	6.8%
2009	12.2%
2010	4.3%
2011	−6.9%
2012	14.7%

The company's geometric mean for sales from 2008 to 2012 is *closest* to:

A. 5.9% ✓

B. 6.2%

C. 6.8%

Explanation:

Geometric mean

$$G + 1 = \sqrt[n]{(1 + x_1)(1 + x_2)\ldots(1 + x_n)}$$

Calculating an **average growth rate** for an asset over **multiple periods** requires the use of **compounding**, since the growth rate each year is affected by the growth rates in previous years. In Year 2, the asset will have already grown by the rate in Year 1. In Year 3, the asset will have already grown by the rate in both Years 1 and 2. This is true for asset returns and financial figures like sales growth and earnings per share growth.

The **geometric mean** calculates the average growth rate over multiple periods of time by taking compounding into account. Because of this, the geometric mean is used for the time series for the company's sales growth since sales each year are affected by the growth in previous years. The geometric mean helps to calculate the terminal value of an asset or, in this case, sales.

Steps	Calculations
Insert the five growth rates into the equation	$G + 1 = \sqrt[5]{(1 + 0.068)(1 + 0.122)(1 + 0.043)(1 - 0.069)(1 + 0.147)}$
Multiply the five values	$G + 1 = \sqrt[5]{1.3346}$
Calculate the 5th root	$G + 1 = 1.0594$
Subtract 1 from each side	$G = 1.0594 - 1$ $= 5.94\%$

The company's five-year compounding annual growth rate rounds to 5.9%.

(Choice B) The arithmetic mean, or simple average, of the company's sales growth is 6.2%. The arithmetic mean does not take compounding into consideration.

(Choice C) The median of the company's sales growth is 6.8%. Since there is an odd number of years being calculated, the median is the middle data point.

Things to remember:
The geometric mean calculates the average growth rate over multiple periods of time by taking compounding into account.

Question 4 of 9

An analyst reviews the performance of three exchange-traded funds (ETFs):

ETF Annual Returns	Large-cap	Mid-cap	Small-cap
Year 1	13.0%	17.0%	10.0%
Year 2	14.0%	−1.5%	18.5%
Year 3	−0.1%	9.6%	−5.3%

Which ETF had the highest three-year annual geometric mean return?

A. Large-cap ETF ✓

B. Mid-cap ETF

C. Small-cap ETF

Explanation:

Geometric mean return

$$G = \sqrt[n]{(1 + x_1)(1 + x_2)\ldots(1 + x_n)} - 1$$

n = number of periods

The **geometric mean return (GMR)** formula is used to convert actual performance data over multiple periods into a comparable format. The GMR produces a **constant periodic return**, usually annual, from actual periodic returns. The actual periodic returns, which are geometrically linked, produce an actual total return. The GMR breaks this multiperiod total return into a conforming time period.

Advantages of the **GMR** include allowing for **varying investment levels** and **comparability**. The simple arithmetic mean assumes that each period's level of investment is constant. The GMR is independent of levels of investment, which allows for better performance comparisons between securities such as large-, mid-, and small-cap exchange-traded funds (ETFs).

To calculate the GMR, link the annual returns to produce a three-year total return. Link the returns geometrically (by multiplying), not arithmetically (by adding), then convert the three-year total return to an annualized one-year return. Use the total number of years (3) as the root number when reconstructing the GMR. The large-cap ETF has the highest annual GMR at 8.8%.

Geometric mean returns

Large-cap	Mid-cap	Small-cap
$G + 1 =$	$G + 1 =$	$G + 1 =$
$\sqrt[3]{(1 + 0.13)(1 + 0.14)(1 - 0.001)}$	$\sqrt[3]{(1 + 0.17)(1 - 0.015)(1 + 0.096)}$	$\sqrt[3]{(1 + 0.10)(1 + 0.185)(1 - 0.053)}$
$G = \sqrt[3]{(1.13)(1.14)(0.999)} - 1$	$G = \sqrt[3]{(1.17)(0.985)(1.096)} - 1$	$G = \sqrt[3]{(1.10)(1.185)(0.947)} - 1$
$G = \sqrt[3]{1.2869} - 1$	$G = \sqrt[3]{1.2631} - 1$	$G = \sqrt[3]{1.2344} - 1$
$G = 1.0877 - 1$	$G = 1.0810 - 1$	$G = 1.0727 - 1$
$G = 8.8\%$	$G = 8.1\%$	$G = 7.3\%$

Add 1 to the ETF return to represent the initial investment. Subtracting 1 after calculating the GMR removes the effect of the initial investment. Scaling all initial investments of any dollar amount to a base of 1 allows for comparable performance figures.

(Choice B) This option incorrectly employs the arithmetic mean to find each of the ETFs' returns. The arithmetic mean is *dependent* on the level of investment. It assumes a constant investment.

(Choice C) In this calculation, +1.5% instead of −1.5% is used in Year 2 for the mid-cap ETF. Omitting the minus sign for negative returns is a common computational error.

Things to remember:
The geometric mean return (GMR) is used to convert a security's total return over multiple periods to a more comparable constant return. Use the total number of years, which can be whole or fractional, when reconstructing the GMR. A common mistake is to enter negative returns incorrectly.

Question 5 of 9

A hedge fund manager invested in three investments, A, B, and C, with differing holding periods. The manager collected the following information:

Investment performance	A	B	C
Holding period return	0.5%	50.0%	350.0%
Holding period return	5 days	23 months	6.7 years

Which of the three investments has the *lowest* annualized rate of return?

A. Investment A

B. Investment B ✓

C. Investment C

Explanation:

$$\text{Annualized rate of return} = (1 + \text{holding period return})^{\frac{1}{t}} - 1$$

The **annualized rate** of return is an application of the geometric mean return. It converts a holding period return to an equivalent annualized return. To find the annualized rate of return, determine the time unit (eg, days, months, or years). Identify how many time units occur in one year (eg, days = 365, months = 12, year = 1). Express the time in years. Apply this standardized unit of time to the geometric mean return formula.

	Annualized performance		
	Investment A	Investment B	Investment C
Holding period return (HPR)	0.50%	50.00%	350.00%
Holding period (time)	5 days	23 months	6.7 years
Periods per year	365	12	1
Holding period adjustment	$\frac{5}{365}$ years	$\frac{23}{12}$ years	6.7 years
$(1 + \text{HRP})^{\frac{1}{t}} - 1$	$(1 + 0.005)^{\frac{1}{\left(\frac{5}{365}\right)}} - 1$	$(1 + 0.50)^{\frac{1}{\left(\frac{23}{12}\right)}} - 1$	$(1 + 3.50)^{\frac{1}{(6.7)}} - 1$
Annualized return	43.92%	**23.56%**	25.17%

When the annualized returns of all investments are compared, Investment B has the lowest return, 23.56%.

The annualized returns of Investment A **(Choice A)** and Investment C **(Choice C)** both show higher annualized returns than Investment B's annualized return.

Things to remember:
In all performance calculations, match the time unit of the return (eg. annual, monthly or daily), r, with the corresponding time, t (ie, how many periods in the time unit). Apply this standardized unit of time to the geometric mean return formula.

Question 6 of 9

A portfolio manager has compiled an investment's annual returns for three years:

Time Period	Return (%)
Year 1	−3%
Year 2	11%
Year 3	−5%

The average annual compounded return on the investment over the three-year holding period is *closest* to:

A. 0.76% ✓

B. 1.00%

C. 2.29%

Explanation:

Geometric mean return

$$G = \sqrt[n]{(1 + x_1)(1 + x_2)\ldots(1 + x_n)} - 1$$

n = number of periods

The **compounded periodic rate** of return for an asset is the **geometric** mean return. It is used to measure the periodic average of investment performance over multiple **consecutive periods**.

Compounding assumes that one period's ending investment value is the next period's beginning value (ie, that returns accumulate). The geometric mean is the average periodic rate of return that accounts for compounding and includes accumulated returns in subsequent periods. In this instance, there are three annual periods (ie, N = 3), so the geometric mean is:

$$(1 + R_G)^N = (1 + R_1)(1 + R_2) \ldots (1 + R_N)$$

$$R_G = \sqrt[N]{(1 + R_1)(1 + R_2) \ldots (1 + R_N)} - 1$$

$$= \sqrt[3]{(1 + (-0.03))(1 + 0.11)(1 + (-0.05))} - 1 \approx 0.0076 = 0.76\%$$

(Choice B) 1.00% is the arithmetic mean, which is calculated as the simple average of multiple periodic returns and does not reflect compounding.

(Choice C) 2.29% is the holding period return, which is the total cumulative return received over the three-year period, not the annual return.

Things to remember:
The geometric mean is the average periodic rate of return that accounts for compounding and includes accumulated returns in subsequent periods. It is used to measure investment performance over multiple consecutive periods.

Question 7 of 9

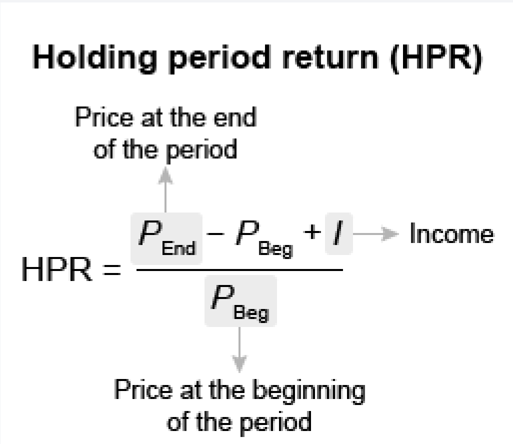
An investor purchases a stock for €100 at the beginning of the year. Six months later, the stock trades at €105. During the year, the stock paid a cash dividend of €2 per share. At the end of the year, the stock trades at €96, and the investor continues to hold the stock. The investor's capital distribution return for the year is *closest* to:

A. −2.00%

B. 1.90%

C. 2.00% ✓

Explanation:



An asset's total return (ie, holding period return) comprises price return (ie, capital appreciation) and capital distributions (ie, income). The price return captures the percent change in the asset's price. The **capital distribution** return captures the **income** as a **percentage of** the asset's **beginning price**. In this scenario, the income return is 2% (= 2 / 100).

(Choice A) −2.00% [= (96 − 100 + 2) / 100] is the holding period return.

(Choice B) 1.90% (= 2 / 105) results from taking the dividends as a percentage of the interim price, instead of the beginning price.

Things to remember:
Capital distribution return measures the income part of a stock's total return. It measures the cash dividends received as a percentage of the stock's beginning price.

Question 8 of 9

Which of the following is *most likely* a macro risk category for an asset's return?

A. Credit risk

B. Industry risk

C. Interest rate risk ✓

Explanation:

Macro vs. micro risks

Macro (systematic) risk Broad economy-wide risks that impact returns across asset classes	**Inflation risk**	Returns deviate from expectations due to inflation or deflation
	Interest rate risk	Uncertainty of returns resulting from interest rate fluctuations
	Market risk	Possibility that market forces will affect returns of all financial assets in an unexpected manner
Micro (unsystematic) risk Risks associated with factors specific to an issuer or sector	**Business risk**	Risk inherent in the nature of the firm's operations
	Financial risk	Risk arising from the use of financial leverage
	Credit risk	Risk that issuer may fail to meet its debt payment obligations
	Sector or industry risk	Risk affecting a specific sector will influence the returns of assets within that sector

Investment risk arises from the uncertainty between expected and realized returns and can reflect either macro (ie, systematic) or micro (ie, unsystematic) factors. **Macro**

risks impact the **broad economy** and cannot be diversified away, whereas **micro risks** specifically **impact** an **issuer**, or the issuer's sector, and can be reduced through diversification.

Three common macro risks are **inflation** risk, **interest rate** risk, and **market risk**. Interest rate risk refers to the uncertainty of returns resulting from interest rate fluctuations. Asset prices generally rise (fall) when interest rates fall (rise), since:

- assets are valued as the present value of their future cash flows, and
- interest rates are a factor in determining the discount rate applied.

(Choice A) Credit risk is an issuer-specific micro risk related to a firm's ability to meet its debt obligations.

(Choice B) Industry risk is a micro risk affecting firms within a particular industry rather than the broader market.

Things to remember:
Macro risks arise from economy-wide factors that influence many asset classes and cannot be diversified away. Three common types of macro risks are inflation risk, interest rate risk, and market risk.

Question 9 of 9

A mutual fund has the following annual rates of return:

Year	20X1	20X2	20X3
Returns	−10%	5%	16%

The mutual fund's annual geometric mean return (%) is *closest* to:

A. 3.1 ✓

B. 3.7

C. 9.6

Explanation:

Geometric mean return

$$G = \sqrt[n]{(1 + x_1)(1 + x_2)\ldots(1 + x_n)} - 1$$

n = number of periods

The **geometric mean return** captures the impact of **compounding** over **multiple periods**. Compounding involves multiplying (ie, "chaining") the returns for each of *n* periods and then taking the *n*th root of the result. The calculation convention is to add 1 to each return before multiplying the series and then subtract 1 from the calculated *n*th root.

In this scenario, the geometric mean is calculated as follows:

$$G = \sqrt[3]{(1 - 0.1) \times (1 + 0.05) \times (1 + 0.16)} - 1$$
$$G = 3.11\%$$

(Choice B) 3.7 is the arithmetic mean (ie, simple average) and does not take into account multiperiod compounding.

(Choice C) 9.6 is the total holding period return, not an annual return.

Things to remember:
The geometric mean return measures a compounded return over multiple periods. Compounding involves multiplying the returns for each of *n* periods and then taking the *n*th root of the result. The calculation convention is to add 1 to each return before multiplying the series and then subtract 1 from the calculated *n*th root.