Question 1 of 25

The real risk-free interest rate is *most appropriately* described as:

A. the nominal rate minus inflation. ✓

B. the expected inflation rate multiplied by the nominal rate.

C. the sum of the real risk-free rate plus inflation and a risk premium.

Explanation:

Comparing real and nominal rates		
Rate	Definition	Formula
Real risk-free rate	Return on a risk-free investment, adjusted for inflation	Nominal rate − Inflation
Nominal rate	Return on a risk-free investment, including inflation	Real risk-free rate + Inflation

The real risk-free interest rate is the single-period interest rate for a completely risk-free security, assuming there is **no inflation**. It represents the compensation required to postpone consumption when inflation is zero. The **nominal risk-free interest rate** represents the sum of the real risk-free interest rate and the inflation premium Additional premiums (eg, default risk, liquidity, maturity premiums) may be added to compensate the investor for specific types of risks.

(Choice B) The real risk-free interest rate is not determined by multiplying the expected inflation rate by the nominal rate. While inflation impacts the nominal rate, it is subtracted (not multiplied).

(Choice C) The sum of the real risk-free rate plus inflation and a risk premium represents the nominal rate, as it includes inflation and risk premiums, which are excluded from the real risk-free interest rate.

Things to remember:
The real risk-free rate reflects the time value of money in an environment with no inflation or risk. The nominal risk-free rate includes expected inflation, while the required rate of return adds various risk premiums. Understanding these components helps investors distinguish between the effects of inflation and the additional compensation required for bearing risk.

Question 2 of 25

At year end, a bondholder reviews performance records of a INR 1,000 par value bond. The holding period return of the security is 3.0%. If the beginning price of the bond was INR 998 and the ending price of the bond was INR 1,008, the bond's interest income for the period was *closest* to:

- **A. INR 19.94 ✓**
- **B.** INR 29.94
- **C.** INR 39.94

Explanation:

Holding period return (HPR)

Price at the end of the period

$$\text{HPR} = \frac{P_{\text{End}} - P_{\text{Beg}} + I}{P_{\text{Beg}}}$$

Income

Price at the beginning of the period

A **holding period return (HPR)** is a **single-period return**, where the single period can represent any timeframe. There are two sources of investment returns to the HPR: capital changes and income. The capital changes reflect the security's market value difference from the beginning to the end of the period. Income during the period is either the result of dividends from stocks or of interest earned from bonds.

For example, a stock or bond's price may move up or down during the period, which creates a capital gain or loss. In addition, most bondholders earn interest income during the period, and some stockholders earn dividend income. HPR compares these combined sources (gain or loss plus income) to the value the security had at the beginning of the period.

To find the interest income of a bond, use the HPR formula and input all but the missing interest income.

$$\text{HPR} = \frac{P_{\text{End}} - P_{\text{Beg}} + I}{P_{\text{Beg}}}$$

$$3.0\% = \frac{1{,}008 - 998 + \text{Interest income}}{998}$$

$$29.94 = 10 + \text{Interest income}$$

$$19.94 = \text{Interest income}$$

(Choice B) This incorrect answer of INR 29.94 is obtained by calculating interest as the HPR multiplied by the beginning price. This ignores the capital gain part of the return and assumes all return is generated from income.

(Choice C) This answer of INR 39.94 incorrectly uses the ending price, rather than the beginning price, in the dominator to find the interest. This is flawed since the interest found is greater than the holding period return.

Things to remember:
A holding period return compares the change in investment value plus interest to the beginning value of the investment to determine performance.

Question 3 of 25

The holding period return (HPR) for an investment purchased for $75 and sold for $85 with dividends of $5 received during the period is *closest* to:

A. 7%.

B. 13%.

C. **20%.** ✓

Explanation:

(Choice A)

$\text{Holding Period Return} = \dfrac{P_{sell} - P_{buy} + D}{P_{buy}} = \dfrac{\$85 - 75 + 5}{\$75} = 20\%$ where *P* = trade principal converted to the investor's currency and *D* = dividends received during the holding period.

Question 4 of 25

An investor creates a portfolio by purchasing one share of each of the following three stocks at the beginning of 20X1, shown below:

	Price (€)	
Stock	**Jan 1, 20X1**	**Dec 31, 20X1**
X	50	54
Y	50	52
Z	50	58

Stock Y pays a dividend of €5 during 20X1. At the end of 20X1, the investor sells Stock Z for €58. Based on this information, the investor's realized return on the portfolio is *closest* to:

A. 3.33%

B. 8.67% ✓

C. 12.67%

Explanation:

$$\textbf{Realized return} = \frac{\text{Realized capital gains or losses + Income}}{\text{Initial investment}}$$

Realized return measures the return **actually earned** during a specified period and includes **realized capital gains** (or losses) and **income distributions** (eg, dividends) received. In contrast, unrealized gains (or losses) reflect changes in market value for positions that remain unsold at the end of the period.

In this scenario, the investor purchases three stocks at the beginning of 20X1, each priced at €50. Therefore, the initial portfolio value equals €150. At the end of the period, the investor's realized return in currency terms is €13:

Realized capital appreciation on Stock Z = 58 − 50 = 8
Dividends from Stock Y = 5
Realized return = 8 + 5 = 13

The portfolio cost the investor €150, so the investor's total realized return is 13 / 150 = 0.0867 = 8.67%

(Choice A) 3.33% (= 5 / 150) is the realized return from dividends only.

(Choice C) 12.67% (= 19 / 150) represents the portfolio's total return, which includes unrealized price appreciation on Stocks X and Y.

Things to remember:
Realized return includes only gains and income actually received during the period, such as realized capital gains and dividends. Price changes on positions that remain unsold (eg, unrealized return) are not included in realized return.

Question 5 of 25

An analyst gathers the following information on three investments with a two-year holding period:

Investment	Type of Return	Returns (%)
X	Holding period return over two years	8.0
Y	Holding period return each year for two years	3.9
Z	Continuously compounded return for two years	7.9

Which investment *most likely* has the highest continuously compounded return over the two-year period?

A. Investment X

B. Investment Y

C. Investment Z ✓

Explanation:

Continuously compounded return ($r_{t,t+1}$)

$$r_{t,t+1} = \ln(1 + R_{t,t+1}) = \ln\left(\frac{S_{t+1}}{S_t}\right)$$

$R_{t,t+1}$ = holding period return
t = beginning time of investment
ln = natural logarithm
$r_{t,t+1}$ = continuously compounded return
$t + 1$ = ending time of investment
S = investment price

For a given holding period, different frequencies of compounding affect an investment's returns. **Many shorter compounding periods** (ie, greater frequecy) result in a **greater return than** a **smaller number of longer**compounding periods for the **same investment period**, all else being equal. An infinite number of infinitesimally small compounding periods during an investment holding period is known as continuous compounding.

A holding period return, $R_{t,t+1}$, can be converted into an equivalent continuously compounded return using the formula *Continuously compounded return* = ln(1 + *holding period return*):

Investment X	Investment Y	Investment Z
$r_{0,2} = \ln(1 + 0.08)$ $r_{0,2} = 7.7\%$	$R_{0,2} = (1 + R_{0,1})^2 - 1$ $R_{0,2} = (1.039)^2 - 1$ $R_{0,2} = 0.0795$ $r_{0,2} = \ln(1.0795)$ $r_{0,2} = 7.65\%$	$r_{0,2} = 7.9\%$

Convert Investment X's two-year holding period return to a continuously compounded return. Investment Y is stated in terms of a one-year holding period return and is held for two years. Calculate the two-year holding period return of Investment Y by compounding the one-year holding period return for two years using $R_{0,2} = (1 + R_{0,1})^2 - 1$. Then use the formula for continuously compounded return to convert the two-year holding period return to a continuously compounded return. All else being equal, **Investment Z** has the **highest continuously compounded return (Choices A and B)**.

Things to remember:
Continuously compounded return involves compounding with infinitesimally small compounding periods over an investment period. Continuously compounded return is equal to the natural log of (1 + holding period return): $r_{t,\, t+1} = \ln(1 + R_{t,\, t+1})$.

Question 6 of 25

If an asset's two-year continuously compounded return is 13.42%, then its holding period return for the same period is *closest* to:

A. 13.00%

B. 13.87%

C. 14.36% ✓

Explanation:

Continuously compounded return = Natural log(1 + Holding period return)

$$r_{t,t+1} = \ln(1 + R_{t,t+1})$$

In a discretely compounded return, an investment's return compounds periodically (eg, monthly, quarterly) for a defined number of periods within a certain time frame. In a **continuously compounded return**, returns compound over **infinite periods** within a certain time frame. The continuously compounded return is the natural log of (1 + Holding period return).

In this scenario, the holding period return is calculated as follows:

Steps	Calculations
1. Substitute variables	$0.1342 = \ln(1 + R_{t,t+1})$
2. Apply exponential function to both sides	$e^{0.1342} = e^{\ln(1+R_{t,t+1})}$
3. Simplify both sides: note that $e^{\ln(X)} = X$, so $e^{\ln(1+R_{t,t+1})} = 1 + R_{t,t+1}$	$1.1436 = 1 + R_{t,t+1}$
4. Rearrange equation and solve	$R_{t,t+1} = 1.1436 - 1$ $= \mathbf{0.1436}$

(Choice A) 13.00% results from incorrectly multiplying the annual geometric return by 2.

(Choice B) 13.87% results from incorrectly dividing the two-year continuously compounded return by 2, and then annually compounding it for 2 years.

Things to remember:
With a continuously compounded return, returns compound over infinite periods within a certain time frame. The continuously compounded return equals the natural log of (1 + Holding period return (HPR). The HPR is the change in price (ie, Ending price − Beginning price + Income) as a percentage of the beginning price.

Question 7 of 25

The price of silver is $14.00 per ounce at the beginning of the year and $14.50 at the end of the year. The continuously compounded rate of return for silver is *closest* to:

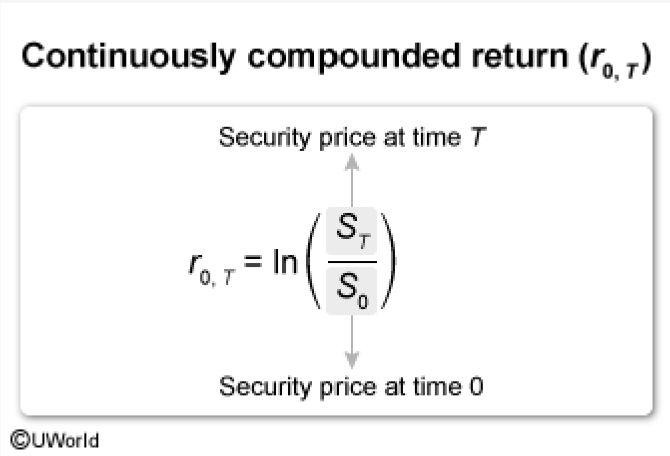
A. 3.51% ✓

B. 3.57%

C. 3.64%

Explanation:

In a discretely compounded return, an investment's return compounds periodically (eg, monthly, quarterly, semiannually) for a defined number of periods within a certain timeframe. In a **continuously compounded return**, an investment's return compounds over **infinite periods** within a certain time frame. The continuously compounded return equals the natural log of ending price over beginning price.



Note that S_T/S_0 is the same as (1 + holding period return)

$$r_{0,T} = \ln\left(\frac{14.5}{14}\right)$$

$$r_{0,T} = 0.0351$$

The beginning price of $14.00 grown continuously over one year at 0.0351 equals $14.50 at year-end.

$$14e^{0.0351} = 14.5$$

(Choice B) 3.57% is the holding period return (14.5/14 − 1), not the continuously compounded rate of return.

(Choice C) 3.64% results from dividing the holding period return by 365 and compounding that daily.

Things to remember:
With continuously compounded return, an investment's return compounds over infinite

periods within a certain time frame. The continuously compounded return equals the natural log of the ending price over the beginning price (or the natural log of [1 + holding period return]).

Question 8 of 25

An investor bought 200 shares at $20.35 per share on February 15 and sold the shares at $17.77 on April 1. He received $50 in dividends on March 2. The investor's holding period return is *closest to*:

A. −13.11%

B. −12.68%

C. −11.45% ✓

Explanation:

Holding period return (HPR)

Price at the end of the period

$$\text{HPR} = \frac{P_{\text{End}} - P_{\text{Beg}} + I}{P_{\text{Beg}}}$$

I → Income

Price at the beginning of the period

The holding period return (HPR) measures the return an investor receives over a specified period. The return comprises both the capital gain (or loss) that the investor realizes plus any income that the investment generates (eg, dividends, interest).

HPR has limited utility for comparing returns of investments across different intervals since it does not assume a common holding period for the investments.

The HPR in this question is calculated as:

$$\text{HPR} = \frac{(200 \times (17.77 - 20.35)) + 50}{200 \times 20.35} \approx -11.45\%$$

(Choice A) −13.11% results from incorrectly calculating the capital loss that the investor incurred.

(Choice B) −12.68% results from not including the dividend received as part of the HPR.

Things to remember:
The HPR measures the return an investor earns over a specific period, including gains or losses and any income received. However, it is less useful for comparing returns across different intervals.

Question 9 of 25

An investor purchased 1,000 shares of stock at $20 per share. After two years, all shares were sold for $15 per share. A dividend of $2 per share was paid for each year during the holding period. What is the total return on each share?

A. –25%

B. –5% ✓

C. 45%

Explanation:

Holding period return (HPR)

$$\text{HPR} = \frac{\overbrace{P_{\text{End}}}^{\text{Price at the end of the period}} - P_{\text{Beg}} + \underbrace{I}_{\text{Income}}}{\underbrace{P_{\text{Beg}}}_{\text{Price at the beginning of the period}}}$$

A holding period return (HPR) is the return earned from holding an asset for a single specified period of time which could be one day, one year, two years, etc. This example uses a two-year period. The following formula is used to find the total return on each share.

$$\text{HPR}\ \frac{(\$15 - \$20 + \$4)}{\$20} = -5\%$$

(Choice A) –25% results from not including the dividend income.

(Choice C) 45% results from incorrectly subtracting the ending value ($15) from the beginning value ($20) and then adding the dividend in the numerator, followed by dividing by the beginning value.

Question 10 of 25

An asset's one-year continuously compounded return is 9.2%. Assuming no income from the asset, if its beginning price is BRL 136.50, then its price at the end of one year (in BRL) is *closest* to:

A. 149.06

B. 149.65 ✓

C. 150.33

Explanation:

$$\text{Continuously compounded return} = \text{Natural log}\left(\frac{\text{Ending price}}{\text{Beginning price}}\right)$$

$$r_{t,t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right)$$

In a discretely compounded return, an investment's return compounds periodically (eg, monthly, quarterly) for a defined number of periods within a certain time frame. In a **continuously compounded return**, an investment's return compounds over **infinite periods** within a certain time frame. The continuously compounded return equals the natural log of the ratio of ending price to beginning price. It should be noted that the ratio of the ending price to the beginning price is equivalent to 1 + Holding period return.

In this scenario, the asset's ending price is calculated through the following steps:

Steps	Calculations
1. Substitute variables	$0.092 = \ln\left(\frac{S_{t+1}}{136.5}\right)$
2. Apply exponential function to both sides	$e^{0.092} = e^{\ln(S_{t+1}/136.5)}$
3. Simplify both sides: note that $e^{\ln(X)} = X$, so $e^{\ln(S_{t+1}/136.5)} = S_{t+1}/136.5$	$1.0964 = \frac{S_{t+1}}{136.5}$
4. Rearrange equation and solve	$S_{t+1} = 1.0964 \times 136.5$ $=$ **149.65**

(Choice A) 149.06 is the result of treating 9.2% as the asset's holding period return. The incorrect setup would have been $r_{t,t+1} = \ln(1 + 0.092)$.

(Choice C) 150.33 is the result of treating 9.2% as the asset's discount yield. The incorrect setup would have been: (Face value − Purchase price) / Face value = 0.092.

Things to remember:
In a continuously compounded return, an investment's return compounds over infinite periods within a certain time frame. The continuously compounded return is the natural

log of the ratio of the ending price to the beginning price. This ratio is equivalent to (1 + Holding period return).

Question 11 of 25

An asset's present value is £1,150 and its annual continuously compounded rate of return is 5.9%. Assuming no income, if the asset is sold in 4 years, its value (in £) is *closest* to:

A. 1,220

B. 1,446

C. 1,456 ✓

Explanation:

Ending price (S_T) based on continuously compounded return

$$S_T = S_0 e^{r_{0,1} T}$$

S_0 = beginning price T = total number of periods

$r_{0,1}$ = continuously compounded return per period

The return an investor receives on an investment depends on the rate of return and number of compounding periods. All else being equal, for a given investment holding period **many shorter compounding periods** (ie, more frequent compounding) result in a **greater return than** a **smaller number** of **longer**compounding periods (ie, less frequent compounding). An infinite number of infinitesimally small compounding periods during a holding period is known as continuous compounding.

In this scenario, the number of years (T) and the asset's present value (S_0) and annual continuously compounded return ($r_{0,1}$) are given, so the future price after 4 years can be determined by substituting as follows:

$$S_T = S_0 e^{r_{0,1} T}$$

$$S_4 = 1{,}150 \times e^{[(0.059) \times 4]}$$

$$S_4 \approx 1{,}456.10$$

(Choice A) £1,219.89 results from incorrectly treating 5.9% as the continuously compounded return for the entire period, not the annual continuously compounded return.

(Choice B) £1446.38 results from incorrectly treating 5.9% as an annually compounded return and compounding for 4 years.

Things to remember:

An infinite number of infinitesimally small compounding periods during a holding period is known as continuous compounding. With continuous compounding, an ending value is equal to its beginning value multiplied by e^{rT}, where r is the annual continuously compounded return and T is the number of years.

Question 12 of 25

An investor purchases stock for $20 and sells it for $14. On the date of sale, the stock paid a $1 dividend. The investor's continuously compounded rate of return for the investment is *closest* to:

A. −25.0%

B. −28.8% ✓

C. −35.7%

Explanation:

Continuously compounded rate of return

$$r_{0,t} = \ln(1 + \text{Holding period return})$$

A **continuously compounded rate of return** measures an asset's **change in value** over a holding period. The assumption is that the changes are compounded continuously over the period. This differs from discrete compounding, which assumes that compounding occurs at evenly spaced intervals, such as weekly or annually. The change in value has two elements:

- Change in price over the period (ie, gain, loss); and/or
- Income received from the asset over the period

The combination of these two elements is known as the total return. The formula for calculating total return is given as:

$$\text{Total return} = \frac{(S_T - S_0 + I)}{S_0}$$

S_T = end price, S_0= beginning price, I = income (eg, dividends, interest)

The total return for an investment is also its **holding period return** (HPR).

In this question, the ending price is less than the beginning price, so the investor incurs a loss; even after adding in the dividend income, the HPR is negative:

$$\text{Total return} = \frac{14 - 20 + 1}{20} = -\frac{5}{20} = -25\%$$

The investor's continuously compounded rate of return is the natural logarithm (ln) of (1 + HPR), or ln(0.75), which is approximately −0.2877.

(Choice A) −25.0% is the holding period loss for the investment.

(Choice C) −35.7% is the approximate continuously compounded return when neglecting to account for the dividend. Although the investment generated an overall loss, the dividend reduced the loss.

Things to remember:
The holding period return (HPR) for an investment is its total return. Total return is the

sum of price appreciation plus income received on the investment. The continuously compounded return is the natural logarithm of (1 + HPR).

Question 13 of 25

A manager buys 500 shares of a stock at $15.00 per share at the beginning of the year. At the end of the year, the stock's price appreciates to $18.50. If this stock pays its annual dividend of $0.50 at the end of the year, then the manager's holding period return is *closest* to:

A. 21.6%

B. 23.3%

C. 26.7% ✓

Explanation:

Holding period return (HPR)

Price at the end of the period

$$HPR = \frac{P_{End} - P_{Beg} + I}{P_{Beg}}$$

Income

Price at the beginning of the period

A **holding period return** (HPR) is a **single-period** measurement of return. The period can represent a day, a year, or any other timeframe. The difference between the security's ending price and its beginning price is its capital gain or loss during the holding period. Also affecting the HPR are company distributions to bondholders (interest) or shareholders (dividends). These distributions are the income part of a security's return and increase HPR.

$$HPR = \frac{18.50 - 15.00 + 0.50}{15.00} = 26.7\%$$

(Choice A) This answer incorrectly applies the HPR using the ending price, rather than the beginning price, to find the HPR.

(Choice B) This incorrect choice does not include the stock's dividend and therefore results in an understatement of HPR.

Things to remember:
A holding period return is a performance measurement that compares the beginning and ending values of an investment and includes any income earned.

Question 14 of 25

In portfolio management, it is assumed that the expected return is:

A. never equal to the historical mean return.

B. equal to the historical mean return over the next 30 years.

C. equal to the historical mean return over a future period of unknown length. ✓

Explanation:

The historical mean return is the actual return that an investor earned in the past. Since assets are risky, it is unlikely that this exact return will be earned again over a specific period in the future.

However, in portfolio management it is assumed that given a **long enough**, although unspecified, period, it is reasonable to **expect** that a future **return** will be **equal** to the **historical mean return**. Although this assumption may not prove accurate, in practice the historical mean return is often used as a reasonable estimate of an asset's expected return **(Choice A)**.

(Choice B) It is incorrect to say that historical mean return is equal to expected return over any *specific* period, be it 30, 50, or 100 years, since the period may not be long enough.

Things to remember:
Given a long enough, although unspecified, period, it is reasonable to expect that a future return will be equal to the historical mean return. Although this assumption may not prove accurate, in practice the historical mean return is often used as a reasonable estimate of an asset's expected return.

Question 15 of 25

An analyst compares portfolios in which the amount invested changes each year. Which of the following is the *most appropriate* measure for the analyst to use in comparing the portfolios' returns over a multiyear period?

A. Holding period return

B. Arithmetic mean return

C. Geometric mean return ✓

Explanation:

Geometric mean return

$$G = \sqrt[n]{(1 + x_1)(1 + x_2)\ldots(1 + x_n)} - 1$$

n = number of periods

A portfolio's value can change during a period due to contributions, withdrawals, and/or income earned. Changes in portfolio value should be included in the beginning value in each subsequent period so that those changes are compounded.

The **geometric mean return** (*G* in the image above) provides a **better** representation of a portfolio's return since it **allows** for the **compounding** of returns and the **varying amounts invested**. The geometric mean return produces an average **periodic return** from actual periodic returns (eg, three-year average annual return from annual returns over three years). Analysts prefer the geometric mean return since it allows for better performance **comparisons** between investments with different amounts invested each year.

(Choice A) A holding period return is an asset's return earned over a specific period of time such as a day, week, or year. However, this return will be inaccurate if the amounts invested change during the period. It is best used as a periodic return when the amount invested remains the same.

(Choice B) An arithmetic mean return (ie, simple average return) assumes an equal base amount invested at the beginning of each period.

Things to remember:
The geometric mean return converts performance data from multiple periods into a comparable format and produces a constant periodic return. It provides a better representation of an portfolio's return since it allows for the compounding of returns and the varying base amounts invested.

Question 16 of 25

The following table shows an investment's closing price on each day:

Date	Closing Price
March 1	50
April 1	74
May 1	65

Based only on this information, the continuously compounded return of the investment from March 1 to May 1 is *closest* to:

A. 23.1%

B. 26.2% ✓

C. 30.0%

Explanation:

Continuously compounded return ($r_{t,t+1}$)

$$r_{t,t+1} = \ln(1 + R_{t,t+1}) = \ln\left(\frac{S_{t+1}}{S_t}\right)$$

$R_{t,t+1}$ = holding period return
t = beginning time of investment
ln = natural logarithm
$r_{t,t+1}$ = continuously compounded return
$t + 1$ = ending time of investment
S = investment price

A **continuously compounded rate of return** is one way to measure an asset's **change in value** over a holding period, assuming that changes in value are compounded continuously. This differs from discrete compounding, which assumes that compounding occurs at evenly spaced intervals, such as weekly or annually. The asset's change in value has two elements:

- Change in price over the period (ie, gain, loss)
- Income received from the asset over the period

These two elements are used to calculate the asset's total return. The formula for calculating total return is:

$$\text{Total return} = \frac{(S_T - S_0 + I)}{S_0}$$

S_T = end price, S_0= beginning price, I = income (eg, dividends, interest)

The total return is used to calculate the **holding period return** (HPR), which is (Total return − 1).

In this scenario, the investment does not pay interest or a dividend, so its HPR from March 1 to May 1 is (65 / 50) − 1 = 0.3, or 30%. The continuously compounded return over this period is the natural logarithm of (1 + HPR) = the natural logarithm of (Total return), or ln(1 + 0.3) ≈ 26.2%.

(Choice A) 23.1% is the approximate result incorrectly calculating the HPR as the *average* of the three prices divided by the beginning price, and then calculating the natural logarithm of (1 + result), as shown above.

(Choice C) 30.0% is the HPR, not the continuously compounded return, for the asset over the time period.

Things to remember:
An asset's continuously compounded return is the natural logarithm of [1 + Holding period return (HPR)]. The HPR is the asset's (Total return − 1). When an asset does not pay income (eg, interest, dividends) over the period, total return is the ending price divided by the beginning price.

Question 17 of 25

An investment fund holds investments X, Y, and Z. An analyst reviews the following information:

	20X8		20X9	
	Allocation	Annual Return	Allocation	Annual Return
Investment X	0.40	0.30	0.30	0.25
Investment Y	0.05	0.10	0.10	0.10
Investment Z	0.55	0.20	0.60	−0.10

The geometric mean return for the fund over two years is *closest* to:

A. 12.5% ✓

B. 13.0%

C. 14.0%

Explanation:

Portfolio expected return

Weight of asset

Return of asset

Portfolio return (R_p) = $w_a r_a + w_b r_b + \ldots + w_n r_n$

Asset

For multiple assets: 100% = $w_a + w_b + \ldots + w_n$

For two assets: 100% = $w_a + w_b$ or $w_a = 1 - w_b$

To find the two-year geometric mean return, first determine the portfolio returns for 20X8 and 20X9. The **portfolio return** is the **weighted average return** of the holdings of the portfolio. It is based on each holding's weight and return. Using the portfolio return formula, find the fund's annual return for 20X8 and 20X9.

Annual portfolio return

Return 20X8 = (0.40)(0.30) + (0.05)(0.10) + (0.55)(0.20) = 23.5%
Return 20X9 = (0.30)(0.25) + (0.10)(0.10) + (0.60)(−0.10) = 2.5%

Link these annual returns to determine the two-year geometric mean return using the geometric mean formula. Time-weighted returns nullify the influences created from cash flows and improve comparability of returns.

$$\text{Geometric mean return} = G + 1 = \sqrt{(1 + 0.235)(1 + 0.025)}$$

$$G = \sqrt{1.2659} - 1$$

$$G = 1.1251 - 1 = 12.51\%$$

The geometric mean return for the fund over two years is 12.51%.

(Choice B) This choice correctly finds the portfolio return for 20X8 and 20X9 but uses the *arithmetic mean* instead of the *geometric mean* to find the two-year mean return.

(Choice C) This option correctly uses the geometric mean formula but mistakenly applies a *simple arithmetic return* (arithmetic mean), not a *weighted average return*, for the components of the portfolio return.

Things to remember:
The geometric mean return, derived from a total or holding period return, creates a periodic performance measure. Unlike to other performance measures, it provides comparability of different portfolios.

Question 18 of 25

An analyst records the price of an asset over a two-year period:

Time (years)	Price (AUD)
0	99
1	105
2	113

Based only on price, the asset's Year 1 continuously compounded return is *closest* to:

A. 5.88% ✓

B. 6.06%

C. 6.41%

Explanation:

Continuously compounded return ($r_{t,t+1}$)

$$r_{t,t+1} = \ln(1 + R_{t,t+1}) = \ln\left(\frac{S_{t+1}}{S_t}\right)$$

$R_{t,t+1}$ = holding period return
$r_{t,t+1}$ = continuously compounded return
t = beginning time of investment
$t + 1$ = ending time of investment
ln = natural logarithm
S = investment price

An infinite number of infinitesimally small compounding periods during a holding period is known as continuous compounding. The **continuously compounded return** is related to the holding period return; it is the **natural log of** the **ratio** of the **ending value** to the **beginning value** of the investment.

In this scenario, solve to determine the Year 1 continuously compounded return using the price at the beginning (S_t) and price at the end (S_{t+1}) of the one-year holding period, which are both given.

$$r_{t,t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right) = \ln\left(\frac{S_1}{S_0}\right)$$

$$= \ln\left(\frac{105}{99}\right) \approx \ln(1.0606)$$

$$\approx 5.88\%$$

Note that the continuously compounded return in this scenario is lower than the holding period return (6.06%). This makes sense due to the fact that compounding at a *lower* rate *more frequently* results in the same return as compounding at a *higher* rate *less* frequently.

(Choice B) 6.06% is the Year 1 holding period return.

(Choice C) 6.41% is the geometric mean of the two-year, continuously compounded return.

Things to remember:
An infinite number of infinitesimally small compounding periods during a holding period is known as continuous compounding. The continuously compounded return is related to the holding period return; it is the natural log of the ratio of the ending value to the beginning value of the investment.

Question 19 of 25

One year ago, an individual taxpayer invested SGD 25 million in a portfolio of technology stocks. The investor has just sold this entire portfolio. The following data apply to this investor and transaction:

Percentage of investment financed	20.00%
Cost of loan (annual rate)	4.50%
Capital gains tax rate	35.00%
Holding period rate of inflation	2.90%
Investment return	17.34%

Ignoring any fees or commissions, the investor's holding period, leveraged, after-tax, real return on this investment is *closest* to:

A. 8.14%

B. 10.17% ✓

C. 11.15%

Explanation:

Holding period leveraged pre-tax return

$$R_{\text{LPTR}} = R_p + \frac{V_B}{V_E}(R_p - r_D)$$

R_{LPTR} = Leveraged pre-tax return
R_p = Investment return (before leverage)
V_B = Amount of borrowed funds
V_E = Amount of portfolio equity
r_D = Loan interest rate (borrowing cost)

Leveraged after-tax return

$$R_{\text{LATR}} = [R_{\text{LPTR}} \times (1 - r_{\text{TAX}})]$$

R_{LATR} = Leveraged after-tax return
R_{LPTR} = Leveraged pre-tax return
r_{TAX} = Tax rate

Leveraged after-tax real return

$$R_{\text{LATRR}} = \left(\frac{1 + R_{\text{LATR}}}{1 + r_{\text{INF}}}\right) - 1$$

R_{LATRR} = Leveraged after-tax real return
R_{LATR} = Leveraged after-tax return
r_{INF} = Inflation rate

To make effective comparisons, investors often calculate the real return, net of fees and taxes, accounting for the effects of leverage and inflation. Note that the calculations are done in order:

- First, account for the effects of leverage; if cost of borrowing < gross return, leverage increases return;
- Next, deduct fees and taxes;
- Finally, account for the effects of inflation; if inflation rate > 0, inflation decreases return.

In this scenario, the effects of leverage, taxes, and inflation must be accounted for. The investor borrowed 20% of the SGD 25 million investment at an interest rate of 4.50%. The holding period leveraged pre-tax return considers the impact of the loan, which is deducted from the gross holding period return. In addition, the investor's profits are taxed at the capital gains rate of 35.00%.

Steps	Calculations
Calculate holding period leveraged pre-tax return	$R_{\text{LPTR}} = 17.34\% + \dfrac{5{,}000{,}000}{20{,}000{,}000}(17.34\% - 4.50\%)$ $R_{\text{LPTR}} = 17.34\% + 3.21\% = 20.55\%$
Calculate leveraged after-tax return	$R_{\text{LATR}} = 20.55\% \times (1 - 0.35)$ $R_{\text{LATR}} \approx 13.36\%$
Calculate *real* (inflation-adjusted) leveraged after-tax return	$R_{\text{LATRR}} = \left(\dfrac{1 + 0.1336}{1 + 0.0290}\right) - 1$ $R_{\text{LATRR}} = 1.10165 - 1$ $R_{\text{LATRR}} = 0.10165 \approx 10.17\%$

Thus, the leveraged after-tax real (inflation-adjusted) return of this portfolio is approximately 10.17%.

(Choice A) A return of 8.14% incorrectly adjusts the leveraged return for inflation before applying the tax rate.

(Choice C) A return of 11.15% mistakenly fails to account for the additional return from the use of leverage (net of the cost of debt).

Things to remember:
To calculate a real return on investment, net of fees, taxes, and the impact of leverage and inflation, an investor must account (in order) for leverage, then fees and taxes, and then inflation. If the total investment return earned on a leveraged portfolio is higher than the cost of the funds borrowed, then leverage enhances the return. Taxes paid reduce the return. Additionally, inflation (unless negative) further reduces the portfolio's real return.

Question 20 of 25

A US-based taxpayer invests USD45,000 into a small-cap equity fund on January 1, 20X4. On December 31, 20X4, when the position is worth USD50,103, the investor liquidates the investment. Inflation for the year 20X4 is 3.10%.

If taxes are paid immediately upon sale at a 20% capital gains tax rate, the investor's after-tax real return for the year is *closest* to:

A. 5.79% ✓

B. 5.97%

C. 9.07%

Explanation:

After-tax real return

$$R_{ATR} = \left(\frac{1 + R_{AT}}{1 + r_{INF}}\right) - 1$$

R_{ATR} = After-tax real return
R_{AT} = After-tax nominal return
r_{INF} = Inflation rate

Investment returns are generally stated as pre-tax nominal returns—that is, no adjustment has been made for taxes or inflation, unless specifically stated. Investors, however, are often concerned about the **impact of taxes** and **inflation** since these factors further **reduce returns**. The **after-tax nominal return** is calculated as the return **net of taxes paid on** dividends, interest income, and **realized gains**.

The real return accounts for the effect of inflation. The **after-tax real return** is the **change** in the **purchasing power** of the **investor's wealth, net of taxes**. In this example, inflation of 3.10% further reduces the after-tax return:

Steps	Calculations (in USD)
Calculate investor's return	$\frac{50{,}103 - 45{,}000}{45{,}000} = 0.1134 = 11.34\%$
Calculate percentage tax effect	$\frac{(50{,}103 - 45{,}000) \times 0.20}{45{,}000} = 0.02268 \approx 2.27\%$
Calculate after-tax nominal return	$11.34\% - 2.27\% = 9.07\%$
Calculate after-tax real (inflation-adjusted) return	$R_{ATR} = \left(\frac{1 + 0.0907}{1 + 0.0310}\right) - 1 = 0.0579 = 5.79\%$

Thus, the after-tax real return is 5.79%.

Note: The after-tax nominal return can also be calculated as:

Investor's return − (Capital gains tax rate × Investor's return)

(Choice B) 5.97% results from a simple subtraction of the inflation rate. Since inflation and investment returns compound over time, the formula for calculating the after-tax *real* return must account for inflation's and investment's geometric returns.

(Choice C) 9.07% is the after-tax *nominal* return, which does not account for inflation.

Things to remember:
Investment returns are generally stated as pre-tax nominal returns with no adjustment for taxes or inflation. The after-tax real return accounts for the impact of taxes and inflation by calculating the change in the purchasing power of the investor's wealth, net of taxes.

Question 21 of 25

An analyst gathers data on two portfolios:

			Annual Returns	
Portfolio	% Debt	Borrowing Cost	Year 1	Year 2
1	None	N/A	20.0%	−20.0%
2	60%	5.0%	20.0%	−20.0%

Including the effects of leverage, which portfolio's 2-year holding period return is *most likely* greater?

A. Portfolio 1 ✓

B. Portfolio 2

C. Both portfolios' returns are equal

Explanation:

Holding period leveraged return

$$R_{LR} = R_p + \left(\frac{V_B}{V_E}\right)(R_p - r_D)$$

R_{LR} = Leveraged return

R_p = Investment return (before leverage)

V_B = Amount of borrowed funds

V_E = Amount of portfolio equity

r_D = Loan interest rate (borrowing cost)

Leverage arises when an investor's **equity position** in an asset is **less** than the **total investment**. This is the outcome of borrowing money to purchase the investment. Leverage amplifies returns, both positive and negative, versus an investment without leverage: leveraged positive returns are greater, and leveraged negative returns result in greater losses, than their respective unleveraged returns. The greater the proportion of debt, the greater this effect will be. The calculation of a leveraged return includes both the proportion of leverage used and its cost.

For this scenario, both portfolios' holding period returns (ie, horizon returns) may be calculated as follows:

Steps	Calculations
Calculate Portfolio 1's holding period return	$[(1.20)(0.80)]^{0.5} - 1 = -0.0202 = -2.02\%$
Calculate Portfolio 2's leveraged return for Year 1	$R_{LR} = 0.20 + \frac{0.60}{0.40}(0.20 - 0.05) = 0.425$
Calculate Portfolio 2's leveraged return for Year 2	$R_{LR} = -0.20 + \frac{0.60}{0.40}(-0.20 - 0.05) = -0.575$
Portfolio 2's 2-year holding period return:	$[(1.425)(0.425)]^{0.5} - 1 = -0.2218 = -22.2\%$

As shown, Portfolio 2's positive performance in Year 1 was boosted by the effect of leverage, while its negative performance in Year 2 was further exacerbated by the effect of leverage. The overall result was that Portfolio 1 outperformed Portfolio 2 during the 2-year holding period **(Choices B and C)**.

Things to remember:
Leverage amplifies both positive and negative returns, and the greater the proportion of leverage that is used, the greater this effect will be. A leveraged return accounts for the proportion of leverage as well as its cost.

Question 22 of 25

An analyst gathered the following information about a company:

Current stock price = $55

Expected growth rate = 6%

Cost of equity = 13%

The company's current dividend is *closest to*:

A. $3.85.

B. $3.63. ✓

C. $3.54.

Explanation:

(Choice A)

$P = D_1/k_e - g$

$D_1 = 55 \times (0.13 – 0.06) = \$3.85$

Current dividend = 3.85 / 1.06 = $3.63

Question 23 of 25

An analyst has gathered the following data to calculate the expected return of an asset:

Selected data (%)	
Economic growth rate	2.0
Risk-free rate	3.0
Expected risk premium	4.0
Expected inflation	1.8

The asset's expected return is *closest* to:

A. 7.99%

B. 8.80%

C. 9.05% ✓

Explanation:

$$\textbf{Expected return} = \left[\left(1 + \begin{matrix}\text{Real risk-free}\\ \text{rate}\end{matrix}\right) \times \left(1 + \begin{matrix}\text{Expected}\\ \text{inflation}\end{matrix}\right) \times \left(1 + \begin{matrix}\text{Expected}\\ \text{risk premium}\end{matrix}\right)\right] - 1$$

An investment's return is the investor's compensation for delaying consumption and bearing risk. Therefore, *expected* return is a function of:

- Real risk-free rate (ie, compensation for delaying consumption)
- Expected inflation rate (ie, compensation for the loss of purchasing power)
- Expected risk premium (ie, compensation for bearing the risks of an investment)

The expected return in this scenario is calculated as:

$$\text{Expected return} = [(1 + 0.03) \times (1 + 0.018) \times (1 + 0.04)] - 1$$
$$= 9.05\%$$

(Choice A) 7.99% is the result of incorrectly using the economic growth rate instead of the real rate of return to calculate expected inflation.

(Choice B) 8.80% is the result of adding the real risk-free rate, expected risk premium, and expected inflation. Although that is one way to approximate expected return, it does not result in the exact expected return.

Things to remember:
The *expected* return of an asset is the investor's compensation for postponing consumption and bearing risks. Expected return = [(1 + real risk-free rate) × (1 + expected inflation) × (1 + expected risk premium)] − 1.

Question 24 of 25

An analyst gathers annual returns for an equity index:

20X4	7.0%
20X5	8.2%
20X6	7.5%

The average annual inflation rate was 2%, and the average real risk-free rate was 1%. Based on this information, the real rate of return on the equity index in geometric terms is *closest* to:

A. 4.46%

B. 5.46% ✓

C. 7.57%

Explanation:

Nominal and real returns

Nominal return = (1 + Real return) × (1 + Inflation) − 1

$$\text{Real return} = \frac{1 + \text{Nominal return}}{1 + \text{Inflation}} - 1$$

An index's **nominal return includes** the effects of **inflation**. The **real** rate of **return** measures the increase in purchasing power after **adjusting for inflation**.

In order to determine the equity index's real return, first calculate its geometric return. The real return can then be derived by adjusting for inflation. In this question, the real return is approximately 5.46%:

Steps	Calculations
Calculate geometric nominal return	$\text{Return}_{geom} = [(1.07) \times (1.082) \times (1.075)]^{1/3} - 1 \approx 0.0757$
Calculate real return	$\text{Return}_{real} = \frac{1.0757}{1.02} - 1 \approx 0.0546 \approx 5.46\%$

(Choice A) 4.46% is the real equity risk premium, which results from subtracting the real risk-free rate from the equity's real return.

(Choice C) 7.57% is the simple average of the equity index returns.

Things to remember:
An index's nominal return includes the effects of inflation. The real rate of return measures the increase in purchasing power after adjusting for inflation.

Question 25 of 25

An analyst gathers the following information for an equity security:

- Required rate of return = 8.7%
- Real risk-free rate = 1.5%
- Expected inflation = 3.0%
- Value risk factor exposure = 1.0

Assuming no other risk premiums, the value risk premium is *closest* to:

A. 4.2% ✓

B. 5.7%

C. 7.2%

Explanation:

Required rate of return

$$r_i \approx r_{rf} + \text{infl} + \left(\sum_{j=1}^{n} f_{ij} \times rp_j\right)$$

n = Number of risk premiums

f_{ij} = Exposure of security i to the j^{th} risk factor

rp_j = Specific risk premium associated with the j^{th} risk factor

r_{rf} = Risk-free rate of return

infl = Inflation

The **required** rate of **return** comprises the **real risk-free rate**, expected **inflation**, and a set of **risk premiums** specific to the investment. For an equity investment, examples of common risk premiums include premiums for size and value.

With a value **risk factor** exposure of **1.0**, the value risk premium **contributes** its **full value** to the required **return**, and the formula simplifies to:

$$r_i \approx r_{rf} + \text{infl} + rp_{\text{value factor}}$$

Rearranging to solve for the value risk premium:

$$rp_{\text{value factor}} \approx r_i - r_{rf} - \text{infl} \approx 8.7\% - 1.5\% - 3.0\% \approx 4.2\%$$

A value risk premium of 4.2% represents the additional return the equity security must provide above the real risk-free rate and inflation to compensate investors for the security's exposure to the value risk factor.

(Choice B) 5.7% results from omitting the real risk-free rate from the calculation.

(Choice C) 7.2% results from omitting expected inflation from the calculation.

Things to remember:
The required rate of return is the sum of the real risk-free rate, the inflation rate, and all applicable risk premiums. With a value risk factor exposure of 1.0, the value risk premium contributes its full value to the required return.